A Lepton deep inelastic scattering asymmetries from $\gamma - Z^0$ interef-erence

In this section we present parton-model expressions for asymmetries measurable in lepton deep inelastic scattering that arise from the $\gamma - Z^0$ interference, including both PVDIS and charge asymmetries for both the proton and the deuteron. Details of the derivation will be given in Appendix F so as not to bore the readers.

A.1 PVDIS Asymmetries In the Parton Model

The PVDIS asymmetry for a proton target, counting u, d, c, s quark flavors and using $C_{1c,2c} = C_{1u,2u}$, $C_{1s,2s} = C_{1d,2d}$, is:

$$A_{RL,p}^{e^-,\mathrm{PVDIS}} \equiv \frac{\sigma_R^- - \sigma_L^-}{\sigma_R^- + \sigma_L^-} \tag{31}$$
$$= |\lambda| \frac{\sqrt{2} G_F Q^2}{4\pi\alpha} \frac{\frac{2}{3}[(u^+ + c^+)C_{1u} - \frac{1}{3}(d^+ + s^+)C_{1d}] + Y[\frac{2}{3}(u_V + c_V)C_{2u} - \frac{1}{3}(d_V + s_V)C_{2d}]}{\frac{4}{9}[u^+ + c^+] + \frac{1}{9}[d^+ + s^+]}$$

where the $-$ sign in σ^- represents electron scattering, $|\lambda|$ is the magnitude of the incident beam's polarization and the parton distributions are $q^+ \equiv q(x) + \bar{q}(x)$ and $q_V \equiv q(x) - \bar{q}(x)$ ($q = u, d, c, s$). The kinematic function Y is defined as

$$Y(y) \equiv \frac{1 - (1-y)^2}{1 + (1-y)^2} \,. \tag{32}$$

For the deuteron or any isoscalar target and ignoring nuclear effects,

$$A_{RL,d}^{e^-,\mathrm{PVDIS}}$$
$$= |\lambda| \frac{3 G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{2(1+R_C)C_{1u} - (1+R_S)C_{1d} + Y[2C_{2u}(1+\epsilon_c) - C_{2d}(1+\epsilon_s)]R_V}{5 + 4R_C + R_S} \tag{33}$$

where

$$R_V(x) \equiv \frac{u_V + d_V}{u^+ + d^+} \,, \;\; R_C(x) \equiv \frac{2(c+\bar{c})}{u^+ + d^+} \,, \;\; R_S(x) \equiv \frac{2(s+\bar{s})}{u^+ + d^+} \,, \tag{34}$$

and the ϵ's account for $c - \bar{c}$ and $s - \bar{s}$ which are often set to zero in PDF sets:

$$\epsilon_c \equiv \frac{2(c-\bar{c})}{u^+ + d^+} \,, \;\; \epsilon_s \equiv \frac{2(s-\bar{s})}{u^+ + d^+} \,. \tag{35}$$

If counting only the light quarks u and d then

$$A_{RL,d}^{e^-,\mathrm{PVDIS}} \approx |\lambda| \frac{3 G_F Q^2}{10\sqrt{2}\pi\alpha} \left[(2C_{1u} - C_{1d}) + R_V Y (2C_{2u} - C_{2d})\right] \,. \tag{36}$$

A.2 Lepton vs. Anti-Lepton Asymmetries In the Parton Model

The asymmetry between right-handed e^+ and left-handed e^- DIS off a proton target, assuming $c = \bar{c}$ and $s = \bar{s}$ [4], is

$$
\begin{aligned}
A_{RL,p}^{e^+e^-} &\equiv \frac{\sigma_R^+ - \sigma_L^-}{\sigma_R^+ + \sigma_L^-} \\
&= \frac{3G_F}{2\sqrt{2}\pi\alpha} Q^2 Y(y) \frac{2|\lambda| u_V C_{2u} - |\lambda| d_V C_{2d} - 2u_V C_{3u} + d_V C_{3d}}{4(u^+ + c^+) + 1(d^+ + s^+)} \,. \qquad (37)
\end{aligned}
$$

For the deuteron or any isoscalar target and ignoring nuclear effects:

$$
A_{RL,d}^{e^+e^-} = \frac{3G_F}{2\sqrt{2}\pi\alpha} Q^2 Y(y) R_V \frac{|\lambda|(2C_{2u} - C_{2d}) - (2C_{3u} - C_{3d})}{5 + 4R_C + R_S} \,. \qquad (38)
$$

And if only u, d are considered then

$$
A_{RL,d}^{e^+e^-} \approx \frac{3G_F}{10\sqrt{2}\pi\alpha} Q^2 Y(y) R_V \left[|\lambda|(2C_{2u} - C_{2d}) - (2C_{3u} - C_{3d})\right] \,. \qquad (39)
$$

All Eqs. (37) through (39) can be extended to LR, RR, and RL cases: for $A_{LR}^{e^+e^-}$ we let $|\lambda| \to -|\lambda|$; for $A_{RR}^{e^+e^-}$ asymmetry we substitute $VA \to AV$ or $Y(y) q_V C_{2q} \to q^+ C_{1q}$ for the proton and $Y(y) R_V C_{2q} \to C_{1q}$ for the deuteron; and for $A_{LL}^{e^+e^-}$ asymmetry we substitute $|\lambda| \to -|\lambda|$ from $A_{RR}^{e^+e^-}$.

Finally, for $A^{e^+e^-}$ (unpolarized beams) we let $|\lambda| = 0$ in Eqs. (37) through (39):

$$
A_p^{e^+e^-} = \frac{3G_F}{2\sqrt{2}\pi\alpha} Q^2 Y(y) \frac{-2(u_V) C_{3u} + (d_V) C_{3d}}{4(u^+ + c^+) + 1(d^+ + s^+)} \,. \qquad (40)
$$

For the deuteron or any isoscalar target and ignoring nuclear effects:

$$
\begin{aligned}
A_d^{e^+e^-} &= \frac{3G_F}{2\sqrt{2}\pi\alpha} Q^2 Y(y) R_V \frac{-(2C_{3u} - C_{3d})}{5 + 4R_C + R_S} \qquad (41) \\
&\approx -\frac{3G_F}{10\sqrt{2}\pi\alpha} Q^2 Y(y) R_V (2C_{3u} - C_{3d}) \ \ (u, d \text{ only}) \qquad (42)
\end{aligned}
$$

The asymmetry measured at CERN is $B_+ \equiv \frac{\sigma^+(-|\lambda|) - \sigma^-(+|\lambda|)}{\sigma^+(-|\lambda|) + \sigma^-(+|\lambda|)}$ on a ^{12}C target and thus is our $A_{LR,d}^{e^+e^-}$. Substituting $|\lambda| \to -|\lambda|$ from Eq. (38), we obtain:

$$
B = -\frac{3G_F}{2\sqrt{2}\pi\alpha} Q^2 Y(y) R_V(x) \frac{(2C_{3u} - C_{3d}) + |\lambda|(2C_{2u} - C_{2d})}{5 + 4R_C + R_S} \,. \qquad (43)
$$

For SoLID we can use unpolarized beam (higher intensity) and measure

$$
A_d^{e^+e^-} = -\frac{3G_F}{2\sqrt{2}\pi\alpha} Q^2 Y(y) R_V \frac{(2C_{3u} - C_{3d})}{5 + 4R_C + R_S} \,. \qquad (44)
$$

[4] if considering $c \neq \bar{c}$ and $s \neq \bar{s}$, change $u_V \to u_V + c_V$ and $d_V \to d_V + s_V$ in all proton results, and multiply $C_{2u(d)}$ and $C_{3u(d)}$ by $(1 + \epsilon_c)$ $((1 + \epsilon_s))$ – see Eq. (94) – in all deuteron results throughout this section. No change to the C_{1q} term if calculating $A_{LL,RR}^{e^+e^-}$.

A.3 Target mass and longitudinal photon terms

After establishing the parton-model expression for asymmetries, we now consider adding the target-mass correction and longitudinal photon terms. The PVDIS asymmetry on the deuteron can be written as [39]

$$A_{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[a_1(x,Q^2)Y_1(x,y,Q^2) + a_3(x,Q^2)Y_3(x,y,Q^2)\right] , \tag{45}$$

The kinematic factors $Y_{1,3}$ are defined as

$$Y_1 = \left[\frac{1+R^{\gamma Z}}{1+R^{\gamma}}\right]\frac{1+(1-y)^2 - y^2\left[1-\frac{r^2}{1+R^{\gamma Z}}\right] - xy\frac{M}{E}}{1+(1-y)^2 - y^2\left[1-\frac{r^2}{1+R^{\gamma}}\right] - xy\frac{M}{E}} \tag{46}$$

and

$$Y_3 = \left[\frac{r^2}{1+R^{\gamma}}\right]\frac{1-(1-y)^2}{1+(1-y)^2 - y^2\left[1-\frac{r^2}{1+R^{\gamma}}\right] - xy\frac{M}{E}} , \tag{47}$$

where $r^2 = 1 + \frac{Q^2}{\nu^2}$, and $R^{\gamma(\gamma Z)}(x,Q^2)$ is the ratio of the longitudinal to transverse virtual photon electromagnetic absorption cross sections ($\gamma - Z^0$ interference cross sections). With some algebra, one can express the xyM/E term by r^2 and y^2 and Eqs.(46,47) become [40]:

$$Y_1 = \left[\frac{1+R^{\gamma Z}}{1+R^{\gamma}}\right]\frac{1+(1-y)^2 - \frac{y^2}{2}\left[1+r^2-\frac{2r^2}{1+R^{\gamma Z}}\right]}{1+(1-y)^2 - \frac{y^2}{2}\left[1+r^2-\frac{2r^2}{1+R^{\gamma}}\right]} \tag{48}$$

$$Y_3 = \left[\frac{r^2}{1+R^{\gamma}}\right]\frac{1-(1-y)^2}{1+(1-y)^2 - \frac{y^2}{2}\left[1+r^2-\frac{2r^2}{1+R^{\gamma}}\right]} . \tag{49}$$

To a good approximation $R^{\gamma Z}$ can be assumed to be equal to R^{γ}, resulting in $Y_1 = 1$.

The $a_{1,3}$ terms in Eq. (45) are

$$a_1(x) = 2g_A^e\frac{F_1^{\gamma Z}}{F_1^{\gamma}} , \tag{50}$$

$$a_3(x) = g_V^e\frac{F_3^{\gamma Z}}{F_1^{\gamma}} , \tag{51}$$

where the structure functions, $F_{1,3}^{\gamma,\gamma Z}$, can be written in terms of PDFs at the parton model level:

$$F_1^{\gamma}(x,Q^2) = \frac{1}{2}\sum Q_q^2\left[q(x,Q^2)+\bar{q}(x,Q^2)\right] , \tag{52}$$

$$F_1^{\gamma Z}(x,Q^2) = \sum Q_q g_V^q\left[q(x,Q^2)+\bar{q}(x,Q^2)\right] , \tag{53}$$

$$F_3^{\gamma Z}(x,Q^2) = 2\sum Q_q g_A^q\left[q(x,Q^2)-\bar{q}(x,Q^2)\right] . \tag{54}$$

Here, Q_q denotes the quark's electric charge and the summation is over the quark flavors $u, d, s\cdots$. Equations (51,54) show that the $a_3(x, Q^2)$ term involves the chirality of the quark (g_A^i) and therefore is suppressed by the kinematic factor Y_3 due to angular momentum conservation. It vanishes at the forward angle $\theta = 0$ or $y = 0$, and increases with θ or y at fixed x. In most world parameterizations, it is common to fit the structure functions F_2 and R simultaneously to cross-section data. They are related through

$$F_2^{\gamma(\gamma Z)} = \frac{2xF_1^{\gamma(\gamma Z)}(1+R^{\gamma(\gamma Z)})}{r^2} . \quad (55)$$

We now extend the above formalism to the e^+e^- asymmetries. Note: this is not a proof, but only a quick "identify and substitute method". A thorough proof is needed for the validity of Eqs. (56,57,58), and that the Y_3 from Eq. (49) remains the same for these asymmetries. We note that the γ structure function F_1^γ is multiplied by the common denominator of Eqs. (48) and (49); the γZ structure function $F_1^{\gamma Z}$, related to $g_V^q(q+\bar{q})$, is multiplied by the numerator of Eq. (48); the γZ structure function $F_3^{\gamma Z}$, related to $g_A^q(q-\bar{q})$, is multiplied by the numerator of Eq. (49). Applying these factors to Eq. (38), we see that $Y(y) \to Y_3$:

$$A_{RL,d}^{e^+e^-} = \frac{3G_F}{2\sqrt{2}\pi\alpha}Q^2 Y_3 R_V \frac{|\lambda|(2C_{2u}-C_{2d})-(2C_{3u}-C_{3d})}{5+4R_C+R_S} , \quad (56)$$

and to Eq. (41):

$$A_d^{e^+e^-} = -\frac{3G_F}{2\sqrt{2}\pi\alpha}Q^2 Y_3 R_V \frac{-(2C_{3u}-C_{3d})}{5+4R_C+R_S} . \quad (57)$$

While for $A_{RR,LL}^{e+e-}$, the C_{1q} terms needs to be multiplied by Y_1:

$$A_{RR,d}^{e^+e^-} = \frac{3G_F}{2\sqrt{2}\pi\alpha}Q^2 \frac{|\lambda|Y_1(2C_{1u}-C_{1d})-Y_3R_V(2C_{3u}-C_{3d})}{5+4R_C+R_S} , \quad (58)$$

References

[1] P. A. Zyla et al. (Particle Data Group), Review of Particle Physics, Prog. Theor. Exp. Phys. **2020**, 083C01 (2020). doi:10.1093/ptep/ptaa104

[2] J. Erler and S. Su, "The Weak Neutral Current," Prog. Part. Nucl. Phys. **71**, 119 (2013). doi:10.1016/j.ppnp.2013.03.004 [arXiv:1303.5522 [hep-ph]].

[3] D. Androić *et al.* [Qweak], "Precision measurement of the weak charge of the proton," Nature **557**, no.7704, 207-211 (2018) doi:10.1038/s41586-018-0096-0 [arXiv:1905.08283 [nucl-ex]].

[4] D. Wang et al. (PVDIS collaboration), "Measurement of parity violation in electron–quark scattering," Nature **506** no.7486, 67 (2014). doi:10.1038/nature12964

[5] D. Wang, K. Pan, R. Subedi, *et al.* (PVDIS collaboration) "Measurement of Parity-Violating Asymmetry in Electron-Deuteron Inelastic Scattering," Phys. Rev. C **91**, no.4, 045506 (2015) doi:10.1103/PhysRevC.91.045506 [arXiv:1411.3200 [nucl-ex]].

[6] D. Becker et al. "The P2 experiment," doi:10.1140/epja/i2018-12611-6 [arXiv:1802.04759 [nucl-ex]].

[7] J. Erler, "Theoretical Implications of Precision Measurements," doi:10.1142/9789811207402_0018 [arXiv:1710.06503 [hep-ph]].

[8] J. P. Chen, et al. (SoLID collaboration), "A White Paper on SoLID (Solenoidal Large Intensity Device)," [arXiv:1409.7741 [nucl-ex]].

[9] A. Argento, et al. "Electroweak Asymmetry in Deep Inelastic Muon - Nucleon Scattering," Phys. Lett. B **120**, 245 (1983). doi:10.1016/0370-2693(83)90665-2

[10] X. Zheng, J. Erler, Q. Liu and H. Spiesberger, [arXiv:2103.12555 [nucl-ex]].

[11] J. Erler and M. J. Ramsey-Musolf, "Low energy tests of the weak interaction," Prog. Part. Nucl. Phys. **54**, 351-442 (2005). doi:10.1016/j.ppnp.2004.08.001

[12] A. Accardi, A. Afanasev, I. Albayrak, S. F. Ali, M. Amaryan, J. R. M. Annand, J. Arrington, A. Asaturyan, H. Avakian and T. Averett, *et al.* "e^+@JLab White Paper: An Experimental Program with Positron Beams at Jefferson Lab," [arXiv:2007.15081 [nucl-ex]].

[13] H. Spiesberger, RADIATIVE CORRECTIONS TO DEEP INELASTIC e P SCATTERING AT HERA, in XIII International School of Theoretical Physics: The Standard Model and Beyond, DESY-89-175, 145–171 (1989).

[14] A. Kwiatkowski, H. Spiesberger and H. J. Mohring, Comput. Phys. Commun. **69**, 155-172 (1992) doi:10.1016/0010-4655(92)90136-M

[15] K. Charchula, G. A. Schuler and H. Spiesberger, Comput. Phys. Commun. **81**, 381-402 (1994) doi:10.1016/0010-4655(94)90086-8

[16] S. M. Berman and J. R. Primack, "WEAK NEUTRAL CURRENTS IN ELECTRON AND MUON SCATTERING," Phys. Rev. D **9**, 2171 (1974); [erratum: Phys. Rev. D **10**, 3895 (1974). doi:10.1103/PhysRevD.9.2171

[17] C. Y. Prescott et al. "Parity Nonconservation in Inelastic Electron Scattering," Phys. Lett. B **77**, 347 (1978). doi:10.1016/0370-2693(78)90722-0

[18] C. Y. Prescott et al. "Further Measurements of Parity Nonconservation in Inelastic electron Scattering," Phys. Lett. B **84**, 524 (1979). doi:10.1016/0370-2693(79)91253-X

[19] SoLID pre-Conceptual Design Report, (2019) url:https://hallaweb.jlab.org/12GeV/SoLID/files/solid-precdr-Nov2019.pdf

[20] L. A. Harland-Lang, A. D. Martin, P. Motylinski and R. S. Thorne, "Parton distributions in the LHC era: MMHT 2014 PDFs," Eur. Phys. J. C **75** (2015) no.5, 204 doi:10.1140/epjc/s10052-015-3397-6 [arXiv:1412.3989 [hep-ph]].

[21] T. J. Hou, J. Gao, T. J. Hobbs, K. Xie, S. Dulat, M. Guzzi, J. Huston, P. Nadolsky, J. Pumplin and C. Schmidt, *et al.* "New CTEQ global analysis of quantum chromodynamics with high-precision data from the LHC," Phys. Rev. D **103** (2021) no.1, 014013 doi:10.1103/PhysRevD.103.014013 [arXiv:1912.10053 [hep-ph]].

[22] J. Arrington, P. G. Blunden and W. Melnitchouk, Prog. Part. Nucl. Phys. **66**, 782-833 (2011) doi:10.1016/j.ppnp.2011.07.003 [arXiv:1105.0951 [nucl-th]].

[23] Manuscript in preparation for EPJA, still word-smithing.

[24] A. A. Sokolov and I. M. Ternov, "On polarization and spin effects in the theory of synchrotron radiation," Sov. Phys. Dokl. **8**, no.12, 1203-1205 (1964)

[25] L. S. Cardman, "The PEPPo method for polarized positrons and PEPPo II," AIP Conf. Proc. **1970**, no.1, 050001 (2018) doi:10.1063/1.5040220

[26] S. Golge (2010), Ph.D. Thesis, Old Dominion University, Norfolk (VA, USA), JLab number JLAB-ACP-10-1301. FEASIBILITY AND CONCEPTUAL DESIGN OF A C.W. POSITRON SOURCE AT CEBAF

[27] J. Grames, E. Voutier et al. SOURCE(2018), Jefferson Lab LOI12-18-004, "Physics with Positron Beams at Jefferson Lab 12 GeV," [arXiv:1906.09419 [nucl-ex]].

[28] Y. Roblin (2017), Cont. to the JPos17 International Workshop on Physics with Positrons at Jefferson Lab, Newport News (USA, VA).

[29] C. Morean, "Target Boiling Studies Using 2018 F_2-XEM Data" (2020).

[30] OPERA, ELECTROMAGNETIC AND ELECTROMECHANICAL SIMULATION, https://www.3ds.com/products-services/simulia/products/opera/

[31] D. Day, P. Degtiarenko, S. Dobbs, R. Ent, D. J. Hamilton, T. Horn, D. Keller, C. Keppel, G. Niculescu and P. Reid, *et al.* "A Conceptual Design Study of a Compact Photon Source (CPS) for Jefferson Lab," Nucl. Instrum. Meth. A **957**, 163429 (2020) doi:10.1016/j.nima.2020.163429 [arXiv:1912.07355 [physics.ins-det]].

[32] P. Degtiarenko, J. Grames, J. Kowal, J. Musson, S. Philip, M. Tiefenback, K. Welch, Private Communication (2020).

[33] P. Valente, M. Belli, B. Bolli, B. Buonomo, S. Cantarella, R. Ceccarelli, A. Cecchinelli, O. Cerafogli, R. Clementi and C. Di Giulio, *et al.* "Linear Accelerator Test Facility at LNF: Conceptual Design Report," [arXiv:1603.05651 [physics.acc-ph]].

[34] https://www.bergoz.com/en/mx-bpm

[35] W. Barry, "A General analysis of thin wire pickups for high frequency beam position monitors," Nucl. Instrum. Meth. A **301**, 407-416 (1991) doi:10.1016/0168-9002(91)90004-A

[36] F. Halzen and A. D. Martin, *Quarks and Leptons*, 1st edn. (Wiley)

[37] Griffiths, Introduction to Elementary Particles.

[38] Peskin, Schroeder, Quantum Field Theory.

[39] R. N. Cahn and F. J. Gilman, Phys. Rev. D **17**, 1313 (1978).

[40] L. T. Brady, A. Accardi, T. J. Hobbs and W. Melnitchouk, Phys. Rev. D **84**, 074008 (2011) [Erratum-ibid. D **85**, 039902 (2012)].

[41] S. Alekhin, S. A. Kulagin and R. Petti, AIP Conf. Proc. **967**, 215 (2007).

[42] J. D. Bjorken, Phys. Rev. D **18**, 3239 (1978).

[43] L. Wolfenstein, Nucl. Phys. B **146**, 477 (1978).

[44] S. Mantry, M. J. Ramsey-Musolf and G. F. Sacco, Phys. Rev. C **82**, 065205 (2010).

[45] A. V. Belitsky, A. Manashov and A. Schafer, Phys. Rev. D **84**, 014010 (2011).

[46] C. -Y. Seng and M. J. Ramsey-Musolf, Phys. Rev. C **88**, no. 1, 015202 (2013).

[47] A. Accardi, L. T. Brady, W. Melnitchouk, J. F. Owens and N. Sato, Phys. Rev. D **93**, no.11, 114017 (2016) doi:10.1103/PhysRevD.93.114017 [arXiv:1602.03154 [hep-ph]].