Angular Momentum Sum Rule in the Deuteron

Simonetta Liuti
University of Virginia

Int. Workshop on the Frontier of QCD
Xiaotong University,Shanghai
Nov. 2th - 4th, 2012

Developing a Sum Rule for Angular Mometum

1990 A Sum Rule was constructed which identified components of the Energy Momentum Tensor (EMT) with the Angular Momentum carried by quarks and gluons. (Jaffe&Manohar (JM))

$$\langle p', s|T_{\mu\nu}(0)|p, s\rangle = \boxed{A_0(k^2)} P_\mu P_\nu + \boxed{iA_1(k^2)}(\epsilon_{\mu\alpha\beta\sigma}P_\nu + \epsilon_{\nu\alpha\beta\sigma}P_\mu)k^\alpha P^\beta s^\sigma + O(k^2), \quad (6.9)$$

$\rightarrow$ $A(0) = \frac{1}{2}$.

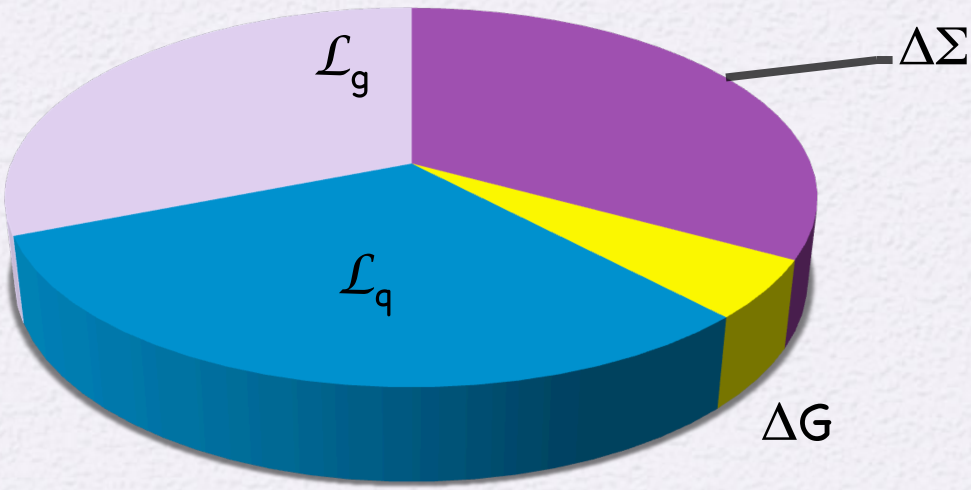
where $P^\mu = \frac{1}{2}(p+p')^\mu$. Both of these terms are conserved ($k^\mu P_\mu = 0$), symmetric, and have the correct parity. An example of a form factor which is ignored is $(k^2 g_{\mu\nu} - k_\mu k_\nu)\boxed{A_2(k^2)}$ It makes a vanishingly small contribution near $k^2 = 0$ since

Partonic picture:

- work directly in $A^+=0$ gauge
- quark and gluon spin components are identified with the n=1 moments of spin dependent structure functions from DIS, $\Delta\Sigma$ and ΔG.

$$M^{+12} = \underbrace{\frac{1}{2} q_+^\dagger \gamma^5 q_+}_{\Delta\Sigma} + \frac{1}{2} i q_+^\dagger (\vec{x}\times\partial)^3 q_+ + \underbrace{Tr(\varepsilon^{+-ij}F^{+j}A^j)}_{\Delta G} + 2iTrF^{+j}(\vec{x}\times\partial)A^j$$

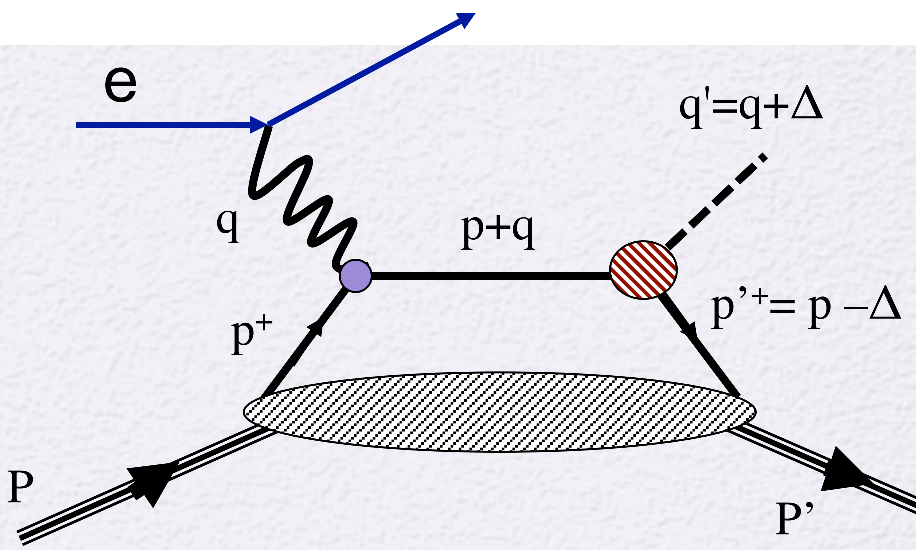
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$



1997 New Sum Rule (X.Ji)

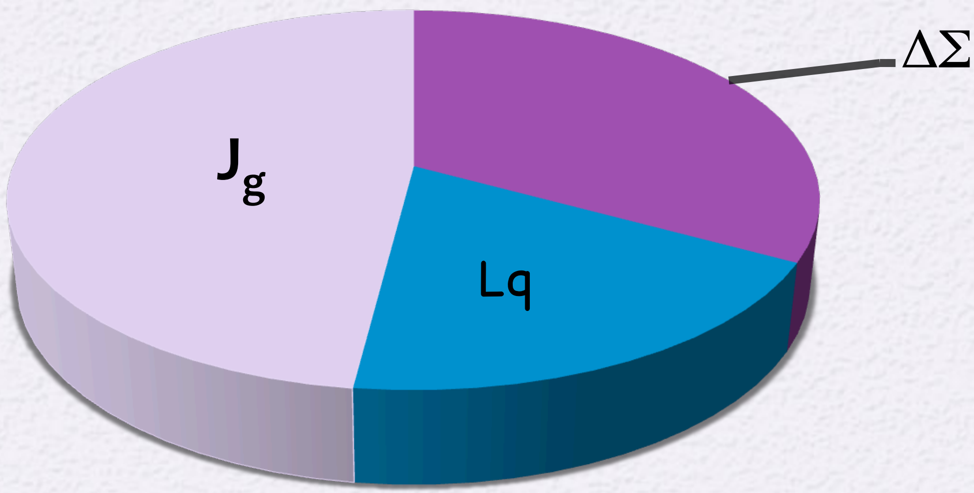
$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')\left[\boxed{A_{q,g}(\Delta^2)}\gamma^{(\mu}\overline{P}^{\nu)} + \boxed{B_{q,g}(\Delta^2)}\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + \boxed{C_{q,g}(\Delta^2)}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \boxed{\overline{C}_{q,g}(\Delta^2)}g^{\mu\nu}M\right]U(P),$$

New processes (DVCS ...) were thought of, whose structure functions – the GPDs - admit n=2 moments that were identified with the (spin+OAM) quark and gluon components of the SR (X.Ji)



$$\int_{-1}^{1} \mathrm{d}x\, x[H_q(x,\xi,t) + E_q(x,\xi,t)] = A_q(t) + B_q(t)$$

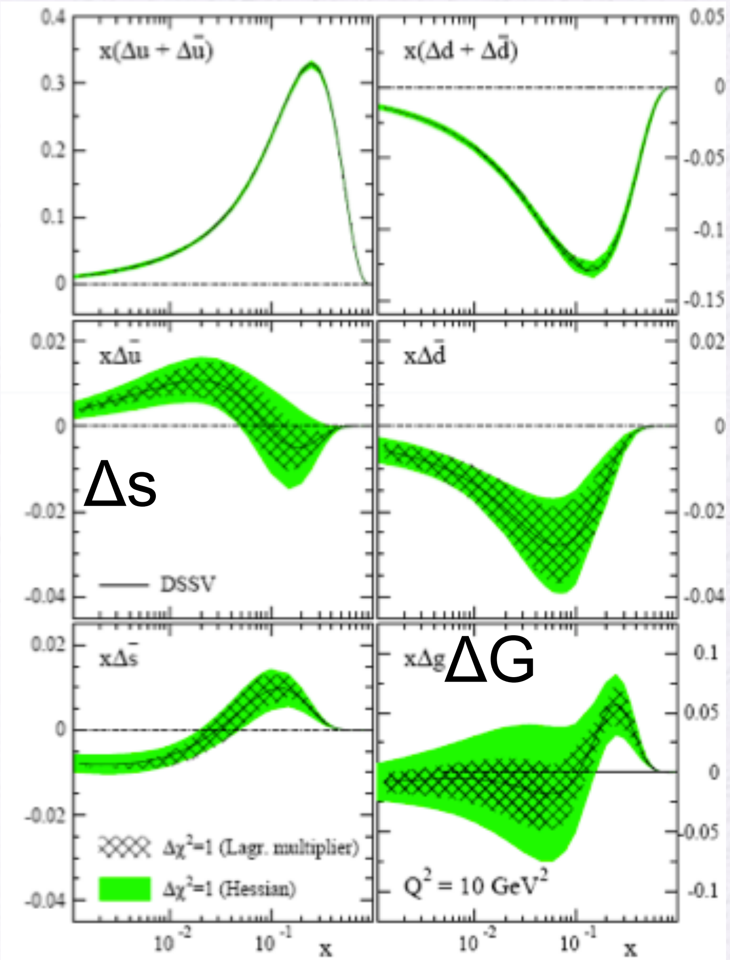
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$



Question of what components do L_q and $\mathcal{L}_q$ measure
(Ji, Xiong, Yuan, 2011, Burkardt 2012)

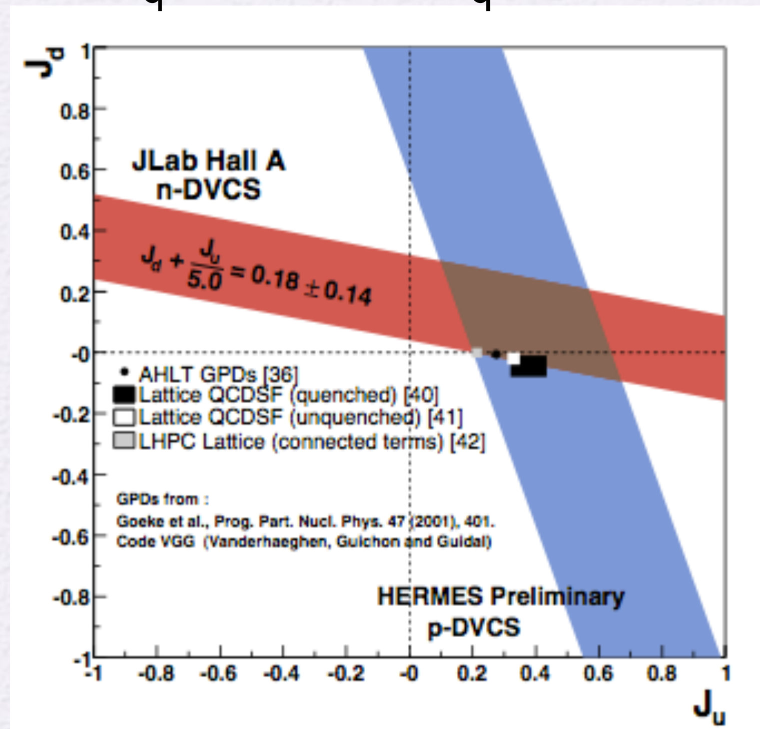
Summary of experimental situation

$\Delta\Sigma, \Delta G$



see S. Taneja's talk, GHP/APS Meeting, 2011

$J_q = 1/2\, \Delta\Sigma + L_q$



Ji $\quad \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$

Jaffe Manohar $\quad \frac{1}{2} = \frac{1}{2}\Delta\Sigma ✔ + \mathcal{L}_q + \Delta G ✔ + \mathcal{L}_g \qquad \mathcal{L}_q, \mathcal{L}_g$?

Need to include TMDs in the picture

Open Questions

- OAM associated with twist-3 GPDs ? (X,Ji et al.)
- Torque term is “missing gauge” piece (M.Burkardt)….?
- Searching for a common origin in the treatment of FSI which become again central (Brodsky, Hwang and Schmidt, Brodsky and Gardner)

- Other spin systems, Spin=1, and Spin=0, might help shed light/add probes, possibilities, new insights

- In particular spin 1 systems, due to the presence of additional L components (D-waves) provide a crucial test the working of the angular momentum sum rules

Interest in Spin 1 targets: deuterium, ^{6}Li, …

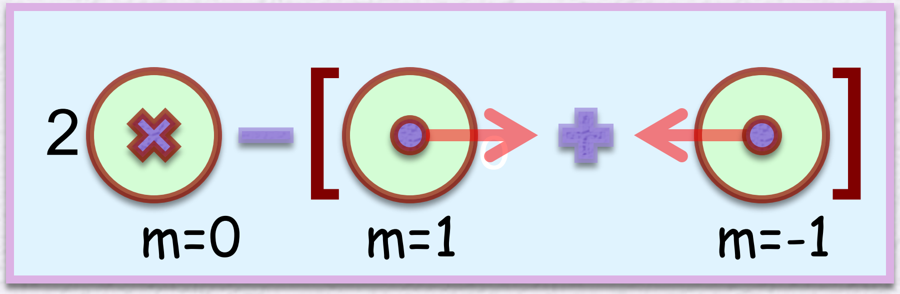
✓ In DIS: Unique possibility to study how the deep inelastic structure of nuclei differs from a system of free nucleons.

➔ One more distribution w.r.t. spin 1/2

Tensor Structure Function

Hoobhoy, Jaffe, Manohar (1989)

$$b_1(x) = \frac{1}{2}\left[2q^0_\uparrow - \left(q^1_\uparrow + q^1_\downarrow\right)\right]$$



$b_1(x) \to 0$ for free nucleons

$b_1(x) \neq 0$ in bound systems

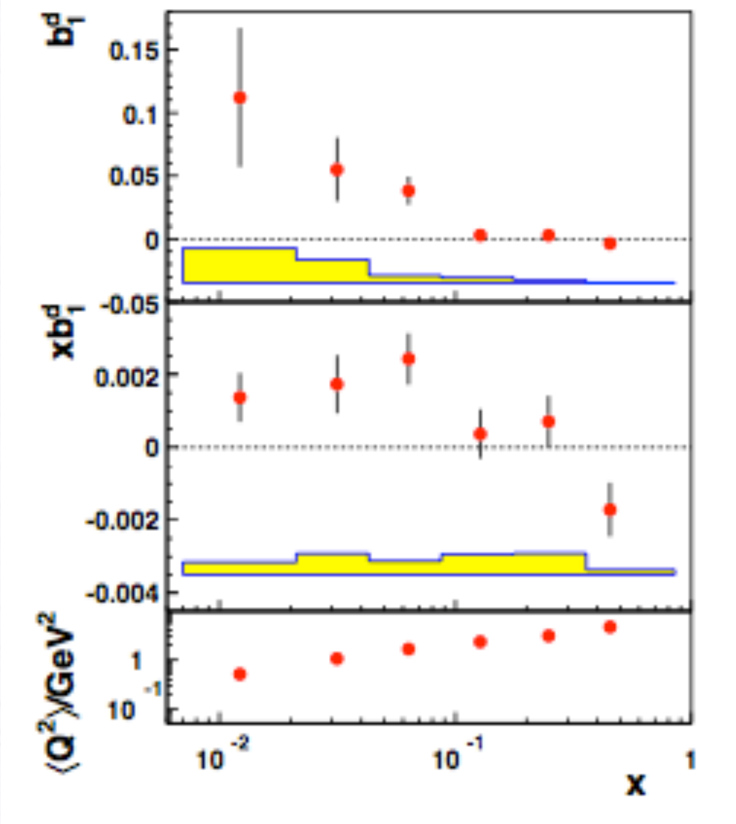
Role of D wave, or non conventional physics!

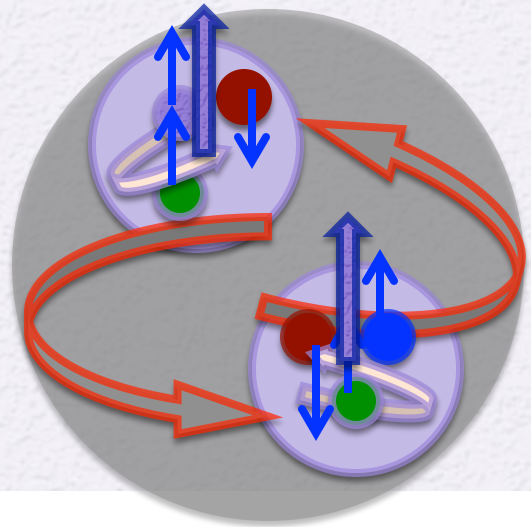
Sum Rule

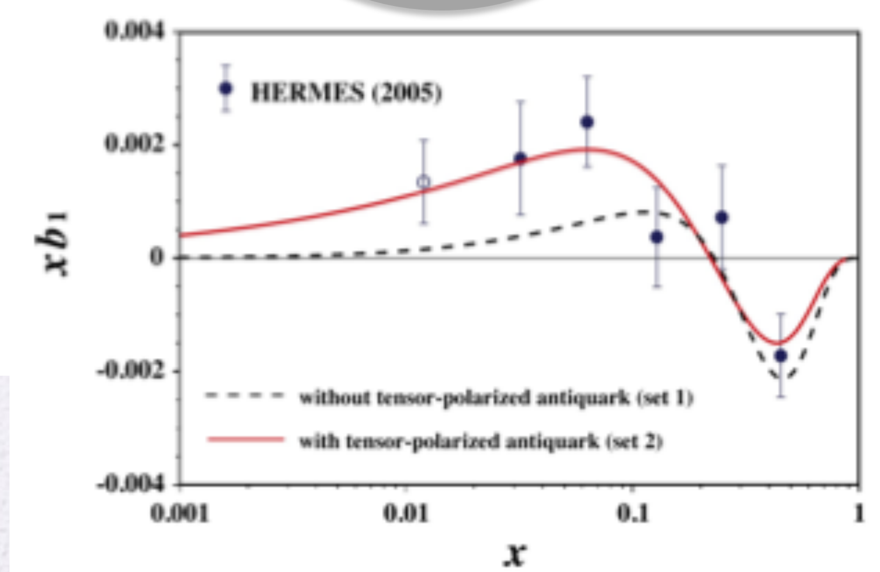
$$\int b_1(x)\,dx = \left(\delta q + \delta\bar{q}\right)_S$$

Close and Kumano (1990)

Hermes 2005







S.Kumano PRD82,2010

Spin One Sum Rule Derivation

OAM Sum Rule: Operators

$$M^{\mu\nu\lambda} = x^{\mu}T^{\nu\lambda} - x^{\lambda}T^{\mu\nu}$$

$$J^{i}_{q,g} = \frac{1}{2}\varepsilon^{ijk}\int d^3x \left(x^{j}T^{0k}_{q,g} - x^{k}T^{0j}_{q,g} \right)^{\mu}$$

$$T^{\mu\nu}_{q} = \frac{1}{2}\left[\bar{\psi}\gamma^{(\mu} i\vec{D}^{\nu)}\psi + \bar{\psi}\gamma^{(\mu} i\overleftarrow{D}^{\nu)}\psi \right]$$

$$T^{\mu\nu}_{g} = \frac{1}{4}g^{\mu\nu}F - F^{\mu\rho}F^{\nu}_{\rho}$$

General rule to count form factors: t-channel J^{PC} q. numbers

Match $\langle P\bar{P}|$ to RHS → $\langle P\bar{P}|\bar{\psi}(0)\Gamma iD^{\{\mu_1}iD^{\mu_2}\cdots iD^{\mu_n\}}\psi(0)|0\rangle$

Haegler, PLB(2004)
Z.Chen&Ji, PRD(2005)

n	$J^{PC}(S;L)$				
0	0^{+-}	$1^{--}(1;0,2)$			
1	$0^{++}(1;1)$	1^{-+}	$2^{++}(1;1,3)$		
2	0^{+-}	$1^{--}(1;0,2)$	2^{+-}	$3^{--}(1;2,4)$	
3	$0^{++}(1;1)$	1^{-+}	$2^{++}(1;1,3)$	3^{-+}	$4^{++}(1;3,5)$
...		...			

TABLE III: J^{PC} of the vector operators with $(S;L,L')$ for the corresponding $N\bar{N}$ state. Where there are no $(S;L,L')$ values there are no matching quantum numbers for the $N\bar{N}$ system.

Deuteron

	$L=0$	1	2	3	4	...
$S=0$	J^{PC} 0^{++}	1^{--}	2^{++}	3^{--}	4^{++}	
$S=1$	1^{+-}	0^{-+}	1^{+-}	2^{-+}	3^{+-}	
		1^{-+}	2^{+-}	3^{-+}	4^{+-}	
		2^{-+}	3^{+-}	4^{-+}	5^{+-}	
$S=2$	2^{++}	1^{--}	0^{++}	1^{--}	2^{++}	
		2^{--}	1^{++}	2^{--}	3^{++}	
		3^{--}	2^{++}	3^{--}	4^{++}	
			3^{++}	4^{--}	5^{++}	
			4^{++}	5^{--}	6^{++}	

TABLE II: J^{PC} of the $d\bar{d}$ states.

Nucleon

	$L=0$	1	2	3	4	...
$S=0$	J^{PC} 0^{-+}	1^{+-}	2^{-+}	3^{+-}	4^{-+}	
$S=1$	1^{--}	0^{++}	1^{--}	2^{++}	3^{--}	
		1^{++}	2^{--}	3^{++}	4^{--}	
		2^{++}	3^{--}	4^{++}	5^{--}	

TABLE I: J^{PC} of the $N\bar{N}$ states.

Both S and L states considered

EMT Matrix Element

$$
\begin{aligned}
\langle p'|T^{\mu\nu}|p\rangle &= -\frac{1}{2}P^\mu P^\nu(\epsilon'^*\epsilon)\boxed{\mathcal{G}_1(t)} \\
&- \frac{1}{4}P^\mu P^\nu \frac{(\epsilon P)(\epsilon'^* P)}{M^2}\boxed{\mathcal{G}_2(t)} - \frac{1}{2}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right](\epsilon'^*\epsilon) \\
&\times \boxed{\mathcal{G}_3(t)} - \frac{1}{4}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\boxed{\mathcal{G}_4(t)} \\
&+ \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) + \epsilon^\mu(\epsilon'^* P)\right)P^\nu + \mu \leftrightarrow \nu\right]\boxed{\mathcal{G}_5(t)} \\
&+ \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) - \epsilon^\mu(\epsilon'^* P)\right)\Delta^\nu + \mu \leftrightarrow \nu\right. \\
&+ \left. 2g_{\mu\nu}(\epsilon P)(\epsilon'^* P) - (\epsilon'^{*\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu)\Delta^2\right]\boxed{\mathcal{G}_6(t)} \\
&+ \frac{1}{2}\left[\epsilon^{*\prime\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu\right]\boxed{\mathcal{G}_7(t)} + g^{\mu\nu}(\epsilon'^*\epsilon)M^2\boxed{\mathcal{G}_8(t)}
\end{aligned}
\tag{5}
$$

7 (conserved f.f.'s $\rightarrow \mathcal{G}_1 - \mathcal{G}_7$) + 1 (non-conserved $\rightarrow \mathcal{G}_8$)

Compare to spin 1/2

$$\langle p', s|T_{\mu\nu}(0)|p, s\rangle = A_0(k^2)P_\mu P_\nu + iA_1(k^2)(\epsilon_{\mu\alpha\beta\sigma}P_\nu + \epsilon_{\nu\alpha\beta\sigma}P_\mu)k^\alpha P^\beta s^\sigma + \mathrm{O}(k^2)\,, \tag{6.9}$$

where $P^\mu = \frac{1}{2}(p+p')^\mu$. Both of these terms are conserved ($k^\mu P_\mu = 0$), symmetric, and have the correct parity. An example of a form factor which is ignored is $(k^2 g_{\mu\nu} - k_\mu k_\nu)A_2(k^2)$. It makes a vanishingly small contribution near $k^2 = 0$ since

JM (1990)

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')\left[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M \right.$$
$$\left. + \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M\right]U(P)\,,$$

Ji (1997)

Compare to spin 0

$$\left\langle p' \middle| T^{\mu\nu}_{q,g} \middle| p \right\rangle = A_{q,g}(t)\bar{P}^{\mu}\bar{P}^{\nu} + C_{q,g}(t)(\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^{2}) / M$$

Only 1 chiral even GPD

A careful wave packet analysis

$$\begin{aligned}
\mathcal{M}^{\mu\nu\lambda} &= \lim_{k\to 0}\int d^3x d^3p d^3p'\phi(p)\phi(p')e^{ikx}\langle p' \mid x^\nu T^{\mu\lambda}(x) - (\lambda \leftrightarrow \nu) \mid p\rangle \\
&= \lim_{k\to 0}\int d^3x d^3p d^3p'\phi(p)\phi(p')x^\nu e^{i(k+p-p').x}\langle p' \mid T^{\mu\lambda}(0) \mid p\rangle - (\lambda \leftrightarrow \nu) \\
&= \lim_{k\to 0}\int d^3x d^3p d^3p'\phi(p)\phi(p')(-i)\frac{\partial}{\partial k_\nu}e^{i(k+p-p').x}\langle p' \mid T^{\mu\lambda}(0) \mid p\rangle - (\lambda \leftrightarrow \nu) \\
&= \lim_{k\to 0}\int d^3p d^3p'\phi(p)\phi(p')(-i)\frac{\partial}{\partial k_\nu}\delta^3(k+p-p')\langle p' \mid T^{\mu\lambda}(0) \mid p\rangle - (\lambda \leftrightarrow \nu) \\
&= \lim_{k\to 0}(-i)\frac{\partial}{\partial k_\nu}\int d^3p d^3p'\phi(p)\phi(p')\delta^3(k+p-p')\langle p' \mid T^{\mu\lambda}(0) \mid p\rangle - (\lambda \leftrightarrow \nu) \\
&= \lim_{k\to 0}(-i)\frac{\partial}{\partial k_\nu}\int d^3p \phi(p+k)\phi(p)\langle p+k \mid T^{\mu\lambda}(0) \mid p\rangle - (\lambda \leftrightarrow \nu) \\
&= \lim_{k\to 0}(-i)\int d^3p \phi(p)\frac{\partial}{\partial k_\nu}[\phi(p+k)\langle p+k \mid T^{\mu\lambda}(0) \mid p\rangle] - (\lambda \leftrightarrow \nu) \\
&= \lim_{k\to 0}(-i)\int d^3p \phi(p)\langle p+k \mid T^{\mu\lambda}(0) \mid p\rangle\frac{\partial}{\partial k_\nu}\phi(p+k) - (\lambda \leftrightarrow \nu) \\
&+ (-i)\int d^3p \phi(p)\phi(p+k)\frac{\partial}{\partial k_\nu}\langle p+k \mid T^{\mu\lambda}(0) \mid p\rangle - (\lambda \leftrightarrow \nu) \\
&= \lim_{\mathbf{k}\to \mathbf{0}}(-\mathbf{i})\int \mathbf{d^3p}\phi(\mathbf{p})\langle \mathbf{p} \mid \mathbf{T}^{\mu\lambda}(\mathbf{0}) \mid \mathbf{p}\rangle\frac{\partial}{\partial \mathbf{k}_\nu}\phi^*(\mathbf{p}+\mathbf{k}) - (\lambda \leftrightarrow \nu) \\
&+ (-\mathbf{i})\int \mathbf{d^3p}|\phi(\mathbf{p})|^2\frac{\partial}{\partial \mathbf{k}_\nu}\langle \mathbf{p}+\mathbf{k} \mid \mathbf{T}^{\mu\lambda}(\mathbf{0}) \mid \mathbf{p}\rangle - (\lambda \leftrightarrow \nu)
\end{aligned}$$

Sum Rules in Deuteron

Momentum

$$\langle p'|\int d^3x\, T^{0i}_{q,g}\,|p\rangle = p^i\langle p'|p\rangle = \mathcal{G}_1^{q,g}\,p^i\int d^3x\, 2p^0$$

$$\Rightarrow \boxed{\mathcal{G}_1^q + \mathcal{G}_1^q = 1}$$

OAM

$$\langle p'|\int d^3x \left(x_1 T^{02}_{q,g} - x_2 T^{01}_{q,g}\right)|p\rangle = \mathcal{G}_5^{q,g}\int d^3x\, p^0$$

$$\Rightarrow \boxed{\frac{1}{2}\mathcal{G}_5^{q,g} = J_z^{q,g}}$$

$$\langle p'|p\rangle = 2p^0\delta^3(p'-p)$$

Use Ji's framework, attention must be payed to singularities (Bakker Leader Trueman), ...

Compare to spin 1/2

Momentum

$$\boxed{\mathrm{A}^q + \mathrm{A}^g = 1}$$

OAM

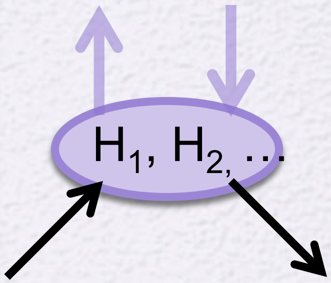
$$\boxed{\frac{1}{2}(\mathrm{A}^{q,g} + B^{q,g}) = J_z^{q,g}}$$

Now see this in terms of Quark-Hadron Helicity Amplitudes

To proceed with the identification of EMT components with n=1 moments of GPDs, start from quark-hadron helicity amplitudes:

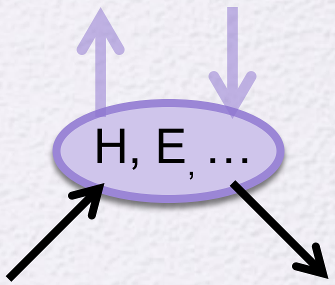
Deuteron

$$V_{\Lambda'\Lambda} = \sum_i \left[\varepsilon^{*\mu}(p',\Lambda') V^i_{\mu\nu} \varepsilon^{\nu}(p,\Lambda) \right] \boxed{H_i(x,\xi,t)}$$



Nucleon

$$F^S_{\Lambda'\Lambda} = \sum_i \left[\bar{U}_{\alpha}(p',\Lambda') O^i_{\alpha\beta} U_{\beta}(p,\Lambda) \right] \boxed{H_i(x,\xi,t)}$$



$$\underline{H_1 = H, \quad H_2 = E,} \qquad O^1 = \gamma^+, O^2 = \frac{-i\sigma^{+\mu}\Delta_\mu}{2M}$$

LC Correlation Function for Deuteron

$$\int \frac{d\kappa}{2\pi} e^{ix\kappa P.n} \langle p', \lambda' | \bar{\psi}(-\kappa n)\, \gamma.n\, \psi(\kappa n)\, | p, \lambda \rangle$$
$$= -(\epsilon'^{*}.\epsilon) \boxed{H_1} + \frac{(\epsilon.n)(\epsilon'^{*}.P) + (\epsilon'^{*}.n)(\epsilon.P)}{P.n} \boxed{H_2}$$
$$- \frac{(\epsilon.P)(\epsilon'^{*}.P)}{2M^2} \boxed{H_3} + \frac{(\epsilon.n)(\epsilon'^{*}.P) - (\epsilon'^{*}.n)(\epsilon.P)}{P.n} \boxed{H_4}$$
$$+ \left\{ 4M^2 \frac{(\epsilon.n)(\epsilon'^{*}.n)}{(P.n)^2} + \frac{1}{3}(\epsilon'^{*}.\epsilon) \right\} \boxed{H_5} \qquad (7)$$

Berger, Cano, Diehl, Pire, PRL(2001)

Compare to spin 1/2

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \overline{\psi}(-\lambda n/2) \gamma^{\mu} \psi(\lambda n/2) | P \rangle = \boxed{H(x, \Delta^2, \xi)}\, \overline{U}(P') \gamma^{\mu} U(P) + \boxed{E(x, \Delta^2, \xi)}\, \overline{U}(P') \frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2M} U(P)$$

Physical Interpretation of the various deuteron GPDs: Form Factors

$$\int H_1(x,\xi,t)\,dx = G_1(t)$$
$$\int H_2(x,\xi,t)\,dx = G_2(t)$$
$$\int H_3(x,\xi,t)\,dx = G_3(t)$$
$$\int H_4(x,\xi,t)\,dx = 0$$
$$\int H_5(x,\xi,t)\,dx = 0$$

$$G_C(t) = G_1(t) + \frac{2}{3}\eta G_Q(t)$$
$$G_M(t) = G_2(t)$$
$$G_Q(t) = G_1(t) - G_2(t) + (1+\eta)G_3(t)$$

$$G_C(0) = 1$$
$$G_M(0) = \frac{M_D}{M_N}\mu_D = 1.714$$
$$G_Q(0) = M_D^2 Q_D = 25.83$$

$$\eta = \frac{t}{2M_D^2}$$

Physical Interpretation of the various deuteron GPDs: PDFs

$$H_1(x,0,0) = \frac{1}{3}\Big(q^{1}(x) + q^{-1}(x) + q^{0}(x)\Big) = f_1(x)$$

$$H_5(x,0,0) = \left(q^{0}(x) - \frac{q^{1}(x) + q^{-1}(x)}{2}\right) = b_1(x)$$

The momentum and OAM SRs are obtained by connecting GPDs Mellin Moment n=2 with the EMT components (X.Ji, J.Phys G, 1998)

<u>Tower of vector operators:</u>

$$O_V^n = \bar{\psi}(0)\gamma^{\{\mu} i\overleftrightarrow{D}^{\mu_1} \dots i\overleftrightarrow{D}^{\mu_n\}} \psi(0)$$

<u>Moments</u>

$$H_{n+1}(\xi,t) \equiv \int_{-1}^{1} dx\, x^n H(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (-2\xi)^i \boxed{A_{n+1,i}(\Delta^2)} + (-2\xi)^{n+1} \boxed{C_{n+1,0}(\Delta^2)}\Big|_{n\ \text{odd}},$$

$$E_{n+1}(\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (-2\xi)^i \boxed{B_{n+1,i}(\Delta^2)} - (-2\xi)^{n+1} \boxed{C_{n+1,0}(\Delta^2)}\Big|_{n\ \text{odd}}.$$

"Parametrization" of spin ½ matrix elements

$$\begin{aligned}
&\langle P'|\bar{\psi}(0)\gamma^{\{\mu} iD^{\mu_1}\cdots iD^{\mu_n\}}\psi(0)|P\rangle \\
&= \bar{U}(P')\Bigg[\sum_{\substack{i=0\\ \text{even}}}^{n}\Big\{\gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\cdots\bar{P}^{\mu_n\}}\boxed{A_{n+1,i}(\Delta^2)} \\
&\quad - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\cdots\bar{P}^{\mu_n\}}\boxed{B_{n+1,i}(\Delta^2)}\Big\} + \frac{\Delta^{\{\mu}\cdots\Delta^{\mu_n\}}}{m}\boxed{C_{n+1,0}(\Delta^2)}\Big|_{n\ \text{odd}}\Bigg]U(P).
\end{aligned}$$

n=2, spin 1/2

"Parametrization" of spin ½ matrix elements

$$\left\langle p' \middle| \bar{\psi}(0)\gamma^{\mu} iD^{\nu}\psi(0) \middle| p \right\rangle = \bar{U}(p',\Lambda')\gamma^{\mu}U(p,\Lambda)\bar{P}^{\nu}\boxed{A_{20}(t)} -$$

$$\bar{U}(p',\Lambda')\frac{i\sigma^{\alpha\mu}\Delta_{\mu}}{2M}U(p,\Lambda)\bar{P}^{\nu}\boxed{B_{20}(t)} + \frac{\Delta_{\mu}\Delta_{\nu}}{M}\bar{U}(p',\Lambda')U(p,\Lambda)\boxed{C_{20}(t)}$$

"Parametrization" of EMT

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\left[\boxed{A_{q,g}(\Delta^2)}\gamma^{(\mu}\bar{P}^{\nu)} + \boxed{B_{q,g}(\Delta^2)}\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_{\alpha}/2M + \boxed{C_{q,g}(\Delta^2)}(\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^2)/M \right.$$
$$\left. + \boxed{\bar{C}_{q,g}(\Delta^2)}g^{\mu\nu}M\right]U(P),$$

$$\left\langle p' \middle| n_{\mu}n_{\nu}\bar{\psi}(0)\gamma^{\mu} iD^{\nu}\psi(0) \middle| p \right\rangle = \sum_i \left[\bar{U}_{\alpha}(p',\Lambda')O^{i}_{\alpha\beta}U_{\beta}(p,\Lambda)\right]\int dx\, x\, H_i(x,\xi,t)$$

$$\boxed{\frac{1}{2}\int dx\, x\Big(H(x,0,0) + E(x,0,0)\Big) = J_z^{q,g}}$$

n=2, spin 1

$$\left\langle p' \middle| n_\mu n_\nu \bar{\psi}(0)\gamma^\mu iD^\nu \psi(0) \middle| p \right\rangle = \sum_i \left[\varepsilon^{*\mu}(p',\Lambda') V^i_{\mu\nu} \varepsilon^\nu(p,\Lambda) \right] \int dx\, x\, H_i(x,\xi,t)$$

"Parametrization" of EMT

$$\begin{aligned}
&\langle p'|T^{\mu\nu}|p\rangle = -\frac{1}{2}P^\mu P^\nu(\epsilon'^*\epsilon)\mathcal{G}_1(t) \\
-&\ \frac{1}{4}P^\mu P^\nu \frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_2(t) - \frac{1}{2}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right](\epsilon'^*\epsilon) \\
\times&\ \mathcal{G}_3(t) - \frac{1}{4}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_4(t) \\
+&\ \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) + \epsilon^\mu(\epsilon'^* P)\right)P^\nu + \mu \leftrightarrow \nu\right]\mathcal{G}_5(t) \\
+&\ \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) - \epsilon^\mu(\epsilon'^* P)\right)\Delta^\nu + \mu \leftrightarrow \nu\right. \\
+&\ \left. 2g_{\mu\nu}(\epsilon P)(\epsilon'^* P) - (\epsilon'^{*\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu)\Delta^2\right]\mathcal{G}_6(t) \\
+&\ \frac{1}{2}\left[\epsilon^{*\prime\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu\right]\mathcal{G}_7(t) + g^{\mu\nu}(\epsilon'^*\epsilon)M^2\mathcal{G}_8(t) \qquad (5)
\end{aligned}$$

Angular Momentum Sum Rule

$$\boxed{\frac{1}{2}\mathcal{G}_5^{q,g} = \frac{1}{2}\int dx\, x H_2(x,0,0) = J_z^{q,g}}$$

Other relations

$$\int dx x[H_1(x,\xi,t) - \frac{1}{3}H_5(x,\xi,t)] = \mathcal{G}_1(t) + \xi^2\mathcal{G}_3(t) \quad (7)$$

Momentum

$$\int dx x H_2(x,\xi,t) = \mathcal{G}_5(t) \quad (8)$$

Angular Momentum

$$\int dx x H_3(x,\xi,t) = \mathcal{G}_2(t) + \xi^2\mathcal{G}_4(t) \quad (9)$$

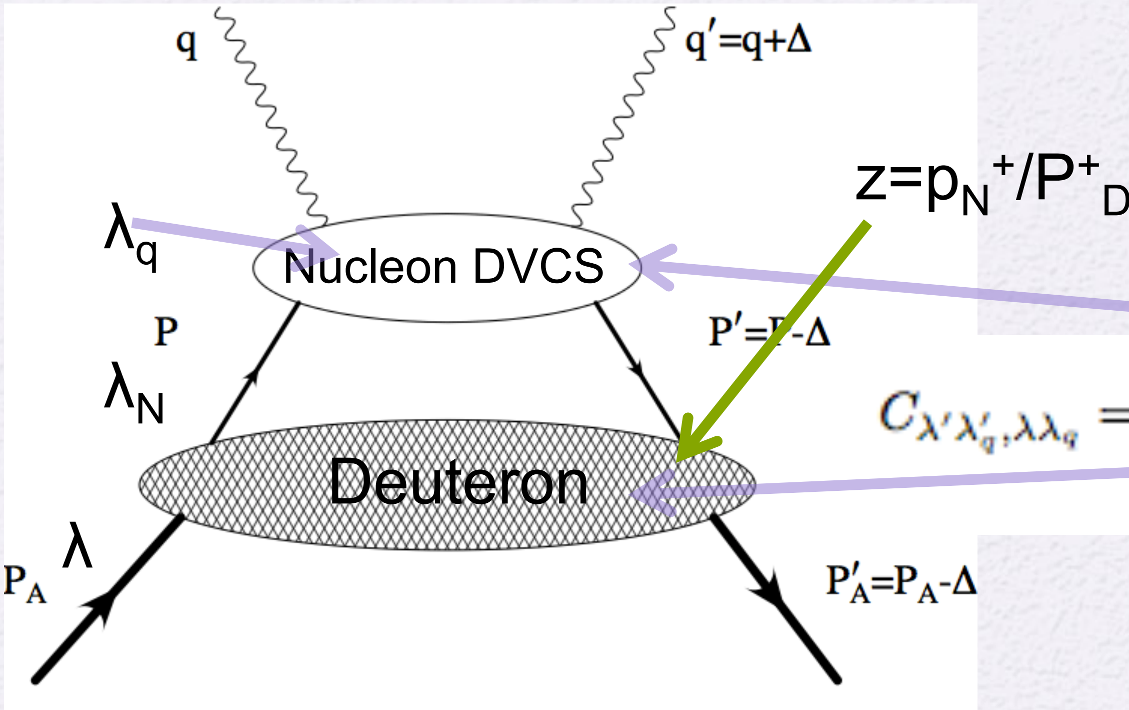
Quadrupole

$$\int dx x H_4(x,\xi,t) = \xi\mathcal{G}_6(t) \quad (10)$$

$$\int dx x H_5(x,\xi,t) = \mathcal{G}_7(t) \quad (11)$$

Connected to b_1 SR

What are the quark and gluon angular momenta in the deuteron?



$$C_{\lambda'\lambda'_q,\lambda\lambda_q} = \sum_{\lambda_N,\lambda'_N} B_{\lambda'\lambda'_N,\lambda\lambda_N} \otimes A_{\lambda'_N\lambda'_q,\lambda_N\lambda_q}$$

$$\sqrt{\frac{t_0-t}{2M^2}}H_2\left(x,0,0\right) = 2\left[\left(C_{1+,1+}+C_{1-,1-}\right)-\left(C_{1+,0+}+C_{1-,0-}\right)\right]$$

$$\Rightarrow H_2 \approx \int_x^{M_D/M_N} dz\left[f^{++}(z)H_{ISO}(x/z,0,0)+f^{0+}(z)E_{ISO}(x/z,0,0)\right]$$

$H_{ISO}=H_u+H_d$ $E_{ISO}=E_u+E_d$

Deuteron LC Momentum distribution

$$f^{++}(z) = 2\pi M \int_{p_{min}(z)}^{\infty} dp\, p \sum_{\lambda_N} \chi_+^{*\lambda'_{N_1}\lambda_{N_2}}(z,p)\chi_+^{\lambda_{N_1}\lambda_{N_2}}(z,p) \tag{14}$$

$$f^{0+}(z) = 4\pi M \int_{p_{min}(z)}^{\infty} dp\, p \sum_{\lambda_N} \chi_0^{*\lambda'_{N_1}\lambda_{N_2}}(z,p)\chi_+^{\lambda_{N_1}\lambda_{N_2}}(z,p). \tag{15}$$

$z = p_N^+/P^+_D$

$\lambda_N = \{\lambda_{N1}, \lambda'_{N1}\lambda_{N2}\}$

Deuteron w.f. (momentum space)

$$\chi_\lambda^{\lambda_{N_1},\lambda_{N_2}}(z,p) = \mathcal{N} \sum_{L,m_L,m_S} \begin{pmatrix} j_1 & j_2 & 1 \\ \lambda_{N_1} & \lambda_{N_2} & m_S \end{pmatrix} \begin{pmatrix} L & S & J \\ m_L & m_S & \lambda \end{pmatrix} \times Y_{Lm_L}\left(\frac{\mathbf{p}}{p} = \frac{M(1-z)-E}{p}\right) u_L(p),$$

Mixture of S and D components , L=0,2

If $f^{++}(z) = f^{+0}(z)=\delta(1-z)$ then $H_2=H+E$

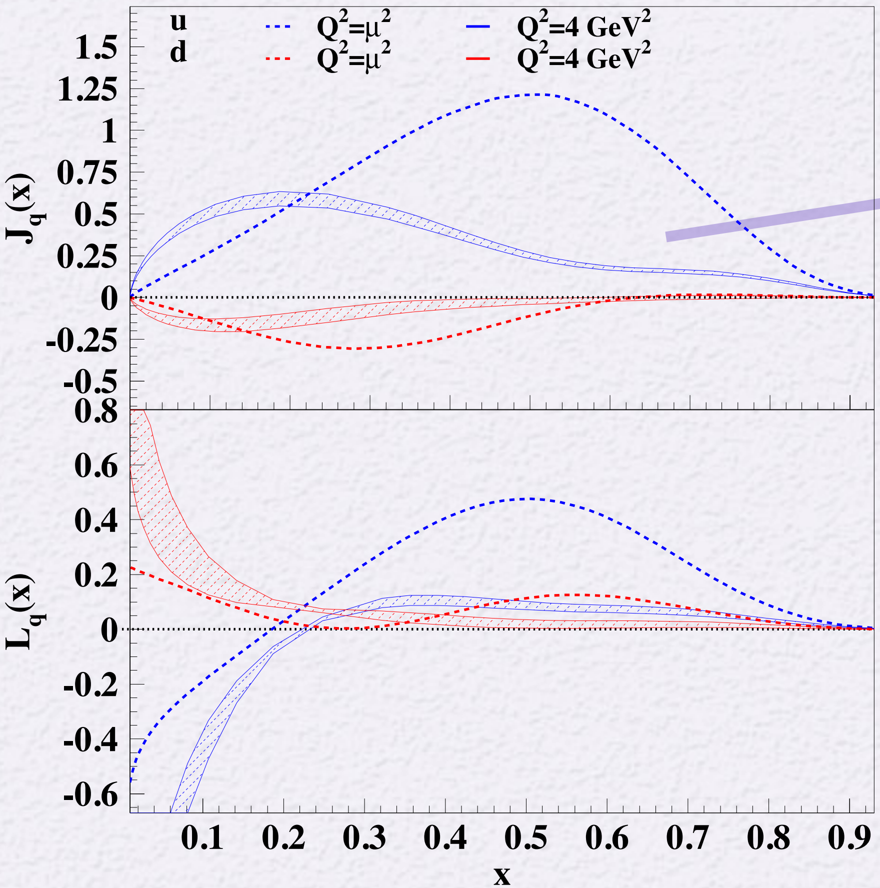
$$J_q = \frac{1}{2}\int dx\, x\,[H_q(x,0,0) + E_q(x,0,0)]\,,$$

$$J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

$F_1+F_2= G_M$

G_M

How does Ji sum rule differ from JM in the deuteron?



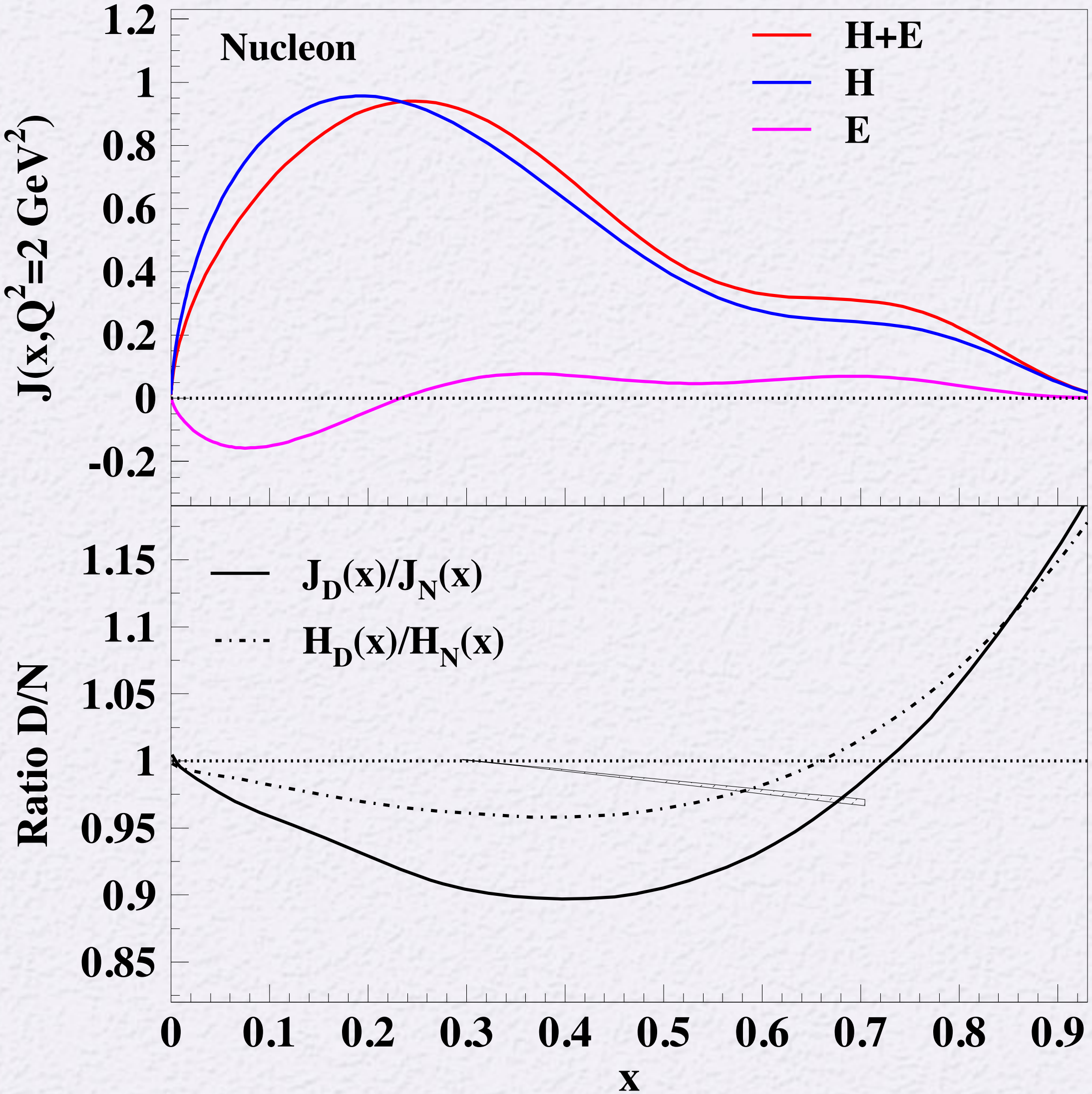
Effect of evolution

Models are important

Model calculations of L with w.f.s' seem to lead to similar conclusions as M.Burkardt, more to explore here… avenue to compare different schemes?

Using GPDs from Goldstein, Gonzalez, SL, PRD84

Nuclear effect much larger than in unpolarized scattering



Needs to be treated systematically…

Observables: DVCS from deuteron

$$A_{UT} \approx -\frac{4\sqrt{D_0}}{\Sigma}\Im m\left[\mathcal{H}_1^*\mathcal{H}_5 + \left(\mathcal{H}_1^* + \frac{1}{6}\mathcal{H}_5^*\right)(\mathcal{H}_2 + \mathcal{H}_4)\right]$$

subleading

Can the deuteron help us understand the role of gluon OAM?
(Brodsky, Gardner, 2006)

By connecting L_g to SSA in $lD \to l'\pi^{\pm}D'$

$$A_{UT}^{\pi^\pm}(\phi,\phi_s) \equiv \frac{1}{|\langle S_p\rangle|}\left(\frac{N_{\pi^\pm}^{\uparrow}(\phi,\phi_s) - N_{\pi^\pm}^{\downarrow}(\phi,\phi_s)}{N_{\pi^\pm}^{\uparrow}(\phi,\phi_s) + N_{\pi^\pm}^{\downarrow}(\phi,\phi_s)}\right) \equiv A_{UT}^{C}\sin(\phi+\phi_S) + A_{UT}^{S}\sin(\phi-\phi_S) + \cdots \approx 0$$

Both L_q and L_g contribute! Since L_q disappears because of isospin symmetry, if A^{UT}_{π} is 0 then L_g is 0

Interest in Spin 0 targets: pion, ^{4}He, …

One less distribution w.r.t. spin $\frac{1}{2}$

Ji's Sum Rule: 0=0

Partonic interpretation of spin and OAM?

L(x) is also 0, no nodes.

Pion production from Spin 0 target (^{4}He) is also very interesting

Quark-Nucleus Helicity Amps.

$$f_{\Lambda_\gamma,0;0,0} = \sum_{\lambda,\lambda'} g_{\Lambda_\gamma,\lambda;0,\lambda'} C_{0,\lambda';0,\lambda}. \tag{1}$$

$\Lambda_\gamma = \pm 1, 0$ being the virtual photon spin. The $C_{0,\lambda';0,\lambda}$ are the "quark-nucleus" helicity amplitudes. that depend on x_{Bj}, t and Q^2 while implicitly containing an integration over unobserved quark and nucleon momenta. They can be written in terms of the quark-nucleon helicity amplitudes:

Chiral-Odd Quark-Nucleon Helicity Amps.

$$C_{0,\lambda';0,\lambda} = \sum_{\Lambda_N,\Lambda'_N} \int d^4P\, B_{0,\Lambda'_N;0,\Lambda_N} A_{\Lambda'_N,\lambda';\Lambda_N,\lambda'} \tag{2}$$

Nucleon-Nucleus Helicity Amps.

Two terms survive:

$$\begin{aligned} T &\Rightarrow g_{1+,0-}\, \overset{\tilde{H}_T}{C_{0-;0+}} && (3)\\ L &\Rightarrow g_{0+,0-}\, C_{0-;0+} && (4) \end{aligned}$$

Both terms contain the same C function. The latter is given by:

$$C_{0,-;0,+} = \int d^4P\, [B_{0+.0-}A_{+-;-+} + B_{0-;0-}A_{--;-+} + B_{0+;0+}A_{+-;-+} + B_{0+.0+}A_{+-;-+}] \tag{5}$$

The B functions are given by:

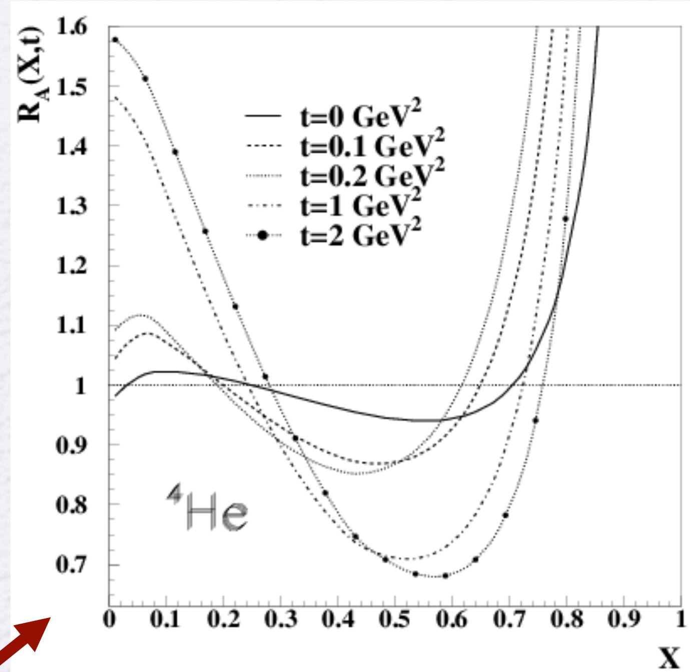
$$B_{0+;0+} = \cos\frac{\theta}{2}\rho_A(P^2, P'^2) \tag{6a}$$

$$B_{0+;0-} = \sin\frac{\theta}{2}\rho_A(P^2, P'^2) \tag{6b}$$

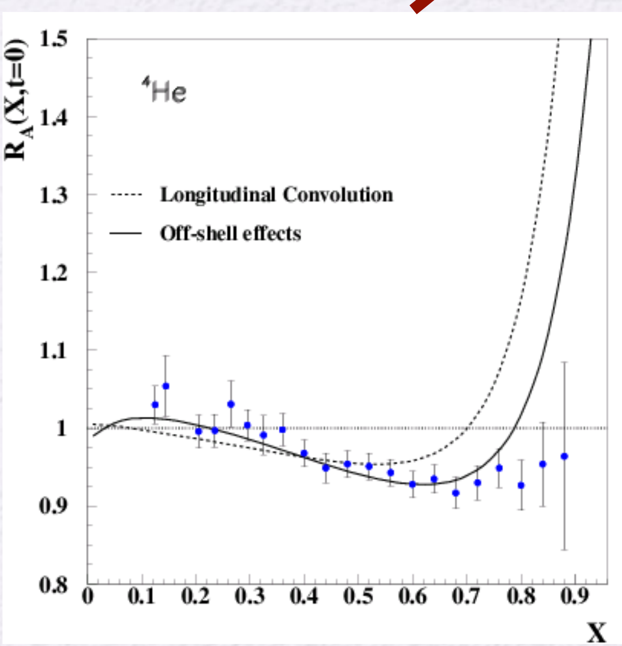
$$B_{0-.0-} = \cos\frac{\theta}{2}\rho_A(P^2, P'^2) \tag{6c}$$

$$B_{0-.0+} = -\sin\frac{\theta}{2}\rho_A(P^2, P'^2) \tag{6d}$$

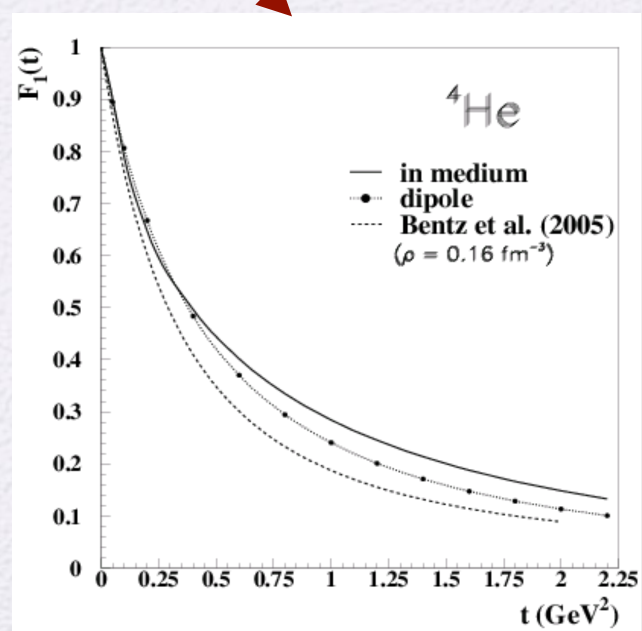
Use φ dependence to disentangle H_T from $\tilde{H}_T$!!



$R_A(X,t)$
t=0 GeV²
t=0.1 GeV²
t=0.2 GeV²
t=1 GeV²
t=2 GeV²
⁴He
X



$R_A(X,t=0)$
⁴He
Longitudinal Convolution
Off-shell effects
X



$F_1(t)$
⁴He
in medium
dipole
Bentz et al. (2005)
(ρ = 0.16 fm⁻³)
t (GeV²)

Angular momentum sum rule for spin one hadronic systems

Swadhin K. Taneja,[1,*] Kunal Kathuria,[2,†] Simonetta Liuti,[2,‡] and Gary R. Goldstein[3,§]

PRD86 (2012)

Conclusions

✓Spin and OAM with Jaffe-Manohar and Ji approaches: interesting relations are obtained by looking at spin 0, spin ½ and spin 1 hadronic systems, these can help disentangling the origin of the discrepancies

✓Deuteron is unique in many respects: as a nuclear target (nuclear effects, b_1), and as a way to determining the gluon spin/OAM content