Measurement of A_{zz}

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Abstract

A quick outline of how to measure A_{zz}.

1 Measurement of A_{zz}

The measured DIS double differential cross section for a spin-1 target characterized by a vector polarization P_z and tensor polarization P_{zz} is expressed as,

$$\frac{d^2\sigma_p}{dxdQ^2} = \frac{d^2\sigma}{dxdQ^2}\left(1 - P_z P_B A_1 + \frac{1}{2} P_{zz} A_{zz}\right), \tag{1}$$

where, σ_p (σ) is the polarized (unpolarized) cross section, P_B is the incident electron beam polarization, and A_1 (A_{zz}) is the vector (tensor) asymmetry of the virtual-photon deuteron cross section. This allows us to write the positive polarized tensor, $0 < P_{zz} \leq 1$, asymmetry using unpolarized electron beam as,

$$A_{zz} = \frac{2}{P_{zz}}\left(\frac{\sigma^1 - \sigma}{\sigma}\right), \tag{2}$$

where σ^1 is the polarized cross section for

$$P_{zz} = \frac{n_+ - 2n_0 + n_-}{n_+ + n_- + n_0}, \text{ for } n_+ + n_- > 2n_0. \tag{3}$$

Here n_m represents the portion of the ensemble in the m state.

Using Eq. 2 the asymmetry A_{zz} compares two different cross sections measured under different polarization conditions of the target, positively tensor polarized and unpolarized. To obtain both relative cross section measurements in the same configuration the same target cup and material will be used at alternating polarization states. In addition the same exact field will be used to keep acceptance consistent within the setability of the supper conducting magnet.
[*The NMR will be used on both to probe polarization. To move from polarized to unpolarized measurements the target polarization will be annihilated using destructive NRM loop field changes and destructive DNP microwave pumping. It is also possible to remove LHe in the nose of the target to remove the polarization by heating. During*

unpolarized data taking the incident electron beam heating is enough to remove the thermal equilibrium polarization.

The NMR measurement will ensure zero polarization. The target material will be kept at ∼1 K for polarized and unpolarized data collection. These consistencies are used to minimize the systematic differences in the polarized and unpolarized data collection. To minimize systematic effect over time the polarization condition will be switched twice in a 24 hour period. This is expected to account for drift in integrated charge accumulation.

(I think we should move this discussion to another section dealing with target physics and the overhead time accounting. Also, I would favor dumping the LHe, and refilling the nose.)]

The expressions for the tensor asymmetry in Eq. 2 needs to be modified to take into account the presence of unpolarized nuclei in the deuterated ammonia ($^{14}N^2H_3$, ND_3 for short) target. Since many of the factors invoved in the cross sections cancel in the ratio, the asymmetry can then be expressed in terms of the charge normalized, efficiency corrected numbers of polarized N^1 and unpolarized N counts,

$$A_{zz} = \frac{2}{fP_{zz}} \left(\frac{N^1 - N}{N} \right). \tag{4}$$

Here f is the dilution factor defined as,

$$f = \frac{N_D \sigma_D}{N_N \sigma_N + N_D \sigma_D + \Sigma N_A \sigma_A}, \tag{5}$$

where N_D is the number of deuterium nuclei in the target and and σ_D is the corresponding inclusive double differential scattering cross section, N_N is the nitrogen number of scattered nuclei with cross section σ_N, and N_A is the numbers of other scattering nuclei of mass number A with cross section σ_A. The denominator of the dilution factor can be written in terms of the relative volume ratio of ND_3 to LHe in the target cell, or the packing fraction p_f. In our case of cylindrical geometry the packing fraction is equivalent to the percent of the cell length filled with ND_3. For the full development of the dilution factor see Appendix A.

The measurement of the tensor asymmetry allows for a calculation of tensor structure function b_1 using the world data on the leading-twist structure function F_1^d,

$$b_1 = -\frac{3}{2} A_{zz} F_1^d. \tag{6}$$

In addition b_1 can be calculated directly using the difference of the two measured cross sections, however the uncertainties will be larger than for A_{zz}.

The time necessary to achieve the desired precision δA is:

$$T = \frac{N_T}{R_D} = \frac{16}{P_{zz}^2 f^2 \delta A_{zz}^2 R_D}. \tag{7}$$

where R_D is the deuteron rate and $N_T = N^1 + N$ is the total estimated number of counts to achieve the uncertainty δA_{zz}. See Appendix B for full details of the statistical uncertainty.

2 Systematic Uncertainty in A_{zz}

The systematic uncertainty of the asymmetry A_{zz} can be estimated based on know relative uncertainties and the systematic effects seen in past experiments.

2.1 Target Polarization

The target positive tensor polarization P_{zz} is calculated using the vector polarization P_z using Boltzmann statistics for spin temperature equilibrium,

$$P_{zz} = 2 - \sqrt{4 - 3P_z^2}. \tag{8}$$

The uncertainty in P_{zz} depends only on the uncertainty in the NMR measurement of P_z. This leads to the expression,

$$\delta P_{zz} = \frac{3P_z}{\sqrt{4 - 3P_z^2}} \delta P_z. \tag{9}$$

Polarization uncertainty for ND_4 have historically been no smaller than 5%. However with new techniques in polarization uncertainty minimization we anticipate to be able to achieve considerable reduction. Here we use the estimate of 4% relative uncertainty in P_z for and average vector polarization of 45% leading to a relative uncertainty in P_{zz} of 7.7%.

2.2 Time dependent factors

Systematic variation in time due to detector drift was studied for transversity JLab experiment E06-010. For 3 months running, all detectors in HRS were stable to about a 1% level. The scintillators, drift chambers, and lead-glass shower detector are stable to ∼2% in 3 months, assuming no significant radiation damage or detector gas loss. For the measurement of A_{zz} we expect no issue with radiation damage being the beam current is comparatively low and in the spectrometer.

2.3 Radiative Corrections

The systematic effect on A_{zz} due to the QED radiative corrections will be quite small. Based on previous data for unpolarized radiative corrections we use a 1.5% uncertainty. The polarized contribution is considered to be negligible for the range in x that we are measuring.

2.4 Charge Determination

The Beam Charge Monitor at low current are estimated to have an uncertainty lower than 5%. Integrating over a reasonable time the charge can be measured to approximately 1%. The Hall A tungsten calorimeter can be used to further reduce this uncertainty.

(#)	source	error (%)
(1)	Target Polarization	8%
(2)	Dilution/Packing fraction	4%
(3)	Detector Drift	1%
(4)	Radiative Corrections	1.5%
(5)	Charge Determination	1%
(6)	Detector resolution and efficiency	1%
	Total	9.2%

Table 1: The systematic error estimates of the A_{zz} asymmetry measurement.

2.5 Total Systematic Uncertainty

Table 1 shows a list of the leading uncertainties contributing to the systematic error in A_{zz}. The resulting estimate in the relative uncertainty of A_{zz} is 9.2%.

A Rendering Dilution Factor

To derive the dilution factor we first start with the ratio of polarized to unpolarized counts. In each case, the number of counts that are actually measured, and neglecting the small contributions of the thin aluminium cup window materials, NMR coils, etc., are

$$N_1 = Q_1 \varepsilon_1 \mathcal{A}_1 l_1 [(\sigma_N + 3\sigma_1)p_f + \sigma_{He}(1 - p_f)], \tag{10}$$

and

$$N = Q\varepsilon\mathcal{A}l[(\sigma_N + 3\sigma)p_f + \sigma_{He}(1 - p_f)]. \tag{11}$$

where Q represents accumulated charge, ε is the dectector efficiency, $\mathcal{A}$ the cup acceptance, and l the cup length.

For this calculation we assume similar charge accumulation such that $Q \simeq Q_1$, and that the efficiencies stay constant, in which case all factors drop out of the ratio leading to

$$\begin{aligned}
\frac{N_1}{N} &= \frac{(\sigma_N + 3\sigma_1)p_f + \sigma_{He}(1 - p_f)}{(\sigma_N + 3\sigma)p_f + \sigma_{He}(1 - p_f)} \\
&= \frac{(\sigma_N + 3\sigma(1 + 2A_{zz}P_{zz}/2))p_f + \sigma_{He}(1 - p_f)}{(\sigma_N + 3\sigma)p_f + \sigma_{He}(1 - p_f)} \\
&= \frac{[(\sigma_N + 3\sigma)p_f + \sigma_{He}(1 - p_f)] + 3\sigma A_{zz}P_{zz}/2}{(\sigma_N + 3\sigma)p_f + \sigma_{He}(1 - p_f)} \\
&= 1 + \frac{3\sigma A_{zz}P_{zz}/2}{(\sigma_N + 3\sigma)p_f + \sigma_{He}(1 - p_f)} \\
&= 1 + \frac{1}{2} f A_{zz} P_{zz},
\end{aligned} \tag{12}$$

where $\sigma_1 = \sigma(1 + 2A_{zz}P_{zz}/2)$ has ben substituted, per eq. (1), with $P_B = 0$. It can be seen that the above result corresponds to eq. (4) in the main text.

B Statistical Uncertainty Calculation

To investigate the statistical uncertainty we start with the equation for A_{zz} using measured counts for polarized data N_1 and unpolarized data N,

$$A_{zz} = \frac{2}{fP_{zz}} \left(\frac{N_1 - N}{N} \right). \tag{13}$$

The absolute error with respect to counts in then,

$$\delta A_{zz} = \frac{2}{fP_{zz}} \sqrt{\left(\frac{\delta N_1}{N} \right)^2 + \left(\frac{N_1 \delta N}{N^2} \right)^2}. \tag{14}$$

To approximate, assume $N_1 \simeq N$, so that twice N is required to obtain the total number of count N_T for the experiment leading to,

$$\delta A_{zz} = \frac{4}{fP_{zz}} \frac{1}{\sqrt{N_T}}. \tag{15}$$