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Letter to the Editor

On the inherent stability of non-isochronous recirculating accelerators

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In a recent paper on polytrons [H. Herminghaus, Nucl. Instr. and Meth. A305 (1991) 1] it was noted rather marginally that non-isochronous recirculation schemes should offer inherently superior energy stability as compared to isochronous schemes. Meanwhile several discussions showed that this beneficial feature does not seem to be recognized widely in the accelerator community. Therefore, in the following, the mechanism involved is briefly described and some numerical examples of its effect are given.

For simplicity we restrict our consideration to the simple scheme of fig. 1 (for more general schemes see ref. [1]). In order to reproduce the rf phase "seen" by a particle from turn to turn, the phase has to be shifted from turn to turn by an integer number ν of wavelengths λ:

$$\Delta E \, \mathrm{d}s/\mathrm{d}E = \nu\lambda, \tag{1}$$

where $\mathrm{d}s/\mathrm{d}E$ is the longitudinal dispersion of the recirculation path and ΔE the energy gain per turn. With isochronous recirculation, $\mathrm{d}s/\mathrm{d}E = 0$ by definition, thus $\nu = 0$ and eq. (1) is fulfilled by any energy gain ΔE. As a consequence, any missetting of the accelerating field will add up, turn by turn.

In a non-isochronous system, $\mathrm{d}s/\mathrm{d}E \neq 0$ by definition and thus $\nu \neq 0$ (usually $\nu = 1$). In that case, the condition (1) is satisfied by one distinct value ΔE_r of ΔE only, well defined by λ and $\mathrm{d}s/\mathrm{d}E$ (the latter being determined by magnetic flux densities in the optical elements of the recirculation path, which are much easier controlled to high accuracy than rf fields). By choosing the amplitude $\Delta\hat{E}$ of the accelerating field somewhat in excess of ΔE_r, a particle will "see" the right field at a certain reference phase angle Φ_r, defined by

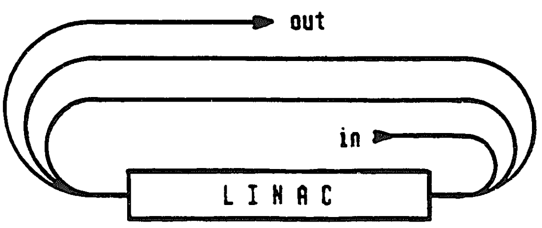

Fig. 1. Simple recirculation scheme considered here.

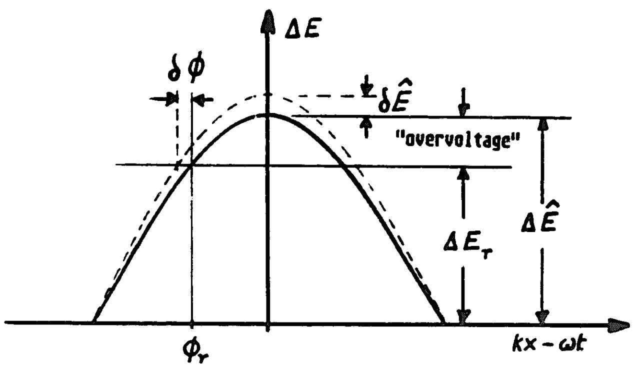

Fig. 2. Energy gain versus rf phase: definition of symbols used.

$$\Delta E_r = \Delta\hat{E} \cos(\Phi_r). \tag{2}$$

The percentage by which $\Delta\hat{E}$ overshoots ΔE_r is called "overvoltage" (see fig. 2). A particle starting at Φ_r with the correct energy will keep this location in longitudinal phase space turn by turn ("fixpoint"). When starting with some offset $\delta\Phi$ and/or δE from this point, the particle will execute synchrotron oscillations about the fixpoint, the Q-value of which in the limit of small amplitudes is given by

$$\cos(2\pi Q) = 1 + \nu\pi \tan(\Phi_r). \tag{3}$$

It is obvious and commonly realized that the deviations in energy gain will smooth out over many oscillations.

Similarly obvious but less commonly realized is an even more powerful stabilisation mechanism which occurs if the number of oscillations is small. If namely Q

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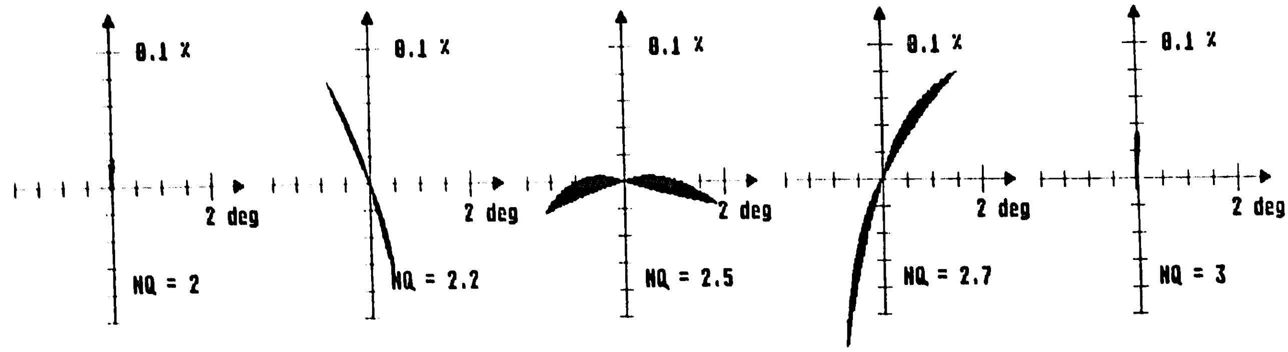

Fig. 3. Phase space response of a non-isochronous recirculator with $N = 8$ linac passages at different longitudinal tunes Q. The total number of synchrotron oscillations, NQ, is given in the figures. Jitter of rf is 0.6% bin in amplitude and 1° in phase. The ordinate refers to relative output energy jitter.

is chosen such that a (small) integer number of oscillations is executed during acceleration, the phase space of the beam at input is closely reproduced at the output, with no regard to minor errors in phase and amplitude. The mechanism implies that the errors stay constant during acceleration, i.e. that the rate of change of rf phase and amplitude is slow as compared to the acceleration time. This supposition should be realistic in cw accelerators since the sources of such errors are usually due to mains noise or mechanical vibrations. The effectiveness of the mechanism is limited by two circumstances. Firstly: if the errors are too large the reproduction of the input phase space will be imperfect due to nonlinearity of the oscillation ("filamentation"). Secondly: in case of amplitude error, Q will change according to eqs. (2) and (3), so that the number of oscillations is no more an integer. This is especially dangerous if Φ_r is small. On the other hand, however, an overvoltage of a few percent is sufficient for Φ_r to be adequate. The imperfections caused by these effects should not accumulate over many periods, so generally the total number of oscillations should be small.

Generally, the factor by which the output energy stability is improved increases with the decrease in rf fluctuations. Typically, a 2% ripple in amplitude is suppressed by a factor of 10, a 2×10^{-4} ripple by a factor of 100. Thus, it may be possible to achieve extremely stable output energy. This could be important e.g. for some FEL applications [2].

Note that, as seen from fig. 2, a jitter of both rf amplitude and phase result in a phase offset of the particles – an energy offset could be caused by an offset of the input energy only. Therefore, if some phase jitter at the output can be tolerated, a half integer number of oscillations may sometimes be used, too. For instance, the output energy of a once recycled linac may be stabilised by choosing $Q = \frac{1}{4}$ (see fig. 4).

Numerous simulations of this mechanism have been computed for different scenarios. For computation, rf amplitude and/or phase are varied stochastically within given limits with rectangular probability distribution. Using this set of parameters, a particle is traced through the accelerator and its position in phase space at the output is marked. This procedure is repeated many times until a significant plot of the density distribution is obtained. A few examples will be given in the following. All phase space plots refer to the midst of the bending system. The range of phase error might also be understood to include the bunch length at input. The percentage given at the ordinates is the energy deviation referred to the total output energy.

Fig. 3 shows the response of a scheme like fig. 1 with $N = 8$ linac passages at different tunes between $Q = \frac{1}{4}$ and $Q = \frac{3}{8}$ (corresponding to $NQ = 2.0$–3.0 synchrotron oscillations and overvoltage 5–14%). The rf field of the linac is assumed to jitter by 0.6% bin in amplitude and 1° in phase, uncorrelated. It is seen that with $NQ =$ integer, both energy and phase jitter are suppressed by more than a factor of ten. At $NQ =$ half-integer, suppression of energy jitter is equally effective, but phase jitter is not suppressed. Without

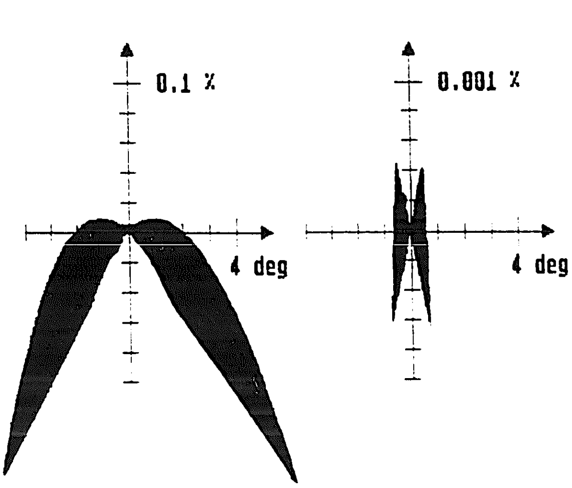

Fig. 4. Response of a once recirculated linac, tuned to $Q = \frac{1}{4}$, at different jitter amplitudes. (a) 2% bin amplitude jitter, 2° bin phase jitter, (b) 0.06% and 0.6°, respectively.

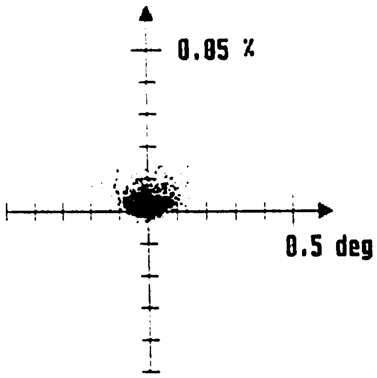

Fig. 5. Response of a polytron with 3 linacs, 5 revolutions, $NQ = 2.0$, rf amplitude jitter 1% bin, phase jitter 2°.

special tune, e.g. at $NQ = 2.2$ or 2.7, energy jitter is still somewhat suppressed, due to the general "smoothing" by synchrotron oscillations.

Fig. 4 shows the response of a once recirculated linac at $Q = \frac{1}{4}$ to different rf jitter amplitudes. In fig. 4a, rf amplitude and phase are assumed to jitter by 2% and 2°, respectively. The resultant output energy jitter is about 0.2%. If rf amplitude and phase jitter are reduced to 0.06% and 0.6° respectively, the output energy will be stabilised to about 1×10^{-5} (fig. 4b).

It should be mentioned that schemes using more than one linac generally show even better response when tuned to an integer number of oscillations. This is not surprising because of the much larger longitudinal bucket of such schemes [1]. As an example, fig. 5 shows the response of a polytron using 3 linacs and 5 revolutions (thus 15 linac passages) [1], all three linacs jittering simultaneously, uncorrelated with respect to each other, by 1% and 2° bin. The tune is $Q = \frac{2}{15}$, so there are two oscillations during acceleration. In that example, the energy stability is improved by a factor of 40–50 as compared to isochronous recirculation.

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References

[1] H. Herminghaus, Nucl. Instr. and Meth. A305 (1991) 1.
[2] A. Schwettman, HEPL Stanford, private communication, Aug. 1991.