Radiative corrections for di-lepton final state processes

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CLAS collaboration
meeting - CNU

12th of July 2023

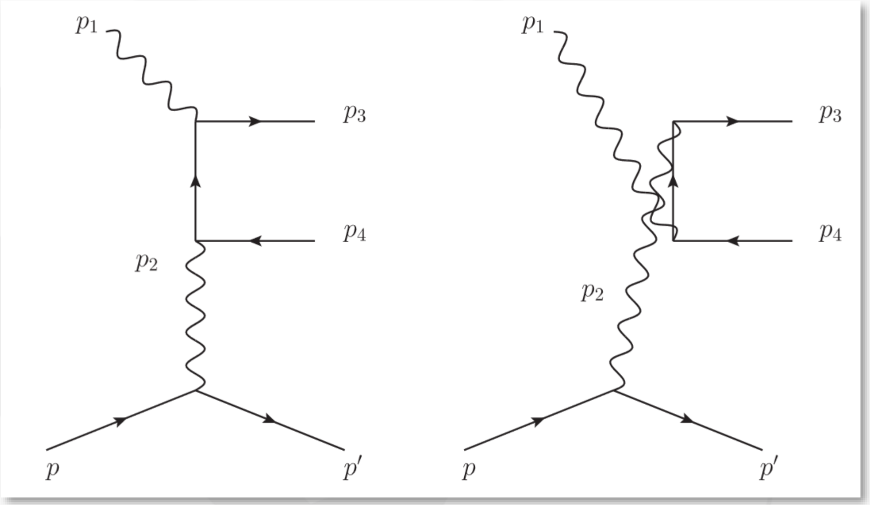
Jefferson Lab

Outline

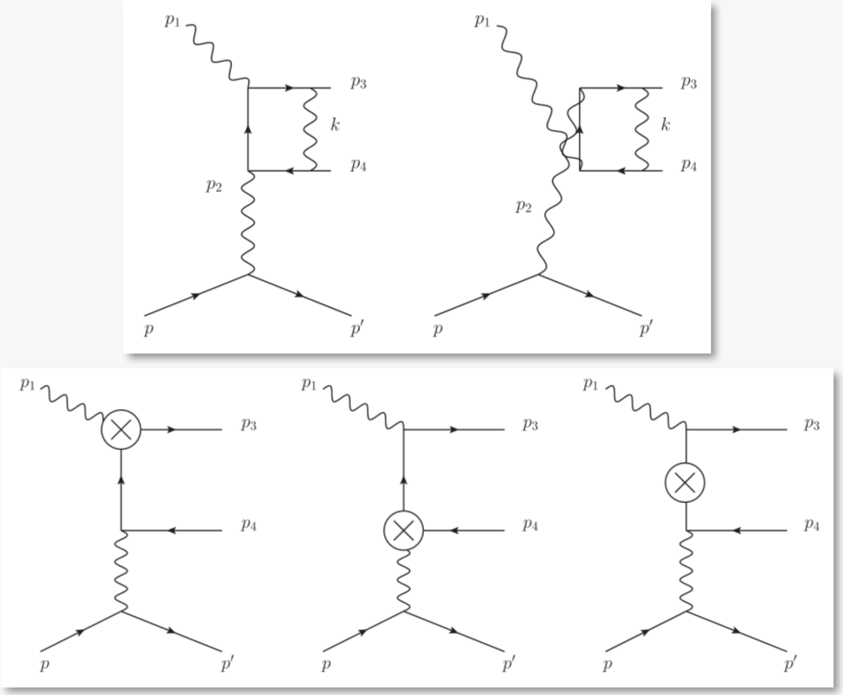
Motivations and background (I)

Matthias Heller, Oleksandr Tomalak, and Marc Vanderhaeghen. Soft-photon corrections to the bethe-heitler process in the $\gamma p \to l^+l^-p$ reaction. Phys. Rev. D, 97:076012, Apr 2018
https://journals.aps.org/prd/abstract/10.1103/PhysRevD.97.076012

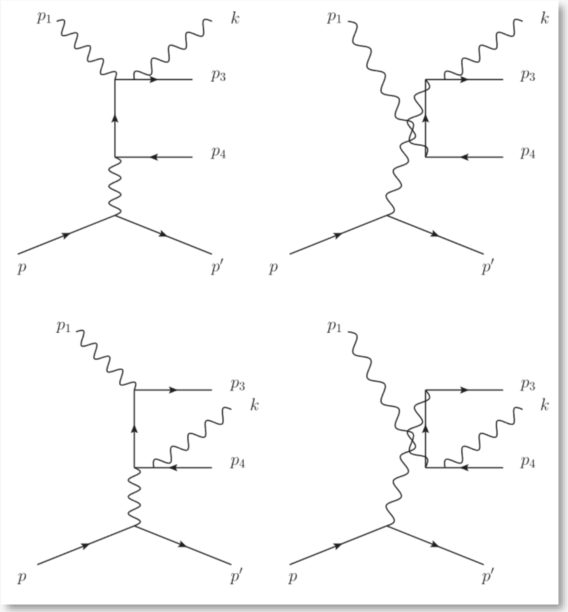
Base diagrams



Loop diagrams



Photon emission diagrams



All figures from Matthias Heller, Oleksandr Tomalak, and Marc Vanderhaeghen. Soft-photon corrections to the bethe-heitler process in the $\gamma p \to l^+l^-p$ reaction. Phys. Rev. D, 97:076012, Apr 2018

Motivations and background (II)

Base equation

$$\left(\frac{d\sigma}{dt\ ds_{ll}}\right)_{rad} = \left(\frac{d\sigma}{dt\ ds_{ll}}\right)_0 (1+\delta)$$

$$\delta = -\left(\frac{\alpha}{\pi}\right)\left\{\left[\ln\left(\frac{4\Delta E_s^2}{m^2}\right) + \ln\left(\frac{1-\beta}{1+\beta}\right)\right]\left[1 + \left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1-\beta}{1+\beta}\right)\right] + \left(\frac{1-\beta}{\beta}\right)\ln\left(\frac{1-\beta}{1+\beta}\right) + \left(\frac{1+\beta^2}{2\beta}\right)\left[4\ \mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) - \pi^2\right]\right\},$$

$$\beta = \sqrt{1 - \frac{4m^2}{s_{ll}}}$$

Large invariant mass approximation

$$s_{ll} \gg 4m^2 \longrightarrow \delta = -\left(\frac{\alpha}{\pi}\right)\left\{\ln\left(\frac{4\Delta E_s^2}{s_{ll}}\right)\left[1 + \ln\left(\frac{m^2}{s_{ll}}\right)\right] - \frac{\pi^2}{3}\right\}.$$

All equations from Matthias Heller, Oleksandr Tomalak, and Marc Vanderhaeghen. Soft-photon corrections to the bethe-heitler process in the $\gamma p \to l^+l^-p$ reaction. Phys. Rev. D, 97:076012, Apr 2018

Motivations and background (III)

Multi-photon emission

$$\left(\frac{d\sigma}{dt\,ds_{ll}}\right)_{s;\mathrm{tot}} = \left(\frac{d\sigma}{dt\,ds_{ll}}\right)_0 \cdot F \exp\left\{-\frac{\alpha}{\pi}\left[\ln\left(\frac{4\Delta E_s^2}{m^2}\right) + \ln\left(\frac{1-\beta}{1+\beta}\right)\right]\left[1 + \left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1-\beta}{1+\beta}\right)\right]\right\}$$
$$\times \left\{1 - \frac{\alpha}{\pi}\left[\left(\frac{1-\beta}{\beta}\right)\ln\left(\frac{1-\beta}{1+\beta}\right) + \left(\frac{1+\beta^2}{2\beta}\right)\left[4\,\mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) - \pi^2\right]\right]\right\}$$
$$\equiv \left(\frac{d\sigma}{dt\,ds_{ll}}\right)_0 (1 + \delta_{\mathrm{exp}}),$$

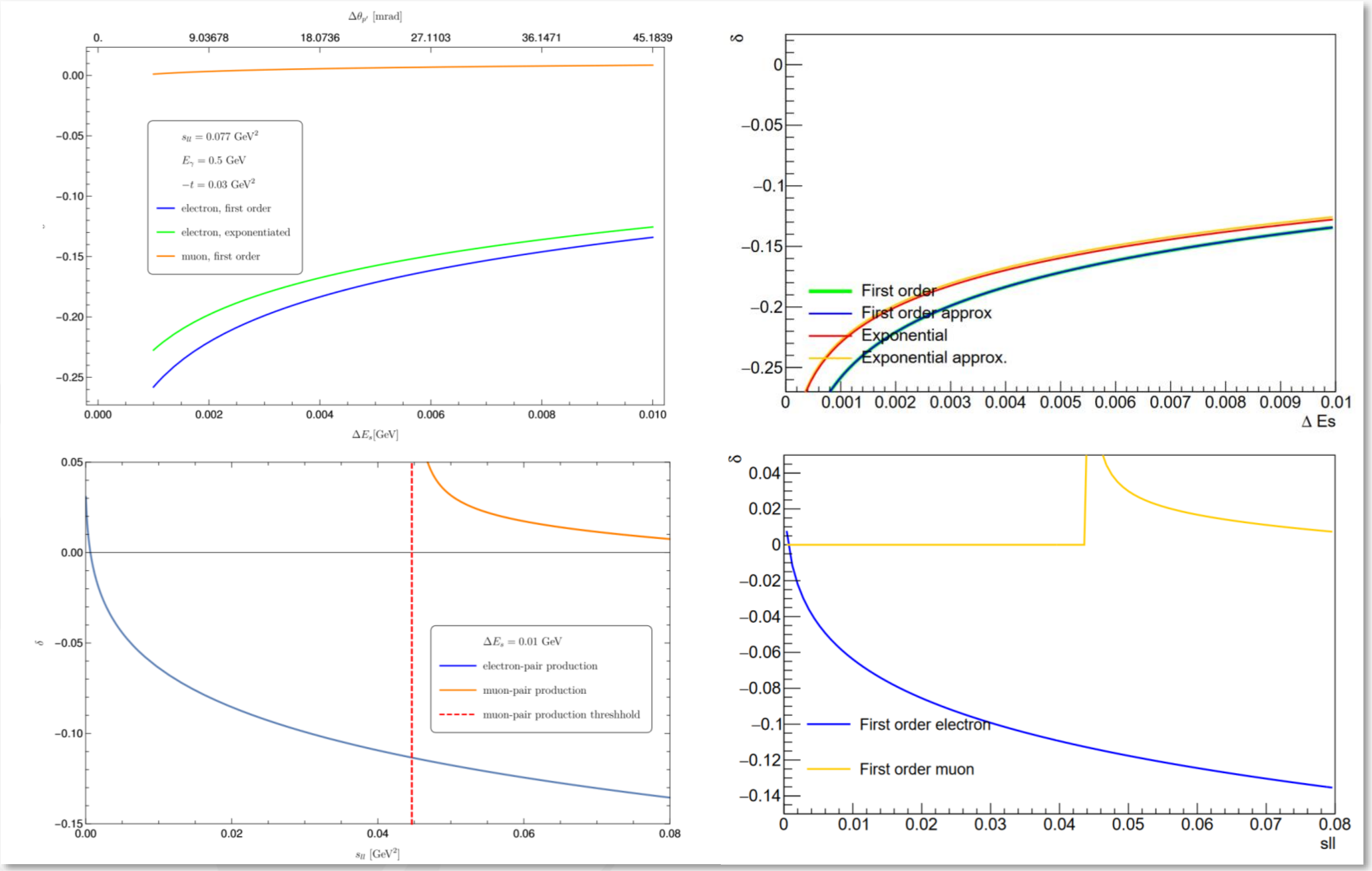
$$F = 1 - \frac{\alpha^2}{3}\left[1 + \left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1-\beta}{1+\beta}\right)\right]^2 + \ldots$$

Final equation

$$\left(\frac{d\sigma}{dt\,ds_{ll}}\right)_{s;\mathrm{tot}} = \left(\frac{d\sigma}{dt\,ds_{ll}}\right)_0 \cdot \exp\left\{-\left(\frac{\alpha}{\pi}\right)\left\{\ln\left(\frac{4\Delta E_s^2}{s_{ll}}\right)\left[1 + \ln\left(\frac{m^2}{s_{ll}}\right)\right] - \frac{\pi^2}{3}\right\}\right\}$$
$$= \left(\frac{d\sigma}{dt\,ds_{ll}}\right)_0 (1 + \delta_{\mathrm{exp.\ approx.}}).$$

Cross-checks and comparison with article results

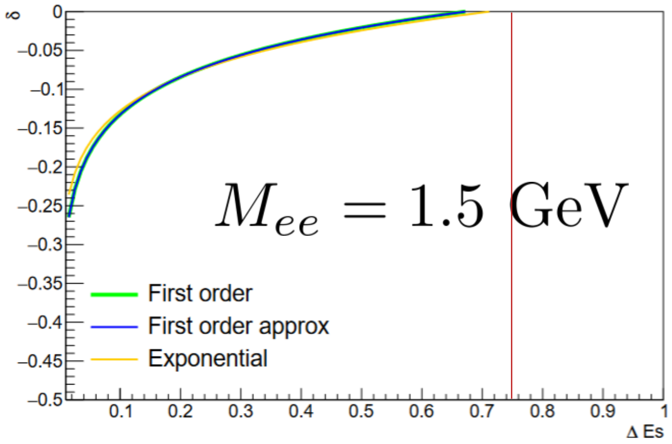
Comparison with article results

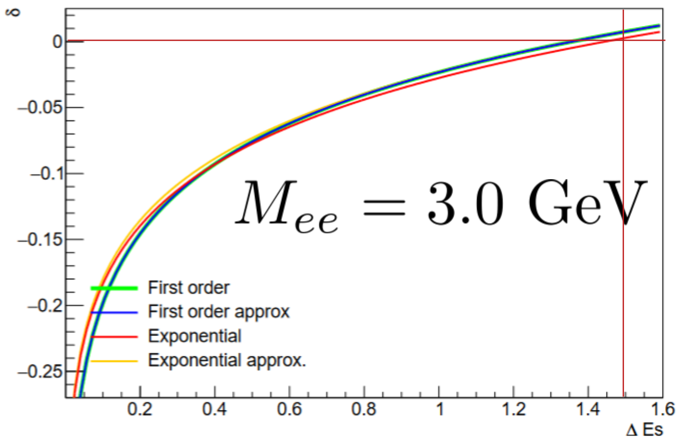


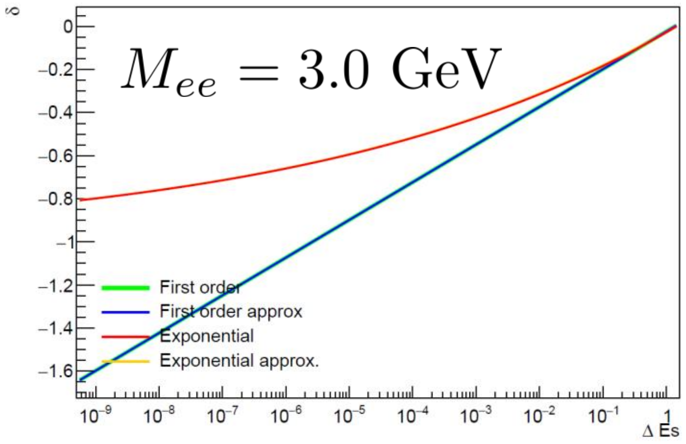
Figures from Matthias Heller et al.

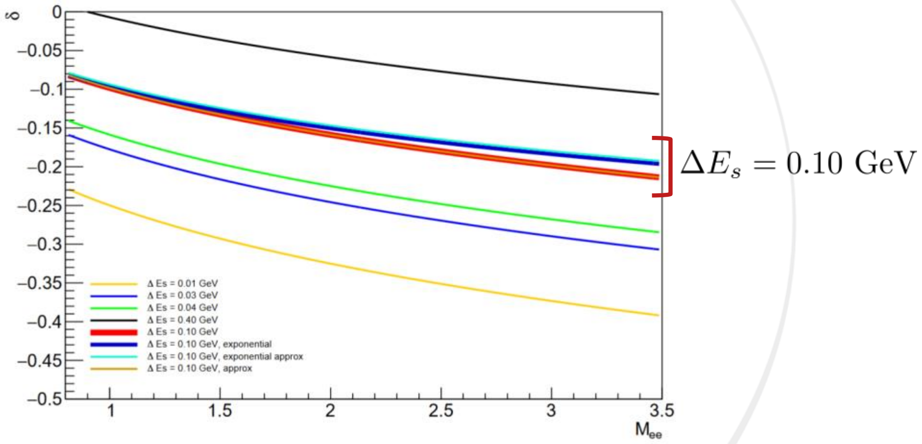
- I have tested the formula against the results shown in the article.
- 1-to-1 agreement is found for the dependence as a function of photon energy and dilepton invariant mass.

Approximation validation and interpretations









- δ is negative for Es below the energy carried by one lepton in the CM frame.
- Approximation holds for both first order and exponential formula.
- Large difference between first order and exponential formula only seen at low photon energy.

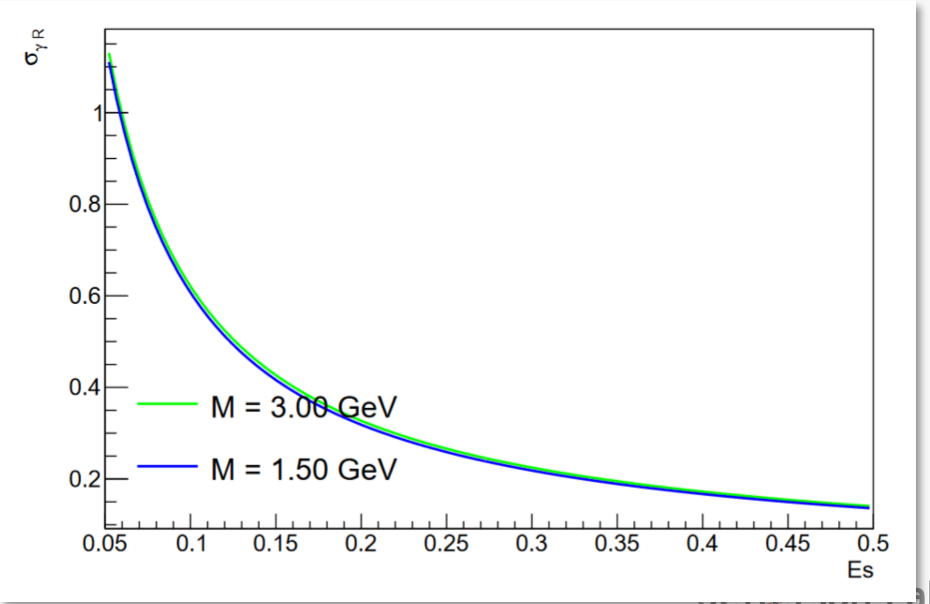
Descriptions of the algorithm (I)

Final equation

$$\left(\frac{d\sigma}{dt\,ds_{ll}}\right)_{s;tot} = \left(\frac{d\sigma}{dt\,ds_{ll}}\right)_0 \cdot \exp\left\{-\left(\frac{\alpha}{\pi}\right)\left\{\ln\left(\frac{4\Delta E_s^2}{s_{ll}}\right)\left[1+\ln\left(\frac{m^2}{s_{ll}}\right)\right]-\frac{\pi^2}{3}\right\}\right\}$$
$$= \left(\frac{d\sigma}{dt\,ds_{ll}}\right)_0 (1+\delta_{\text{exp. approx.}}).$$

Obtain the probability to emit a photon carrying an energy E_s

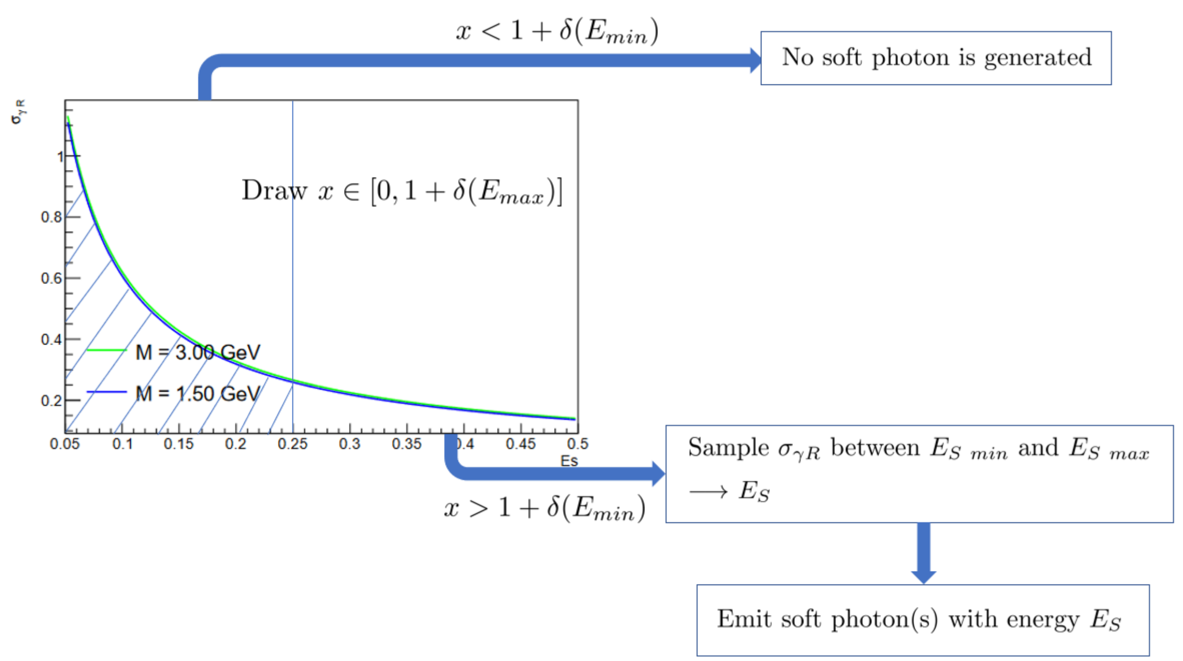
$$\frac{d(1+\delta_{exp})}{dE_s} = \frac{d}{dE_s} exp\left\{-\left(\frac{\alpha}{\pi}\right)\left\{\ln\left(\frac{4E_s^2}{s_{ll}}\right)\left[1+\ln\left(\frac{m^2}{s_{ll}}\right)\right]-\frac{\pi^2}{3}\right\}\right\}$$
$$= \frac{-\alpha}{\pi}\left[1+\ln\left(\frac{m^2}{s_{ll}}\right)\right]\left(\frac{2}{E_s}\right) exp(\delta).$$



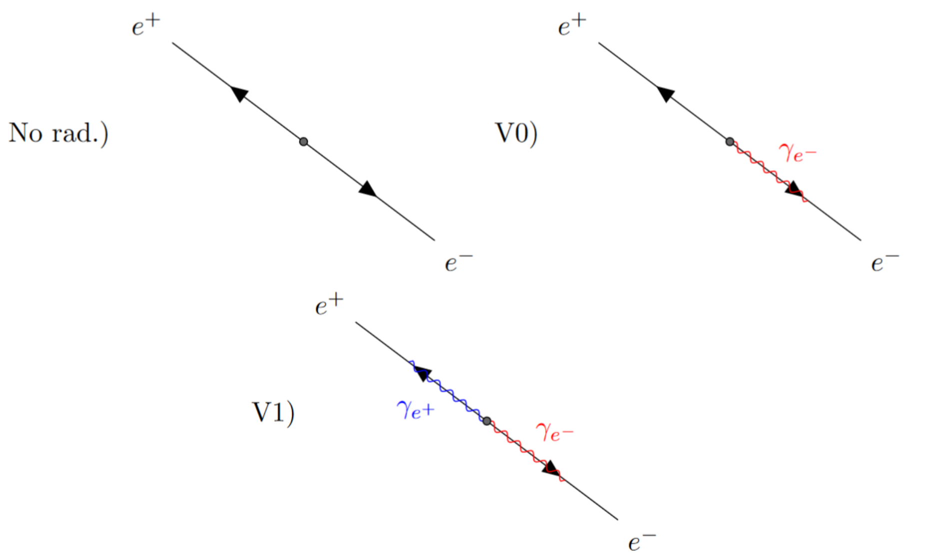
Descriptions of the algorithm (II)

- Set a minimum energy $E_{S\ min}$ under which we consider no photon is emitted
- Set a maximum energy $E_{S\ max}$
- Compute $(1+\delta(E_{S\ min}))$ for this minimal energy, this is the probability to emit a photon with energy below $E_{S\ min}$
- Draw x between 0 and $(1+\delta(E_{S\ max}))$
  - If $x < (1+\delta(E_{S\ min}))$, no photon is generated (the emitted photon energy is too small to be considered)
  - If $x > (1+\delta(E_{S\ min}))$, sample the emission probability between $E_{S\ min}$ and $E_{S\ max}$
- Generate the photon 4-vectors in the CM frame, boost back to the lab frame

Schematic description of the algorithm



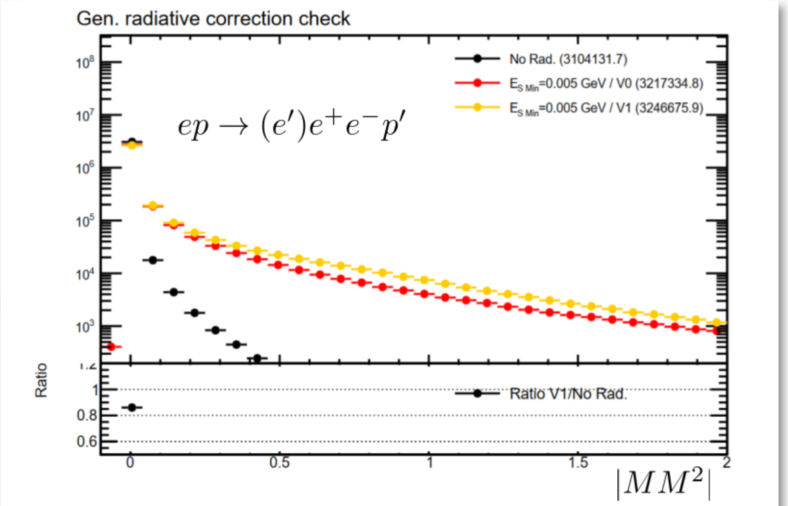
Photon emission choice

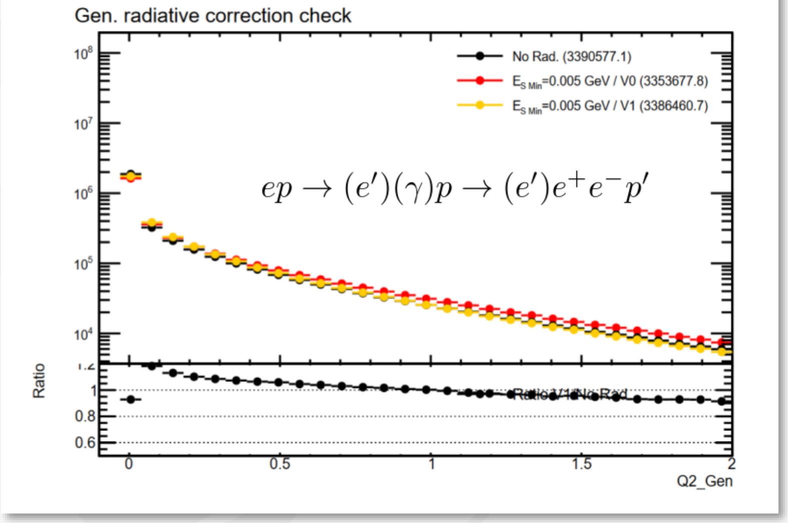


Effect on Bethe-Heilter simulation (Generated) (I)

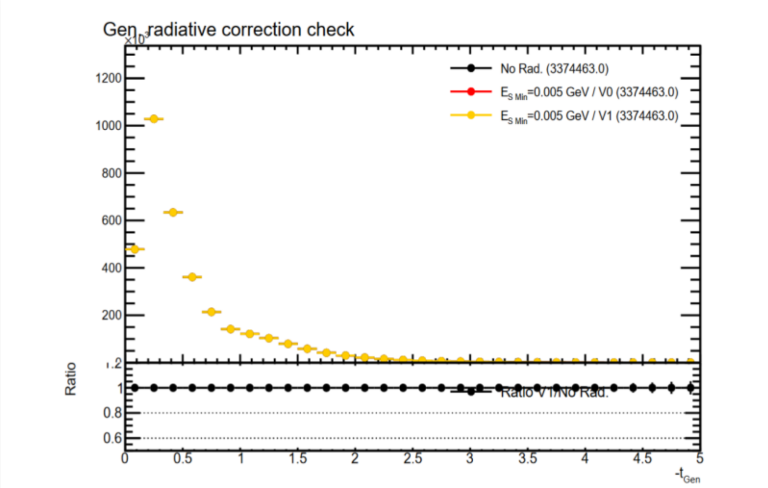
- $E_{S\ min}$ = 0.005 GeV and $E_{S\ max}$ = 0.9 GeV

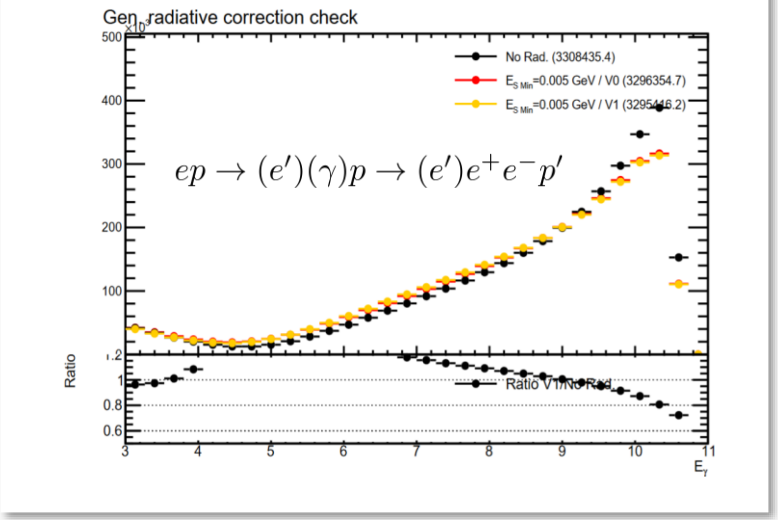
Exclusivity variables



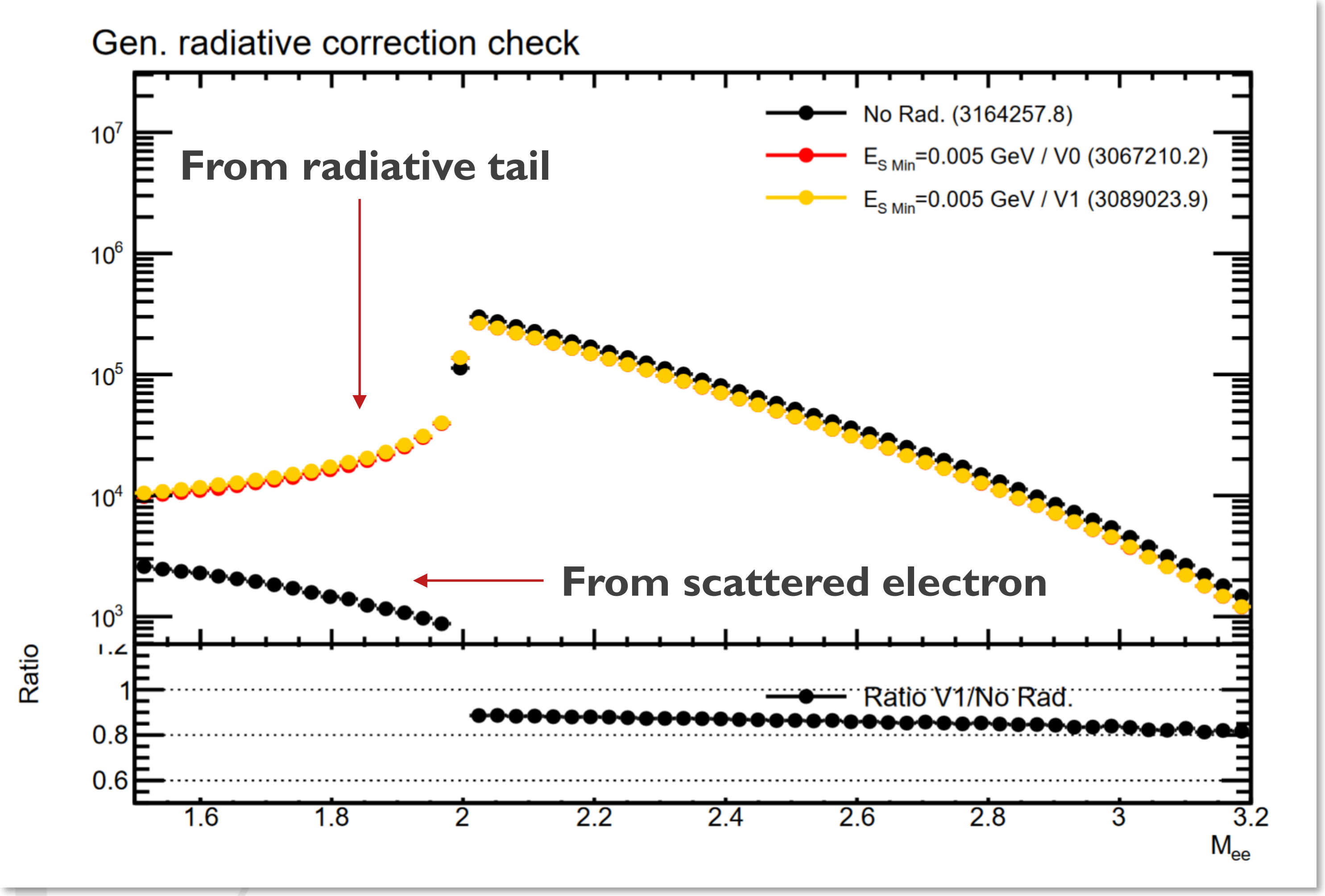


Kinematic variables





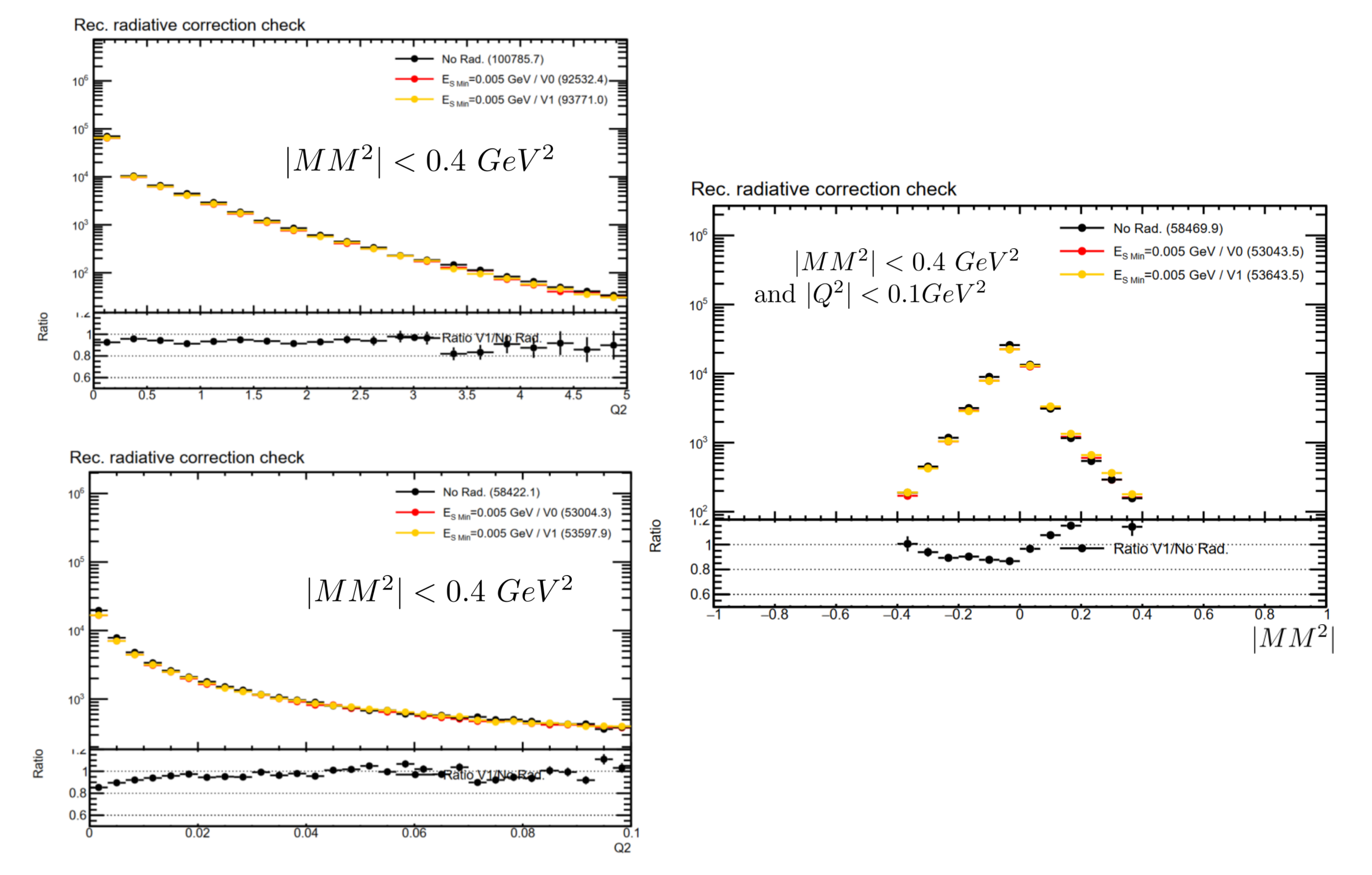
Effect on Bethe-Heilter simulation (Generated) (II)



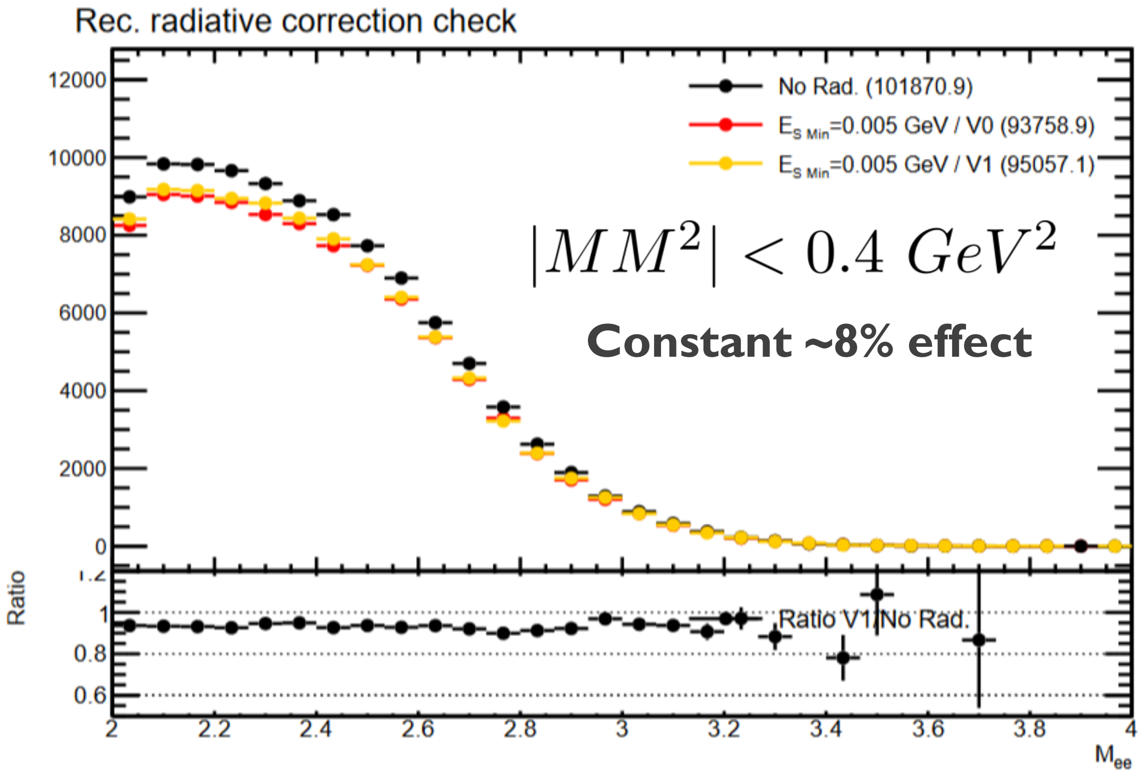
Effect on Bethe-Heilter simulation (Reconstructed) (I)

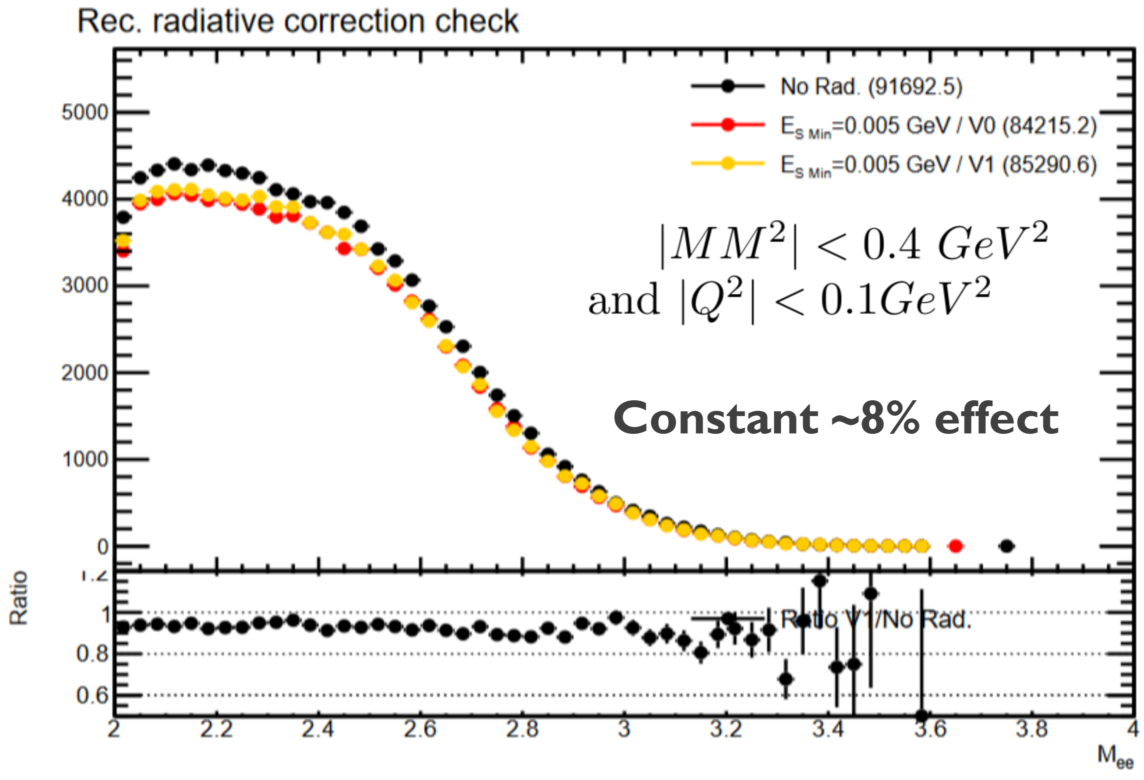
- No significant difference seen between both emission algorithm
- The Q^2 resolution seems mostly driven by detector resolution
- Larger effect on the missing mass

Reconstructed exclusivity variables



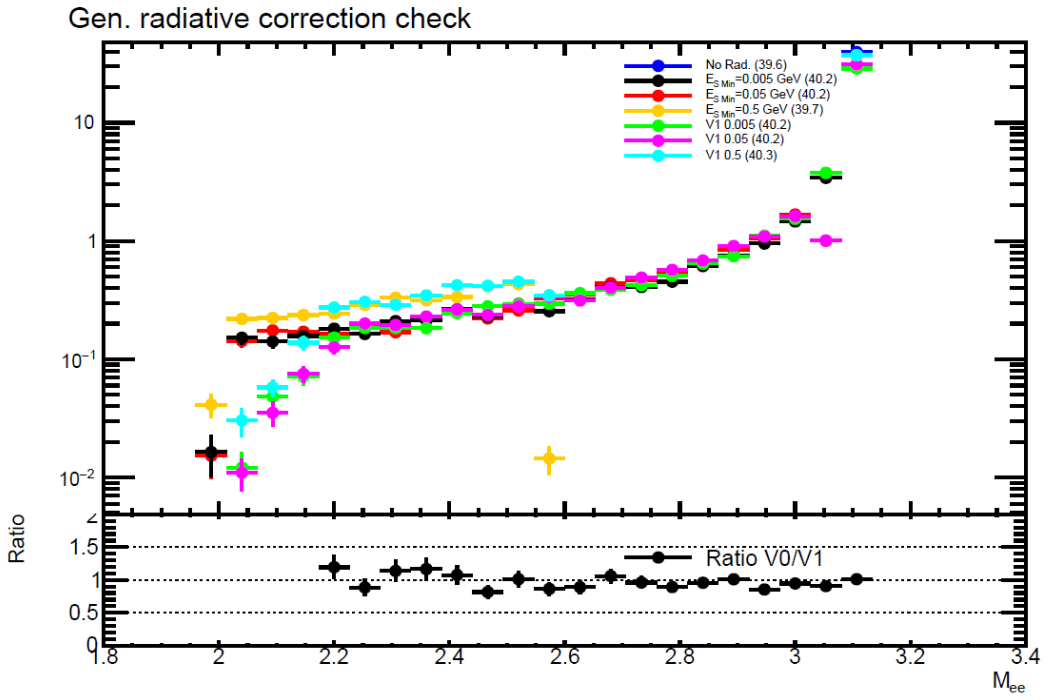
Effect on Bethe-Heilter simulation (Reconstructed) (II)

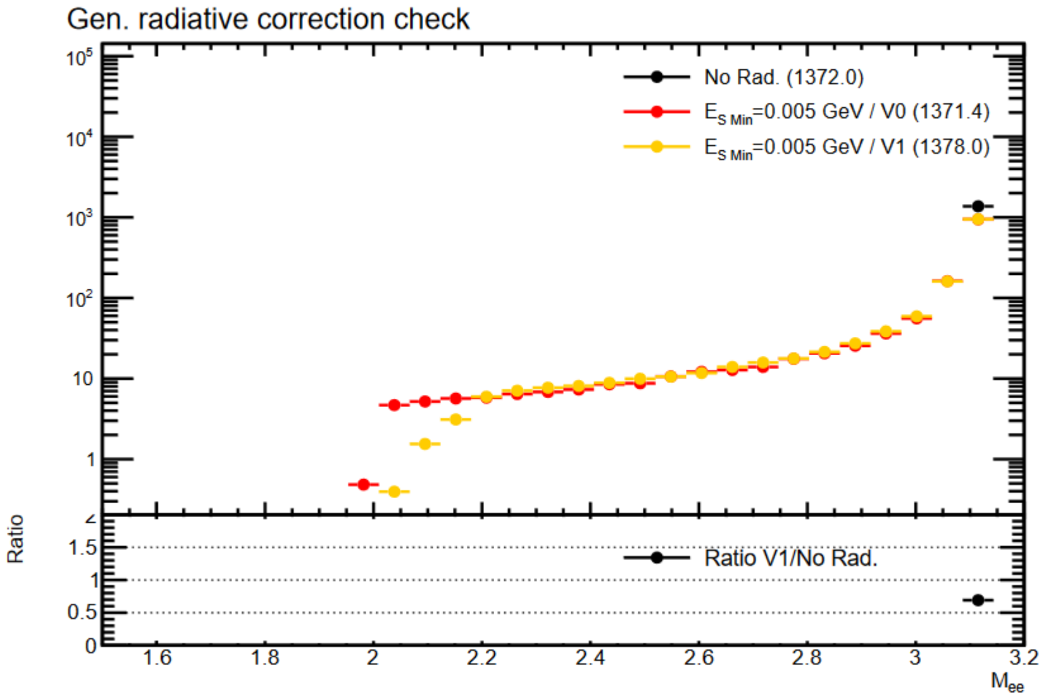




Effect on J/ψ simulation (Generated events, I)

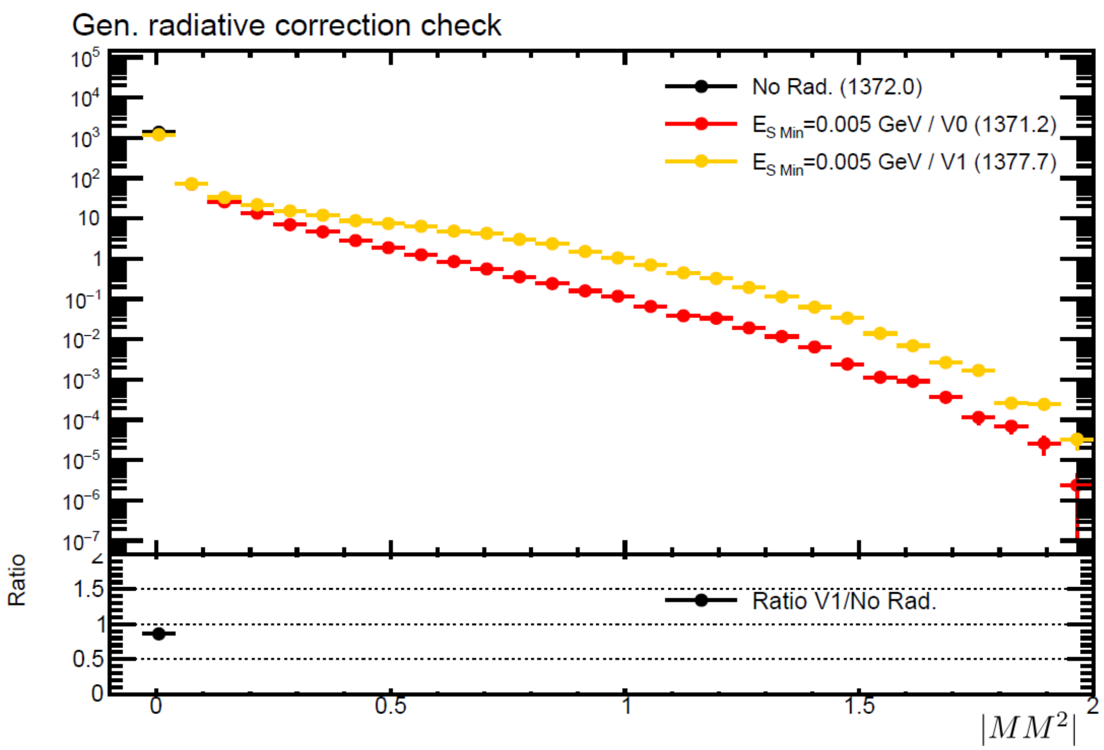
- Assuming the formulae of Matthias Heller, et al. holds for J/ψ photoproduction, one can apply the same algorithm
- While the loop diagram will be different, it seems reasonable to assume the dependence in emitted photon energy holds…
- This needs to be confirm with the authors

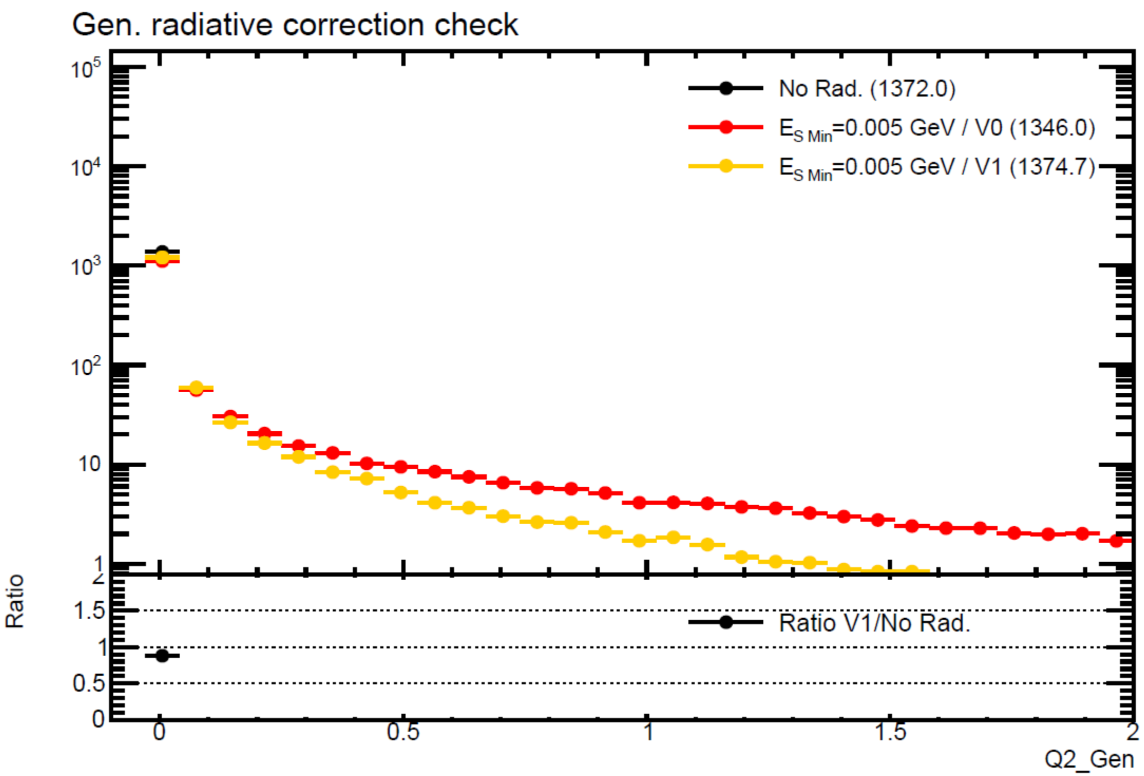




Effect on J/ψ simulation (Generated events, II)

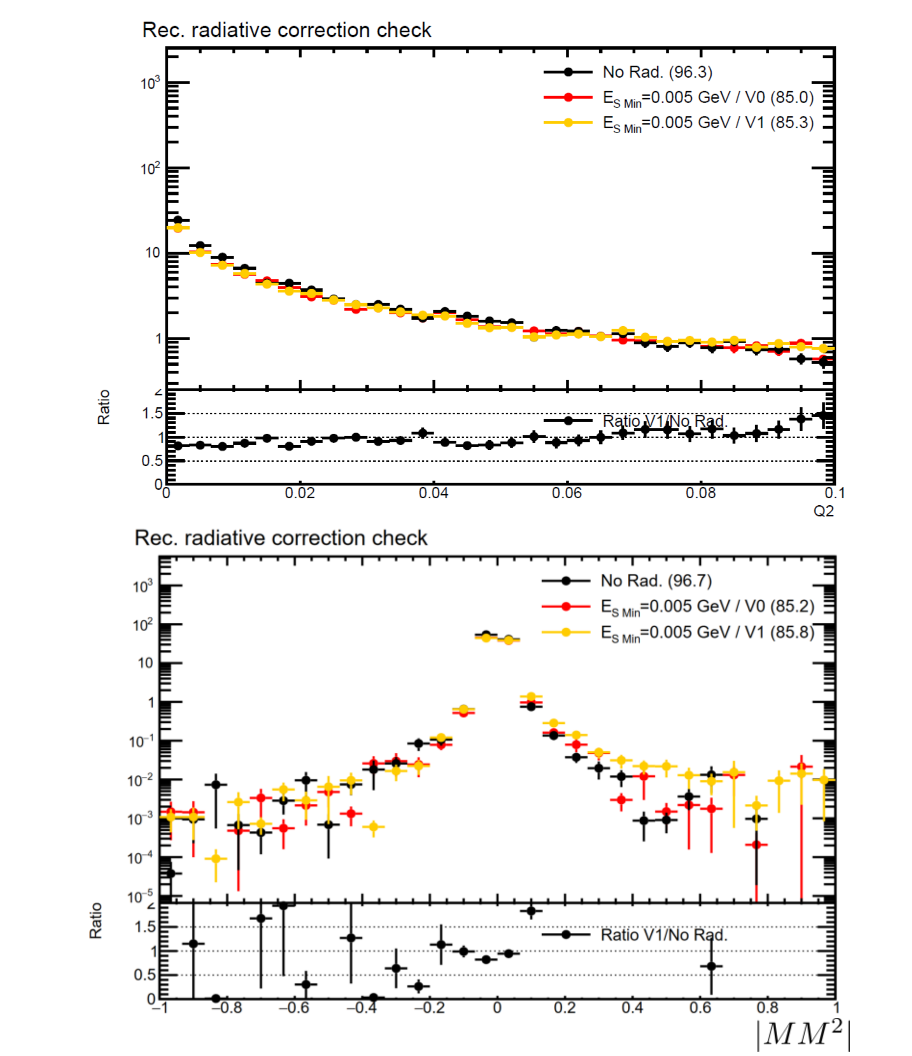
Generated exclusivity variables

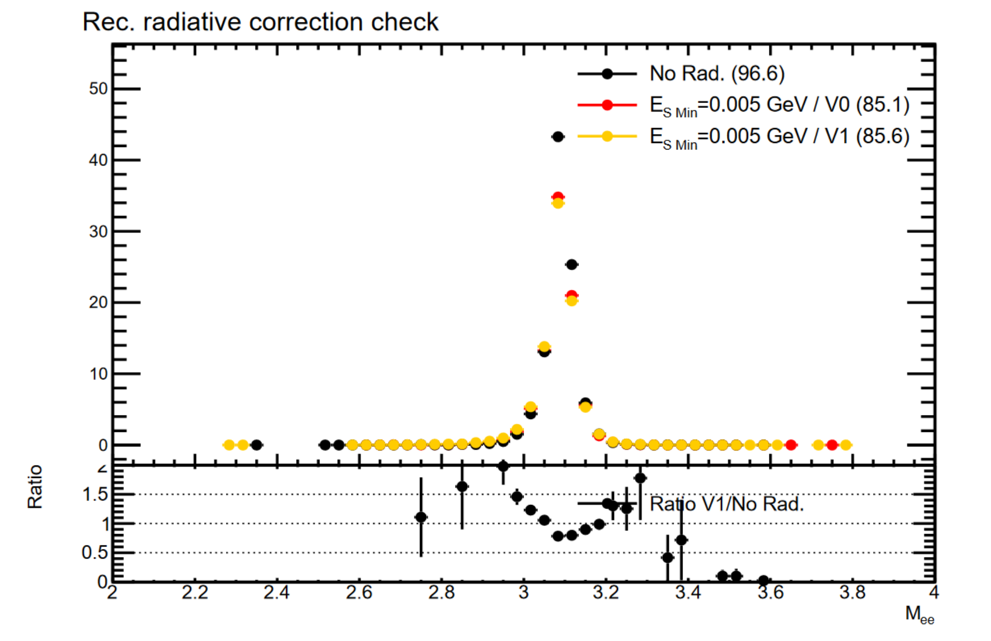




Effect on J/ψ simulation (Reconstructed events)

Reconstructed exclusivity variables





- 12% decrease in number of counts
- Larger width

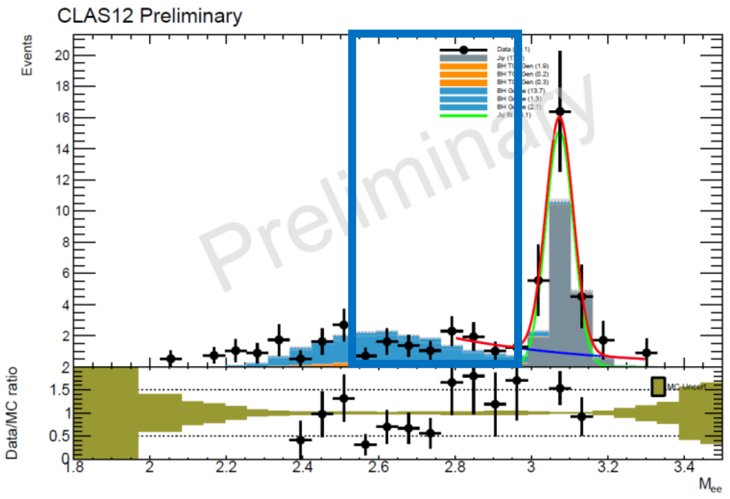
Effect on CS extraction

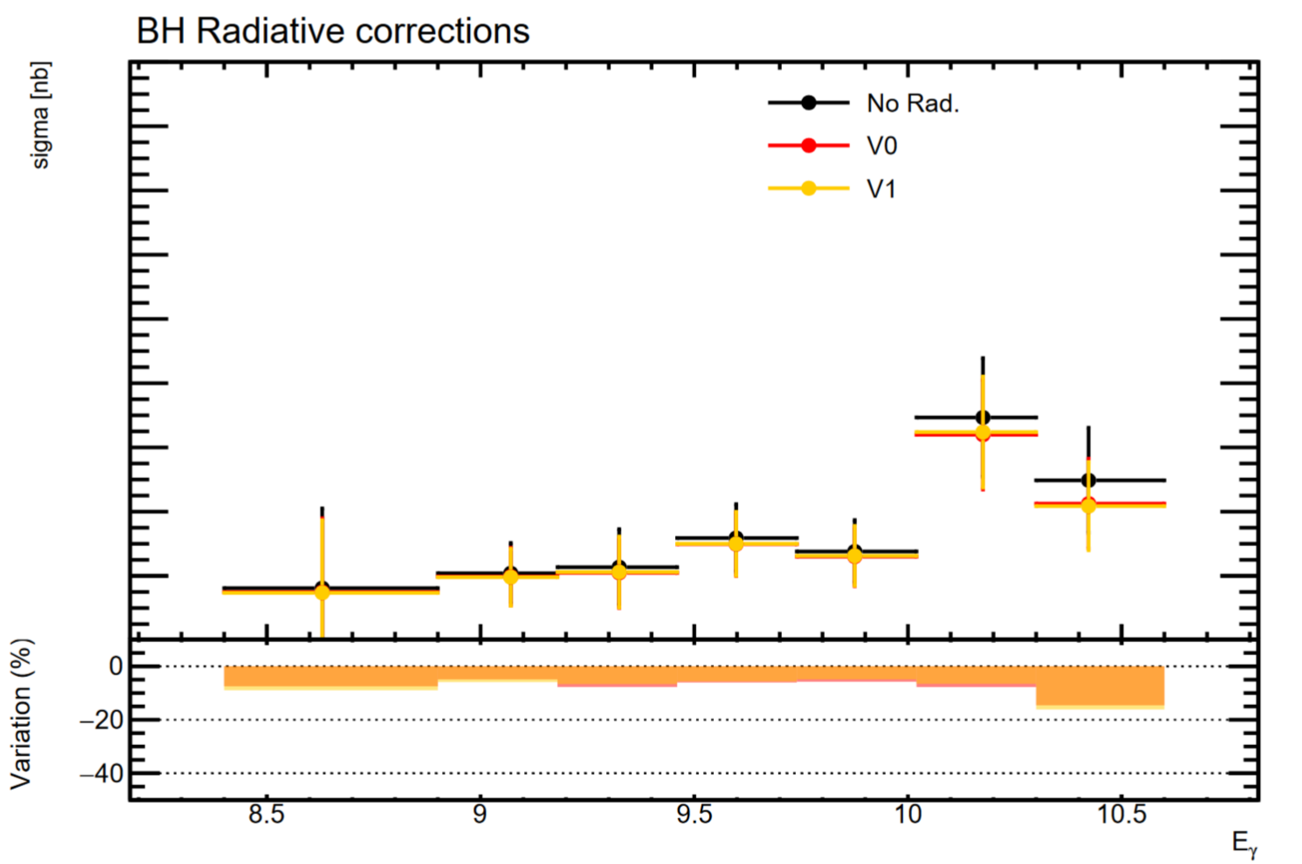
The radiative corrections are expected to play a role in two key ingredient of the cross-section calculation:

- In the acceptance: J/Psi simulation with radiated correction used to account for larger peak width.
- In the normalization factor: Grape generated events are passed through the radiative correction algorithm. Expect to be 10% effect.

Cross-section calculations

$$\sigma_0(E_\gamma) = \frac{N_{J/\psi}}{N_\gamma \cdot n_T \cdot \omega_c \cdot Br \cdot \epsilon(E_\gamma)}$$



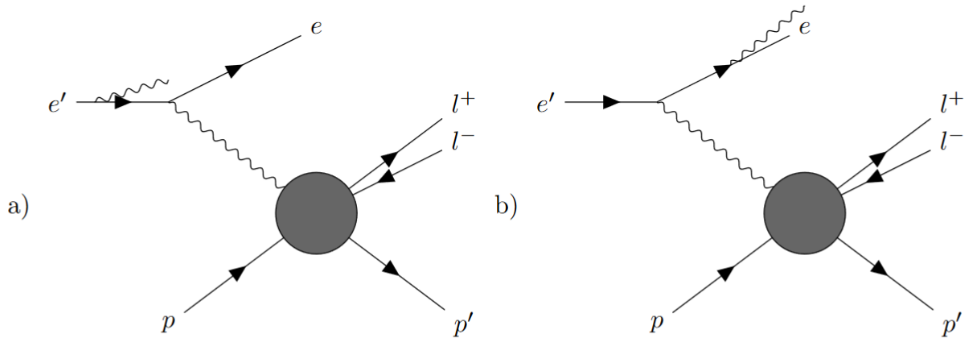


Summary and outlook

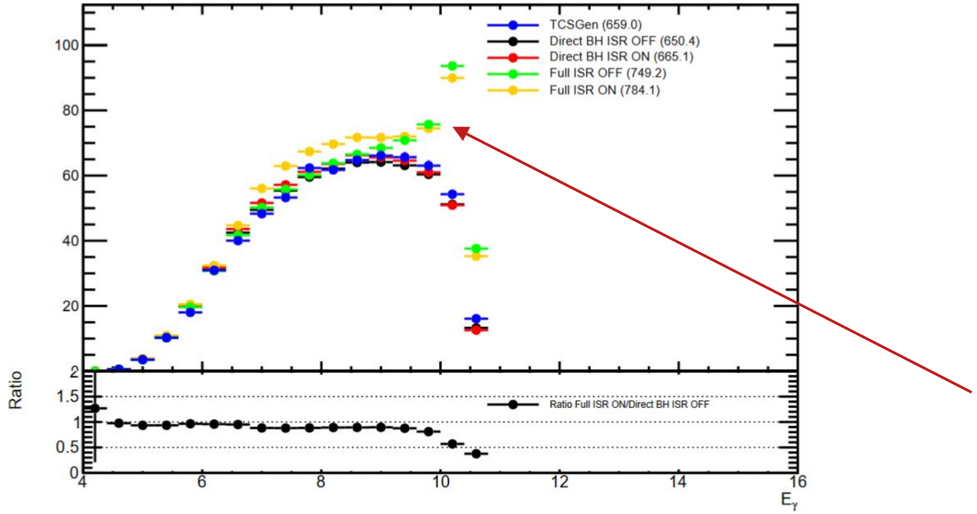
- Based on published work (Matthias Heller, Oleksandr Tomalak, and Marc Vanderhaeghen. Soft-photon corrections to the bethe-heitler process in the $\gamma p \to l^+l^-p$ reaction), a framework to include radiative corrections in dilepton final state processes have been developed.
- The effect is of the order of 8% over the whole invariant mass range of interest. This is consistent with GlueX findings.
- We will contact the authors for further discussion on validation and for validity/implementation of the algorithm for J/ψ photoproduction.
- Cross-check to be done with the GlueX approach using PHOTOS (E. Barberio et al., Comput. Phys. Commun. 79, 291 (1994)).

Back-ups

Initial state radiation

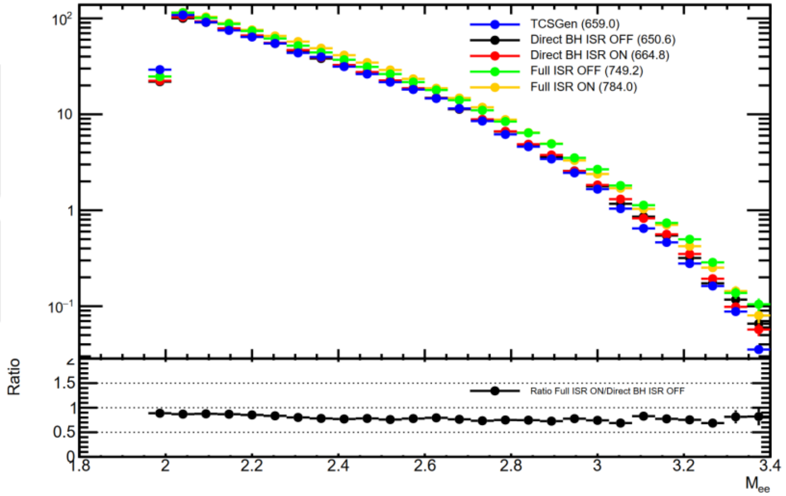


CLAS12 Preliminary - Generator Check



From “trident” diagram

CLAS12 Preliminary - Generator Check



CLAS12 Preliminary - Generator Check