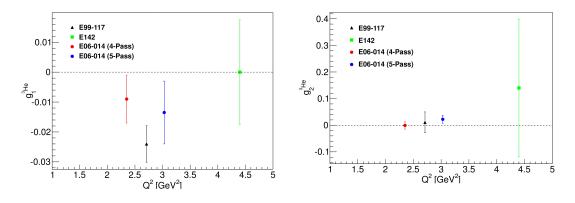
## **8.4.3** $Q^2$ Dependence

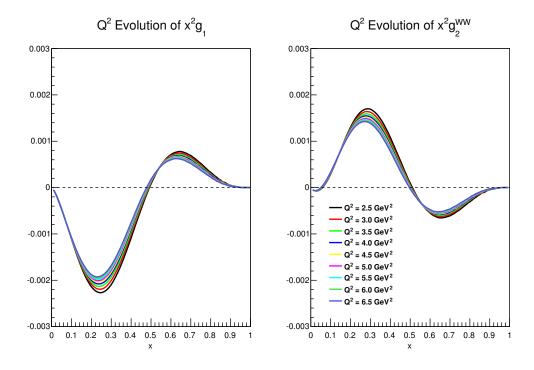
The spin structure functions  $g_1$  and  $g_2$  in general depend on  $Q^2$ , which result in  $d_2$  also depending on  $Q^2$ . Thus a proper evaluation of  $d_2$  should be performed by integrating the spin structure functions over the entire x range at a constant  $Q^2$  value. However, E06-014 only took data at two beam energies, which made a proper interpolation to constant  $Q^2$  impossible. As a result, three approaches were used to estimate the size of the polarized structure functions'  $Q^2$  dependence; which in all cases was found to be small relative to the measured precision of  $g_1$  and  $g_2$ .

One way in which to gauge the polarized structure functions' dependence on  $Q^2$  is to compare their measured values at fixed x. Using the  $^3$ He data from E06-014's two beam energies (E = 4.74 GeV and E = 5.89 GeV), E99-117 [3, 57], and E142 [50] at  $\langle x \rangle = 0.33$ ,  $g_1$  and  $g_2$  were plotted against  $Q^2$  (Figure 8.70). Taking into account the precision of the measurements in Figure 8.70, the  $Q^2$  dependence appears to be minimal.

In addition to comparing with experimental data, one could also use models and fits to world data to investigate the  $Q^2$  dependence of the polarized structure functions. The global analysis fits to polarized parton densities from DSSV [164] were used to evaluate



**Figure 8.70:** Polarized structure functions  $g_1$  (left panel) and  $g_2$  (right panel) on  $^3$ He at mean x of 0.33 as a function of  $Q^2$ .



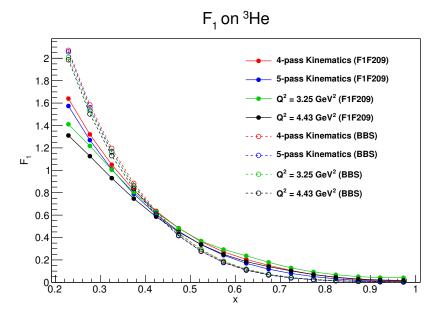
**Figure 8.71:** Polarized structure functions  $x^2g_1$  (left panel) and  $x^2g_2^{WW}$  (right panel) on  $^3{\rm He}$  evaluated from DSSV [164] plotted against x for a range of  $Q^2$  values.

 $g_1$  and  $g_2^{WW}$  <sup>20</sup> for a range of fixed  $Q^2$  values<sup>21</sup>. Figure 8.71 shows  $x^2g_1$  and  $x^2g_2^{WW}$  on <sup>3</sup>He as a function of x at constant  $Q^2$  values ranging from  $Q^2$  = 2.5 to 6.5 GeV<sup>2</sup>. The  $Q^2$  dependence seen in DSSV is largest around x = 0.25 and x = 0.65. Although, compared to the precision of the  $g_1$  and  $g_2$  measured by E60-014, the  $Q^2$  variation is small.

The final method used to assess the  $Q^2$  dependence, also used by SLAC E143 [51], is to assume that  $g_1/F_1$  is  $Q^2$  independent. Then  $g_1$  at a fixed  $Q^2$  value  $(Q_0^2)$  can be determined as

 $<sup>\</sup>overline{\phantom{a}}^{20}g_1$  is formed from polarized quark distributions, whereas  $g_2$  does not have a simple parton interpretation and can not be formed from the polarized quark distributions. As a result  $g_1$  is used to calculate  $g_2^{WW}$  rather than  $g_2$ .

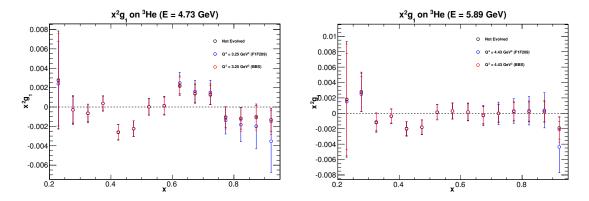
<sup>&</sup>lt;sup>21</sup>Several other global analyses (BBS [165, 166], LSS [167], DNS [168], and GS [169]) were also checked and found to give similar results as DSSV.



**Figure 8.72:** Unpolarized structure function  $F_1$  evaluated using F1F209 [163] and BBS [165, 166] for  $Q^2$  values matching measured E06-014 kinematics and for two constant  $Q^2$  values on  $^3$ He

 $g_1(x, Q_0^2) = \frac{g_1(x, Q^2)}{F_1(x, Q^2)} \cdot F_1(x, Q_0^2).$  (8.65)

The  $^3$ He polarized structure function  $g_1$   $(x,Q^2)$  are the measured E06-014 values. The  $^3$ He unpolarized structure function  $F_1$ , was computed at  $Q^2$  values matching the measured  $g_1$   $(x,Q^2)$  data, in addition to  $F_1$  at  $Q_0^2=3.25$  and 4.43 GeV $^2$ . To evaluate  $F_1$  two different global fits, F1F209 [163] and BBS [165, 166] were used. F1F209 uses fits to world data to determine  $F_1$  and  $F_2$ , and BBS uses a statistical quark model to determine polarized and non-polarized parton distributions. Figure 8.72 shows the results of  $F_1$  as a function of x for both the F1F209 (solid circles, solid lines) and BBS (open circles, dashed lines) fits. The red and blue markers give the  $F_1$  values for x and x values matching E06-014's x E = 4.74 and 5.89 GeV E06-014 datasets. The green and black markers show x values calculated at a constant x of 3.25 and 4.43 GeV $^2$ .



**Figure 8.73:** Compares the evolved  $x^2g_1$  from F1F209 [163] and BBS [165, 166] fits to the measured E06-014  $x^2g_1$  on <sup>3</sup>He for E = 4.74 and 5.89 GeV data sets.

Applying the values of  $F_1$  to the measured  $g_1$ , following Equation 8.65,  $g_1$  can be evolved to a constant  $Q^2$ . A comparison of the evolved  $x^2g_1$  data using F1F209 (blue markers) and BBS (red markers) to the measured  $x^2g_1$  (black marker) can be seen in Figure 8.73, which clearly show a negligible  $Q^2$  dependence within the experimental precision. If one assumes that the  $Q^2$  dependence of  $g_2$  is similar to that of  $g_1$ , then the  $Q^2$  dependence on  $g_2$  is also small.

Considering the size of the  $Q^2$  dependence from the three methods presented above, the  $Q^2$  dependence was neglected in E06-014, and a mean  $Q^2$  value was used when computing  $d_2$  and other  $Q^2$  dependent quantities. While the quantities were not computed at a constant  $Q^2$  for E06-014, the upcoming JLab experiment E12-06-121 [170, 171] will be able to compute the spin structure functions and  $d_2^n$  at constant  $Q^2$ , in addition to providing an extension of the E06-014 measured  $d_2^n$  to higher  $Q^2$  values.

## 8.5 Radiative Corrections

An electron's interaction with materials before and after scattering, as well as with the target itself, will cause it to loose energy. These interactions lead to an alteration of the electron's