Cryogenic Target Thickness Study for EG2

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For the upcoming EG2 experiment, we aim in knowing the cryogenic target thickness to 1% level. In this document we list the error sources of the target thickness and what we have already achieved. For some of them we also give a plan for possible improvement.

1 Overview

The target thickness is product of its density and the target cell length. We calculate the density from both the equation of state and fit to data. The cell length is given by an absolute measurement at room temperature, the thermal contraction of Kapton cell wall from room to operating temperature and the deformation of cell end windows. The bubbling at cell end windows will affect density directly and we propose a method to stop the bubbles from forming.

2 Deuterium Density under Operating Conditions

We use two methods to obtain the deuterium density under operating conditions $P = 1.300$ atm and $T = 22.00$ K. In the first method, the deuterium equation of state is solved and we denote ρ_{equ} its solution. In the second method we perform a two-dimensional fit to liquid state data [1] and interpolate to (P, T). We denote ρ_{fit} the result in this approach. The $P-$ and $T-$ dependence of density, $\partial\rho/\partial P$ and $\partial\rho/\partial T$ can be obtained in both methods. The results ρ_{equ} and ρ_{fit} are very close $(\rho_{equ} - \rho_{fit})/\rho_{fit} \approx 0.01\%$ and is much less than the typical uncertainty $d\rho/\rho < 0.15\%$ of data used in the fit. Details of the calculation and the consistency of the two methods are given in Appendix A.1.

We apply a 0.15% intrinsic uncertainty due to the error in density for the available data set. The temperature sensors that we are using have an advertised calibrated accuracy of ± 50 mK but we record the temperatures to 2 decimal places, therefore the uncertainty in sensor reading is $\Delta T = 0.01$ K. We estimate the difference between the LD_2 temperature and the sensor reading to be 0.2 K (arbitrary value), the total uncertainty in temperature is then $\Delta T = 0.21$ K. The total maximum accuracy specification for the pressure gauge is 0.50% of the the reading and the gauge is calibrated periodically to re-zero the gauge using vacuum. We estimate the uncertainty in pressure due to non-zero offset to be 50 mbar, hence the total uncertainty in pressure is $0.50\% \times 1.300$ atm+50 mbar= 0.056 atm. The uncertainty of density due to temperature and pressure is dominated by the former one. Results of deuterium density and its uncertainty from both methods are given in Table 1

Table 1: Liquid deuterium density at $P = 1.300 \pm 0.056$ atm, $T = 22.00 \pm 0.21$ K and its $P-$, $T-$dependence from equation of state and from fits to data. The uncertainty in density includes those from equation of state (or fits to data) and the uncertainties in temperature and pressure.

Method	ρ (g/cm^3)	$\partial\rho/\partial T$ (g/cm^3/K)	$\partial\rho/\partial P$ (g/cm^3/mbar)
Eq. of State	0.16699 ± 0.00049	-0.002324	0.182224
Fit to NBS data	0.16686 ± 0.00050	-0.002367	0.006184

3   Bubbling at End-caps

Bubbling at the end-caps will be a problem since the gas density inside bubbles are negligible compared to liquid density. The bubbles will get stuck at the location of the entrance window and the vertical height of the bubble is a function of temperature. At saturated condition the height is about 1/2 the diameter of the cell. This would cause the amount of D_2 gas in the beam to be a function of time as the bubbles pulse out of the cell. The bubbles typically form in a continuous stream at the downstream window, starting at about 250 microns diameter. They collect at the top of the end cap to form ≈ 1 mm diameter bubbles that float up the walls of the cone at a rate of ≈ 4/sec. Assuming only single layer of bubbles are formed, they will cause $250 \, \mu$m/2 cm$= 1.25\%$ uncertainty in the target thickness, which is not acceptable for EG2.

There are tests performed on LH_2 sample at JLab [2] which showed that one can stop all the bubbles from forming by adding super insulation and setting the condenser temperature to certain values. More specifically, for LH_2:

- 0 layers: 15.00 K;

- 5 layers: 18.25 K;

- 15 layers: 18.60 K.

I am waiting for similar data for LD_2. Naively one expects the conditions for LD_2 to be similar to that for LH_2. For now I assume with super insulation a 0.5% uncertainty in target thickness due to occasional bubbling.

4   Absolute Measurement of Cell Length

The optical measurements made on the target cell length has an uncertainty of $\approx 80 \, \mu$m. This gives a 0.4% uncertainty in the cell thickness.

5   Cell Thermal Contraction

The wall of EG2 target cell is made of Kapton. The thermal contraction of cell wall will affect the target length directly. We have two sets of data for coefficient of Kapton linear thermal expansion

which differ by 50%. The measurement performed at JLab [2] was performed on a 5-mil thick 10×1 cm Kapton HN film sample and gives a shrinkage of 0.691% from 300K to 78K. The coefficient of linear expansion is therefore 31.13 ppm/K and the shrinkage from 300K to 20K is estimated to be 0.757%. The second measurement is provided by Dupont [3] which gives 0.470% and 0.495% shrinkage for two 3-mil thick HN film samples within the same temperature range. The coefficient is then 21.4 ppm/K and 22.2 ppm/K for the two samples, respectively.

We don't have an estimate of the Dupont measurement, but the JLab crew performed similar measurements for Teflon and Nylon and the results agree with the published data at a level of -8.10% and 11.95%, respectively. If we take the average of JLab and Dupont measurements as the central value and the difference of the two the full uncertainty, the thermal contraction of EG2 target cell wall will be (26.45 ± 4.5) ppm/K$\times(300-20)$ K$= (0.735 \pm 0.13)\%$. This contributes 0.13% to the total uncertainty of target thickness.

6   End-cap Deformation

Tests have been done at JLab to measure the end-cap bulging at 0.5 atm. Averaging all three cells that were measured, the elongigation of the cell due to end-cap deformation is 2×0.14 mm/20 mm/$=1.4\%$ (0.14 mm for each end-cap). Scaling by pressure, we expect a 3.64% change in the cell length at operating pressure 1.300 atm. Assuming a 10% error in this value (corresponds to 15 μm error of the measurement), it contributes a 0.4% uncertainty to the target thickness.

We are planning to do similar measurements at six different pressures from 1.0 to 1.5 atm with 0.1 atm spacing.

7   Summary

Combing all error sources, the total uncertainty in target thickness is 0.84%. However we need to confirm that the super insulation technique works for LD_2 and measure the end-cap deformation up to operating pressure.

A   Deuterium Density from Equation of State and from Fit to Data

A.1   Equation of State for Deuterium

Knowing the temperature and pressure of deuterium, its density can be determined by solving the following Eq. of State, which works for both liquid and vapor states [1].

$$
\begin{aligned}
P = \rho RT \quad &+ \quad (n_1 T + n_2 + n_3/T^2 + n_4/T^4 + n_5/T^5)\rho^2 \\
&+ \quad (n_6 T^2 + n_7 T + n_8 + n_9/T + n_{10}/T^2)\rho^3 \\
&+ \quad (n_{11}T + n_{12})\rho^4 + (n_{13} + n_{14}/T)\rho^5 \\
&+ \quad \rho^3(n_{15}/T^2 + n_{16}/T^3 + n_{17}/T^4)\exp(n_{24}\rho^3) \\
&+ \quad \rho^5(n_{18}/T^2 + n_{19}/T^3 + n_{20}/T^4)\exp(n_{24}\rho^3) \\
&+ \quad \rho^7(n_{21}/T^2 + n_{22}/T^3 + n_{23}/T^4)\exp(n_{24}\rho^3)
\end{aligned}
\tag{1}
$$

Equation (1) represents the $P - \rho - T$ data with good accuracy within the entire temperature and pressure ranges, both in the liquid and vapor phases. It satisfies the critical point values $P = 16.432$ atm, $\rho = 17.328$ g mol/liter and $T = 38.34$ K, where 1 g mol deuterium $= 4.02820$ g based on the ^{12}C$= 12.000$ scale. In addition, the derivatives of this equation are adequate for the calculation of the thermodynamic properties, enthalpy, and entropy. The coefficients are given in Table 2.

Table 2: Coefficients for Deuterium Equation of State 1.

parameter	value	error
R	0.0820535	
n_1	$1.7402845195 \times 10^{-3}$	1.02×10^{-5}
n_2	$-1.7793868011 \times 10^{-1}$	2.90×10^{-3}
n_3	$-2.4001560074 \times 10^{+2}$	$1.51 \times 10^{+1}$
n_4	$8.0781369940 \times 10^{+4}$	$1.32 \times 10^{+4}$
n_5	$9.0331481535 \times 10^{+5}$	$3.90 \times 10^{+5}$
n_6	$-6.2073719586 \times 10^{-8}$	8.17×10^{-9}
n_7	$3.9710560252 \times 10^{-5}$	6.42×10^{-6}
n_8	$4.0521016760 \times 10^{-3}$	1.71×10^{-3}
n_9	$-1.1448326042 \times 10^{-1}$	3.56×10^{-1}
n_{10}	$-1.5518933218 \times 10^{+1}$	$5.52 \times 10^{+0}$
n_{11}	$5.5243829784 \times 10^{-7}$	2.04×10^{-7}
n_{12}	$-1.8459331814 \times 10^{-4}$	7.25×10^{-5}
n_{13}	$2.7872915743 \times 10^{-6}$	1.20×10^{-6}
n_{14}	$2.6676069525 \times 10^{-4}$	1.56×10^{-4}
n_{15}	$-4.6440039399 \times 10^{+1}$	$4.40 \times 10^{+1}$
n_{16}	$5.0142509504 \times 10^{+3}$	$1.81 \times 10^{+3}$
n_{17}	$-8.6823429312 \times 10^{+4}$	$2.91 \times 10^{+4}$
n_{18}	$8.2905182310 \times 10^{-2}$	1.91×10^{-2}
n_{19}	$-3.8202148461 \times 10^{+0}$	$1.22 \times 10^{+0}$
n_{20}	$5.5751492076 \times 10^{+1}$	$3.18 \times 10^{+1}$
n_{21}	$-4.0191132239 \times 10^{-5}$	1.15×10^{-5}
n_{22}	$2.7207301373 \times 10^{-3}$	4.74×10^{-4}
n_{23}	$-2.8208998864 \times 10^{-2}$	1.06×10^{-2}
n_{24}	-1.4670×10^{-3}	$-$

A numerical method was used to solve the equation. For certain value set of (P, T), Eq. (1) has two solutions which satisfy $\partial \rho / \partial P < 0$, the smaller solution of ρ corresponds to gas state while the larger value is for liquid state.

A.2 Fit to Data

In the second method, a two-dimensional fit was used to approximate data and to calculate density for certain values of (P, T). Data of constant pressure are firstly fit using a linear function of temperature and the density at operating temperature is calculated for different pressures. Then the calculated density are fit as a linear function of pressure and the density at operating (P, T)

is calculated.

A.3 Comparison of Two Methods

In Fig. 1 we present the fitted density as a function of T for different pressure from 0.4 atm to

Figure 1: Density as a function of T for constant pressure from deuterium equation of state (blue curve and blue open circle). Also shown are data points from NBS report [1] (red solid circle) and a linear fit of density as function of temperature (green line). The NBS data points are printed with poor quality in the reference and *could be wrong*, this means that the values calculated from equation of state are more reliable :).
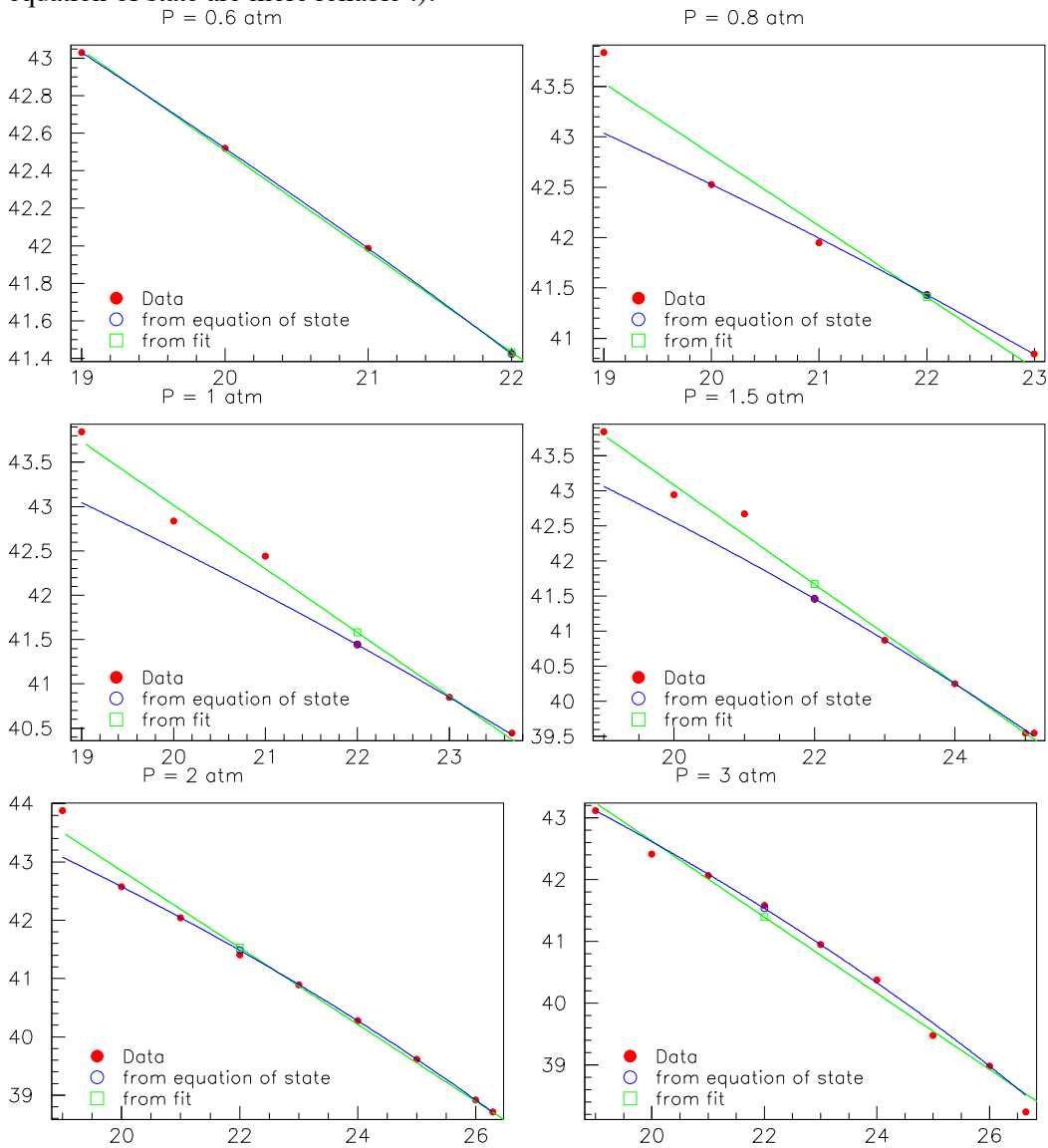
3.0 atm. In each plot the data point with the highest temperature corresponds to phase change.

Also shown is the density from equation of state. Fig. 2 shows the calculated density as a function of P.

Figure 2: Density as function of P at constant T from equation of state (blue open squares) and from fit to data (red solid circles). Also shown are fits of density as a linear function of P and the calculated density at operating conditions $P = 1.300$ atm, $T = 22.00$ K from equation of state (blue line and blue solid star) and from fit to data (red line and red open star).
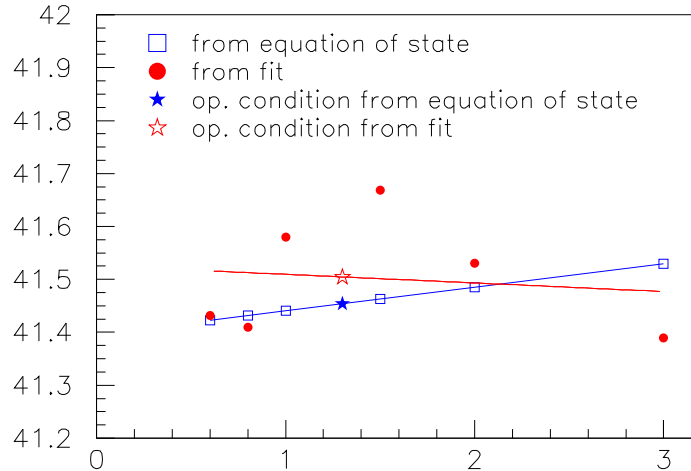
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References

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