

The ${}^4\text{He}$ nucleus as a spin analyzer of meson resonances

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(Submitted June 10, 1975)

Yad. Fiz. 23, 1085-1092 (May 1976)

A possible procedure is discussed for solving some problems of meson spectroscopy in the reactions $\gamma {}^4\text{He} \rightarrow M^* {}^4\text{He}$ and $\pi^* {}^4\text{He} \rightarrow M^* {}^4\text{He}$. The alignment that is provided by the zero spin of the ${}^4\text{He}$ nucleus, as well as other features of these reactions, can be used effectively to determine J^P of meson resonances. Adair distributions are plotted and analyzed in detail for a few concrete examples.

PACS numbers: 25.10.+s, 14.40.-n

1. At the present time it is difficult to determine uniquely the spins and parities of certain meson resonances (see, for example, the Particle Data Group report^[1] and the reviews^[2]). It is especially difficult to distinguish between different hypotheses within a single J^P series. The $X^0(960)$ meson is an example. Despite considerable recent experimental efforts,^[3] it remains difficult to distinguish between $J^P(X^0) = 0^-$ and 2^- , although the observed^[4] anisotropies in the Adair distributions^[5,6] favor $J^P = 2^-$. Here the basic causes of the difficulties are especially clear. In these cases forbiddennesses are indistinguishable or altogether absent from the Dalitz plot, and the analysis of the combined production and decay mechanism requires that events be selected rigorously according to Adair's condition so that spin anisotropies can be manifested.^[1]

In the present paper it is shown that the reactions

$$\gamma {}^4\text{He} \rightarrow M^* {}^4\text{He}, \quad (1)$$

$$\pi^* {}^4\text{He} \rightarrow M^* {}^4\text{He} \quad (2)$$

in which mesons are produced on helium nuclei possess properties that can be utilized to determine the quantum numbers of the resonances, and some concrete examples are discussed.

A. The zero spin of the ${}^4\text{He}$ nucleus provides the maximum degree of alignment in reactions (1) and (2): In the reaction $\gamma {}^4\text{He} \rightarrow M^* {}^4\text{He}$ forward the projections $0, \pm 2, \dots, \pm J$ of the meson spin on the beam momentum direction are forbidden; in $\pi^* {}^4\text{He} \rightarrow M^* {}^4\text{He}$ forward the projections $\pm 1, \pm 2, \dots, \pm J$ are correspondingly forbidden. The alignment leads to the fact that anisotropies in the Adair distributions for particles with $J \neq 0$ cannot fail to be manifested. In (1) and (2) there is forward alignment even for resonances with $J = 1$, which cannot be achieved in $\pi N \rightarrow MN$ reactions, for example. This makes the reactions (1) and (2) useful whenever it is necessary to confirm or exclude spin 1 (in the cases of E , D , and B mesons, for example).

B. Since ${}^4\text{He}$ has zero spin, only a single amplitude of resonance production at small angles remains in (1) and (2). Then the corresponding Adair distributions are determined only by the decay mechanism and do not contain unknown parameters—the elements ρ_{mn} of the resonance spin density matrix. These Adair distributions are more limited and unambiguous than in

$\pi N \rightarrow MN$ reactions, for example, and they can serve as a good additional means to determine the form of the decay matrix element.^[2]

C. The differential cross section for the reaction $\gamma {}^4\text{He} \rightarrow M^0 {}^4\text{He}$ with $J^P(M^0) = 0^-$ vanishes when $\theta_{c.m.s.} \rightarrow 0$ [$d\sigma/d\Omega F^2(t) \sim \sin^2 \theta_{c.m.s.}$], where $\theta_{c.m.s.}$ is the c.m.s. angle of meson production and $F(t)$ is the nuclear form factor; for a meson with spin $J \neq 0$ the cross section does not vanish as $\theta_{c.m.s.} \rightarrow 0$. The behavior of the differential cross section near $\theta_{c.m.s.} = 0$ can therefore possibly be used to distinguish between the hypotheses^[8] $J \neq 0$ and $J^P = 0^-$ (for η , X^0 , and E mesons).^[3]

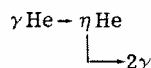
D. In order to provide alignment (and therefore anisotropies in the Adair distributions) it is necessary to select events at small angles according to the condition for applying Adair's analysis^[9]:

$$\theta_A \leq 1/Rk, \quad (3)$$

where θ_A is the c.m.s. angle of production, k is the c.m.s. momentum of the resonance, and R is the interaction radius. The nuclear form factor of ${}^4\text{He}$ [$F(t) = e^{-k^2 t/4}$, $R = 1.37 \text{ F}$] selects events in (1) and (2) at small angles ($t_{\text{nucl}} \leq 1/R^2$), and Adair's condition (3) can be fulfilled in the coherence region for a large number of events. We note that, in the reaction $K^- p \rightarrow X^0 \Lambda$, for the purpose of performing Adair's analysis the total level of statistics had to be reduced by a factor ~ 10 .^[4]

We shall now discuss certain concrete examples in greater detail.

2. $\gamma + \text{He} \rightarrow \eta + \text{He}$. As noted by Buyak *et al.*,^[10] measurement of the differential cross section for $\gamma {}^4\text{He} \rightarrow \eta {}^4\text{He}$ at small angles $\theta_{c.m.s.}$ is an elegant means of proving that the $\eta(550)$ meson is a pseudoscalar particle. The hypothesis $J^P = 2^-$ for $\eta(550)$ is usually excluded by the analysis of the Dalitz plot in $\eta \rightarrow 3\pi$ decay for very large statistics. In the reaction

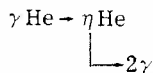


there is a simple possibility of ruling out the hypothesis $J^P = 2^-$ according to the $\eta \rightarrow 2\gamma$ decay, by studying the Adair distribution with respect to the angle θ between the beam momentum and the direction of a γ quantum from the $\eta \rightarrow 2\gamma$ decay in the η -meson rest system. The

Adair distribution of $\eta \rightarrow 2\gamma$ decays for $J^P(\eta) = 2^-$ exhibits the steep angular dependence

$$W(\theta) = \frac{15}{16} \sin^2 2\theta. \quad (4)$$

This formula does not contain unknown parameters characterizing the mechanism of η -meson decay and production, since $\eta \rightarrow 2\gamma$ decay for $J^P(\eta) = 2^-$ and the reaction $\gamma \text{He} \rightarrow \eta \text{He}$ forward are described by a single amplitude. The Adair distribution is isotropic for $J^P = 0^-$. Thus by studying experimentally the behavior of the differential cross section at small angles and of the Adair distribution in



we are able to prove on the basis of small statistics (~ 200 – 300 events) that the η meson is a pseudoscalar particle. We note that the Adair distributions do not depend on the initial beam energy (E_{lab}) and that events at different values of E_{lab} can be summed to improve the statistical reliability of the result.

3. $\gamma \text{He} \rightarrow X^0(960) \text{He}$. The possibility of distinguishing between 0^- and 2^- for $X^0(960)$ from the behavior of the differential cross section was pointed out previously.^[8] We also show here the Adair distributions for the decays $X^0 \rightarrow \rho\gamma$, $X^0 \rightarrow \eta\pi^+\pi^-$, and $X^0 \rightarrow 2\gamma$ with $J^P(X^0) = 2^-$. With $J^P(X^0) = 0^-$ the Adair distributions with respect to the variable $x = \cos\theta$ are isotropic for all the decay modes. For all the X^0 decay modes the Adair distributions do not depend on the production mechanism and are determined only by the decay matrix element; for $X^0 \rightarrow 2\gamma$ they are determined uniquely by (4).

$X^0 \rightarrow \eta\pi^+\pi^-$ decay. For our calculation we used the decay matrix element representing a mixture of amplitudes with $l_\eta = 2$, $l_{\pi\pi} = 0$ and $l_\eta = 0$, $l_{\pi\pi} = 2$, and with the complex mixing parameter w :

$$M_{\lambda} = (q, q_i + wd, d) X_{\lambda i}, \quad (5)$$

where q is the η momentum in the X^0 rest system, d is the π^+ momentum in the c.m.s. of the pions, and $X_{\lambda i}$ is the spin function for $J^P(X^0) = 2^-$.

We present here the moments $\langle P_2 \rangle^{(v)}$ and $\langle P_4 \rangle^{(v)}$ for the Adair distributions with respect to the polar angle between the direction of the beam momentum \mathbf{k} and the spin analyzer \mathbf{v} ($\cos\theta = \mathbf{v}\mathbf{k}/|\mathbf{v}||\mathbf{k}|$), for which we selected \mathbf{q} , \mathbf{d} , and the normal (\mathbf{n}) to the decay plane

$$\begin{aligned} \langle P_2 \rangle^{(n)} &= \frac{1}{7} \left(-\frac{1}{2} + \text{Re } w \frac{\alpha_1}{\alpha_1 + |w|^2 \alpha_2} \right), \\ \langle P_4 \rangle^{(n)} &= -\frac{4}{21} \left(\frac{3}{8} + \frac{5}{12} \text{Re } w \frac{\alpha_1}{\alpha_1 + |w|^2 \alpha_2} \right), \end{aligned} \quad (6)$$

$$\langle P_2 \rangle^{(q)} = \frac{1}{7} \frac{\alpha_1}{\alpha_1 + |w|^2 \alpha_2}, \quad \langle P_4 \rangle^{(q)} = -\frac{4}{21} \frac{\alpha_1}{\alpha_1 + |w|^2 \alpha_2}; \quad (7)$$

$$\langle P_2 \rangle^{(d)} = \frac{1}{7} \frac{|w|^2 \alpha_2}{\alpha_1 + |w|^2 \alpha_2}, \quad \langle P_4 \rangle^{(d)} = -\frac{4}{21} \frac{|w|^2 \alpha_2}{\alpha_1 + |w|^2 \alpha_2}, \quad (8)$$

where α_1 , α_2 , α_3 are the integrals, over the phase

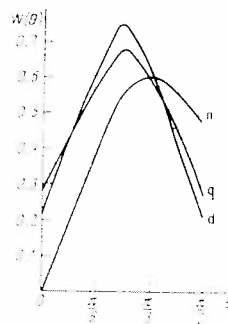


FIG. 1. Adair distributions for $X^0 \rightarrow \eta\pi^+\pi^-$ decay. The spin analyzer \mathbf{v} is directed along the momentum \mathbf{q} of the η meson, \mathbf{d} is the momentum of the π^+ meson in the c.m.s. of the $\pi^+\pi^-$ pair, and \mathbf{n} is the normal to the decay plane; $dN \sim W(\theta)d\cos\theta$ represents the number of events.

space of the decay $X^0 \rightarrow \eta\pi^+\pi^-$, of the quantities q^4 , d^4 , $q^2 d^2$, and $\alpha_1 : \alpha_2 : \alpha_3 = 6.6 : 1 : 1.5$. From recent Brookhaven data^[4] it follows that the parameter w has the almost imaginary value given by $w^{-1} = -0.02 \pm 0.05 \pm i(0.35 \pm 0.02)$. The Adair distributions for $X^0 \rightarrow \eta\pi^+\pi^-$ corresponding to this value of w are shown in Fig. 1.

We note that the Adair distributions for $\eta \rightarrow \pi^+\pi^-\pi^0$ with $J^P(\eta) = 2^-$ are also determined by (6)–(8), where \mathbf{q} is the π^0 momentum in the rest system of the η meson and \mathbf{d} is the π^+ momentum in the c.m.s. of the $\pi^+\pi^-$ pair. The amplitude-mixing parameter $w^{(\eta)}$ can be selected from the condition under which the matrix element in (5) for $J^P(\eta) = 2^-$ correctly describes the π spectrum in the Dalitz plot of $\eta \rightarrow \pi^+\pi^-\pi^0$ decay, which has been well studied experimentally^[22]:

$$|M|^2 = |1 + \alpha Y|^2, \quad (9)$$

$Y = 3(T_{\pi^0}/Q) - 1$, $\alpha = -0.58 \pm 0.01 \pm i(0.00 \pm 0.08)$, and Y is the Dalitz variable. For $J^P(\eta) = 2^-$ the distribution in the Dalitz plot is proportional to $|M_{2-}|^2 = q^4 + |w^{(\eta)}|^2 d^4$ and correctly describes the experimental data with $|w^{(\eta)}| \sim 2.8$. The Adair distributions (7) and (8) then cannot be isotropic, so that they can be used to confirm that the η meson is pseudoscalar also on the basis of $\eta \rightarrow \pi^+\pi^-\pi^0$ decay in reaction (1).

$X^0 \rightarrow \rho\gamma$ decay. We write the decay matrix element for $J^P(X) = 2^-$ in the form

$$M_{\lambda} = X_{\lambda i}([e, \mathbf{q}], q_i + g e_i[\mathbf{q}, \mathbf{d}]), \quad (10)$$

where g is the mixing parameter of $M1$ and $E2$ transition amplitudes, and \mathbf{q} is the γ momentum in the X^0 rest system. We present the moments of the distributions with respect to the polar angle θ between the beam momentum \mathbf{k} and the decay-particle momenta \mathbf{q} ; \mathbf{d} is the π^+ momentum in the ρ rest system and $\mathbf{n} = \mathbf{q} \times \mathbf{d}/|\mathbf{q}||\mathbf{d}|$:

$$\langle P_2 \rangle^{(n)} = \frac{1}{28} \frac{-0.5 + 2.8g + 2.2g^2}{1 + g + 0.7g^2}, \quad \langle P_4 \rangle^{(n)} = -\frac{1}{21} \frac{-0.5 + 0.8g^2}{1 + g + 0.7g^2}; \quad (11)$$

$$\langle P_2 \rangle^{(q)} = -\frac{1}{14} \frac{0.7 + 2.8g + g^2}{1 + g + 0.7g^2}, \quad \langle P_4 \rangle^{(q)} = -\frac{4}{21} \frac{0.2g^2}{1 + g + 0.7g^2}; \quad (12)$$

$$\langle P_2 \rangle^{(d)} = \frac{1}{10} \frac{1 + g - 0.2g^2}{1 + g + 0.7g^2}, \quad \langle P_4 \rangle^{(d)} = 0. \quad (13)$$

A detailed analysis of the Adair distributions for X^0 is given in Ref. 6. All the distributions are anisotropic

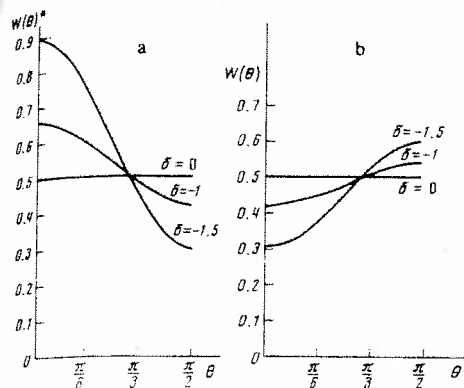


FIG. 2. Adair distributions for (a) $\pi^+{}^4\text{He} \rightarrow A_1{}^4\text{He}$ and (b) $\gamma{}^4\text{He} \rightarrow A_1{}^4\text{He}$, at different values of the anomalous magnetic moment δ of the A_1 meson. The spin analyzer \mathbf{v} is directed along the momentum \mathbf{q} of the ρ meson in the c.m.s. of the A_1 meson.

for $J^P(X^0) = 2^-$ and enable us to distinguish between $J^P(X^0) = 0^-$ and 2^- .

4. A_1 meson. The matrix element of $A_1 \rightarrow \rho\pi$ decay for $J^P(A_1) = 1^+$ corresponds to s and d waves with a mixing parameter w^A :

$$M(A_1 \rightarrow \rho\pi) = (\varepsilon\rho) + w^{(A)}(\varepsilon\mathbf{m})(\rho\mathbf{m}), \quad \mathbf{m} = \mathbf{q}/|\mathbf{q}|, \quad (14)$$

where ε and ρ are the polarization vectors of the A_1 and ρ mesons, and \mathbf{q} is the momentum of ρ in the rest system of A_1 . The Lagrangian of the $A_1\rho\pi$ system in current algebra can be written as

$$\mathcal{L}_{A_1\rho\pi} = -\frac{g_A}{g_{\rho\pi}}(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu) \left\{ \left[1 - \left(\frac{m_\rho}{m_A} \right)^2 (1+\delta) \right] \partial_\mu\pi \times A_\nu + \left[1 - \left(\frac{m_\rho}{m_A} \right)^2 \right] \pi \times \partial_\mu A_\nu \right\}, \quad (15)$$

where δ is the anomalous magnetic moment of A_1 ; $\delta = 0$ corresponds to Schwinger's model; $\delta = -1$ corresponds to Weinberg's model,^[11] and also to Ref. 12.⁵⁾

When it is assumed that $w^{(A)}$ is real the relation

$$\delta = -8w^{(A)} / (1 + 4w^{(A)}) \quad (16)$$

is fulfilled quite accurately: The available data on $A_1 \rightarrow \rho\pi$ decay do not enable us to determine δ uniquely; the data of Frogatt and Ranft^[14] lead to $\delta \approx 0$, while the data of Ballam *et al.*^[15] lead to $\delta \approx -1.5$. The Adair distributions for the reactions $\pi^+{}^4\text{He} \rightarrow A_1{}^4\text{He}$ and $\gamma{}^4\text{He} \rightarrow A_1{}^4\text{He}$ enable us to establish the decay matrix element and to determine δ . We here present moments of the distributions with respect to the polar angle between the beam momentum \mathbf{k} and the analyzer \mathbf{v} ($\cos\theta = \mathbf{v}\mathbf{k}/|\mathbf{v}||\mathbf{k}|$), where as analyzers \mathbf{v} we select the directions of \mathbf{q} (the momentum of ρ in the rest system of A_1), \mathbf{d} (the momentum of the π^+ from the decay of ρ in the c.m.s. of the $\pi^+\pi^-$), and $\mathbf{n} = \mathbf{q} \times \mathbf{d}/|\mathbf{q}||\mathbf{d}|$:

$$\langle P_2 \rangle^{(q)} = -\frac{1}{5}(1-3\rho_{00}) \frac{|w^{(A)}|^2 + 2\text{Re } w^{(A)}}{3|w^{(A)}|^2 + 2\text{Re } w^{(A)}}, \quad (17)$$

$$\langle P_2 \rangle^{(d)} = -\frac{1}{5}(1-3\rho_{00}) \frac{3+2|w^{(A)}|^2 + 2\text{Re } w^{(A)}}{3|w^{(A)}|^2 + 2\text{Re } w^{(A)}}, \quad (18)$$

$$\langle P_2 \rangle^{(n)} = \frac{1}{10}(1-3\rho_{00}), \quad (19)$$

where ρ_{00} is an element of the density matrix of A_1 ; $\rho_{00} = 1$ for $\pi^+{}^4\text{He} \rightarrow A_1{}^4\text{He}$ and $\rho_{00} = 0$ for $\gamma{}^4\text{He} \rightarrow A_1{}^4\text{He}$. The Adair distributions for A_1 production on ${}^4\text{He}$ depend only on the decay mechanism. Figure 2 shows the distributions corresponding to the moments (17) with $\delta = 0, -1$, and -1.5 , respectively, for $\pi^+{}^4\text{He} \rightarrow A_1{}^4\text{He}$ and $\gamma{}^4\text{He} \rightarrow A_1{}^4\text{He}$. The experimental study of these distributions would enable us to determine the behavior of the differential cross section and of the Adair distributions in (1) and (2) also enables us to rule out other hypotheses for the J^P of A_1 if A_1 can be interpreted as a resonance and not as a kinematic peculiarity.

5. $B(1235)$ meson. The quantum numbers of the B meson were studied for different reactions^[16,17] in

$$B \rightarrow \omega\pi$$

decay. There are indications that $J^P = 1^+$ is preferable to $J^P = 2^-$ or 3^- , although it is difficult to fully exclude $J^P = 1^-$ and 2^- .^[16-18]

Tables 1 and 2 give the matrix elements and Adair-distribution moments for all the hypotheses; here the spin analyzers \mathbf{v} are \mathbf{q} (the momentum of the π in the c.m.s. of B), \mathbf{N} (the normal to the decay plane of the ω meson), and $\mathbf{n} = \mathbf{q} \times \mathbf{N}/|\mathbf{q}||\mathbf{N}|$; the density matrix element of B is $\rho_{00} = 1$ for $\pi^+{}^4\text{He} \rightarrow B^+{}^4\text{He}$ and $\rho_{00} = 0$ for $\gamma{}^4\text{He} \rightarrow B^+{}^4\text{He}$. In all instances the distributions depend only on the decay matrix element and when analyzed would enable us to decide among the hypotheses.

6. $\gamma{}^4\text{He} \rightarrow E{}^4\text{He}$. The problem of determining J^P for $E(1420)$ has been studied most thoroughly for the reaction $p\bar{p}$ (at rest) $\rightarrow E\pi\pi$ ^[19] in the decays $E \rightarrow K^*(890)\bar{K}$ ($\bar{K}^*(890)K$) and $E \rightarrow \pi_A(1000)\pi$, and for the reaction $\pi^-p \rightarrow E(1420)n$ ^[20] in the decay $E \rightarrow K(890)\bar{K}$ ($\bar{K}^*(890)K$). The results of these studies lead to the alternatives 0^- ^[19] and 1^+ ^[20] for the J^P of the E meson, although $J^P = 2^-$ cannot be excluded.^[21] We note here that a detailed study of the direct decay channel $E \rightarrow K\bar{K}\pi$ [together with the $E \rightarrow K^*\bar{K}$ (\bar{K}^*K) and $\pi_A\pi$ channels], whose probability is $\sim 40\%$ according to new data, could strengthen the arguments in favor of 0^- or 1^+ . The simplest matrix elements of $E \rightarrow K\bar{K}\pi$ decay are

$$M_0 = \text{const}, \quad J^P = 0^-, \quad (20)$$

$$M_1 = qE_i, \quad J^P = 1^+, \quad (21)$$

$$M_2 = (q_i q_j + w^{(E)} d_i d_j) E_{ij}, \quad J^P = 2^-, \quad (22)$$

where \mathbf{d} is the momentum of K in the c.m.s. of the $K\bar{K}$ pair, \mathbf{q} is the momentum of π in the rest system of the E meson, E_j and E_{ij} are the spin wave functions for $J^P = 1^+$ and 2^- . Adler's self-consistency condition for the hypothesis $J^P(E) = 2^-$ gives $w^{(E)} = 0$ and fixes the decay matrix element. The distributions with respect to the variable

$$Y = \frac{m_\pi + 2m_K}{m_K(m_E - 2m_K - m_\pi)} T_\pi - 1$$

TABLE 1. Moments of the Adair distributions for $\gamma^4\text{He} \rightarrow B^4\text{He}$ and matrix elements for different hypothetical values of $J^P(B)$.

$\langle P_{11} \rangle^0$	$J^P = 1^+$	2^-	1^-	2^+
$\langle P_2 \rangle^{(q)}$	$-\frac{1}{5} \frac{ w_{1+}^B ^2 + 2 \operatorname{Re} w_{1+}^B}{3 + w_{1+}^B ^2 + 2 \operatorname{Re} w_{1+}^B}$	$\frac{1}{7} \frac{4(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 7}{4(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 10}$	$\frac{1}{10}$	$\frac{1}{14}$
$\langle P_4 \rangle^{(q)}$	0	$-\frac{4}{21} \frac{2(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B)}{2(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 5}$	0	$\frac{8}{63}$
$\langle P_2 \rangle^{(N)}$	$-\frac{1}{5} \frac{3 + \frac{2}{3} w_{1+}^B ^2 + 2 \operatorname{Re} w_{1+}^B}{3 + w_{1+}^B ^2 + 2 \operatorname{Re} w_{1+}^B}$	$\frac{1}{7} \frac{7 + \frac{4}{3} w_{2-}^B ^2 + \frac{2}{3} \operatorname{Re} w_{2-}^B}{4(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 10}$	$\frac{1}{10}$	$-\frac{1}{10}$
$\langle P_4 \rangle^{(N)}$	0	0	0	0
$\langle P_2 \rangle^{(n)}$	$\frac{1}{10}$	$-\frac{1}{7} \frac{ w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B + 4}{2(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 5}$	$-\frac{1}{5}$	$\frac{1}{14}$
$\langle P_4 \rangle^{(n)}$	0	$-\frac{1}{21} \frac{3(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 5}{2(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 5}$	0	$\frac{8}{63}$
M_{JP}	$B_1 \left(N_1 + w_{1+}^B \frac{(\mathbf{q} \times \mathbf{N}) \cdot \mathbf{q}_1}{ \mathbf{q} \mathbf{q}_1 } \right)$	$B_{ij} \left(N_i \frac{q_j}{ \mathbf{q} } + w_{2-}^B \frac{(\mathbf{q} \times \mathbf{N}) \cdot \mathbf{q}_i \mathbf{q}_j}{ \mathbf{q} ^2} \right)$	$B_i e_{ijk} N_j \frac{q_k}{ \mathbf{q} }$	$B_{ij} e_{ijk} N_i \frac{q_j q_k}{q^2}$

in the Dalitz plot have the forms

$$|M_0|^2 = \text{const}, \quad J^P = 0^- \quad (23)$$

$$|M_1|^2 = -1 + 0.77(Y + 2.14)^2, \quad J^P = 1^- \quad (24)$$

$$|M_2|^2 = [1 - 0.77(Y + 2.14)^2]^2, \quad J^P = 2^- \quad (25)$$

for $w^{(E)} = 0$.

For the hypotheses 1^+ and 2^- considerable degrees of anisotropy with respect to Y are obtained. If in the Dalitz plot the dependence of $E \rightarrow K\bar{K}\pi$ decay on Y is weak, this is a strong argument for a pseudoscalar E meson. Figure 3 shows the Adair distributions in $\gamma\text{He} \rightarrow E\text{He}$ for $E \rightarrow K\bar{K}\pi$ decay. The analysis of these distributions and of the behavior of the differential cross sec-

tion near $\theta_{\text{c.m.s.}} = 0^\circ$ could clarify the spin of the E meson.

The question as to whether E is pseudoscalar now requires more thorough study since 2^- has been indicated for $X^0(960)$.^[4] A detailed analysis of all the distributions and of the methods of determining the spin and parity of E will be published separately. Similar results in reactions (1) and (2) can be obtained for $D(1285)$ and other meson resonances.

We have here discussed several examples which permit us to conclude that qualitative effects in reactions (1) and (2) contain important information concerning meson resonances. Similar effects are found in the reaction $K^4\text{He} \rightarrow K^*4\text{He}$.

The authors wish to thank T. L. Asatiani, S. B. Gerasimov, S. G. Matinyan, V. A. Tsarev, and especially R. Lednicki and V. I. Ogievetskii for useful discussions.

TABLE 2. Moments of the Adair distributions for $\pi^4\text{He} \rightarrow B^4\text{He}$.

$\langle P_{11} \rangle^{(p)}$	$J^P = 1^+$	$J^P = 2^-$
$\langle P_2 \rangle^{(q)}$	$\frac{2}{5} \frac{ w_{1+}^B ^2 + 2 \operatorname{Re} w_{1+}^B}{3 + w_{1+}^B ^2 + 2 \operatorname{Re} w_{1+}^B}$	$\frac{1}{7} \frac{4(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 7}{2(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 5}$
$\langle P_4 \rangle^{(q)}$	0	$\frac{4}{7} \frac{(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B)}{2(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 5}$
$\langle P_2 \rangle^{(N)}$	$\frac{2}{5} \frac{3 + \frac{2}{3} w_{1+}^B ^2 + 2 \operatorname{Re} w_{1+}^B}{3 + w_{1+}^B ^2 + 2 \operatorname{Re} w_{1+}^B}$	$\frac{1}{7} \frac{7 + \frac{4}{3} w_{2-}^B ^2 + \frac{2}{3} \operatorname{Re} w_{2-}^B}{2(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 5}$
$\langle P_4 \rangle^{(N)}$	0	0
$\langle P_2 \rangle^{(n)}$	$-\frac{1}{5}$	$-\frac{2}{7} \frac{ w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B + 4}{2(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 5}$
$\langle P_4 \rangle^{(n)}$	0	$\frac{1}{7} \frac{3(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 5}{4(w_{2-}^B ^2 + 2 \operatorname{Re} w_{2-}^B) + 10}$

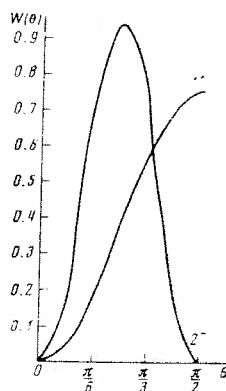


FIG. 3. Adair distributions for $\gamma^4\text{He} \rightarrow E^4\text{He}$ $\rightarrow K\bar{K}\pi$

with different values of $J^P(E)$. The spin analyzer \mathbf{v} is directed along the momentum \mathbf{q} of the pion in the c. m. s. of the E meson.

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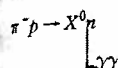
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¹The possibility of determining experimentally the spin and parity of $X^0(960)$ in



and the features of the experimental technique that enable us to determine J^P from the Adair distribution [which can also be useful in reactions (1) and (2)] were discussed in detail in Ref. 7.

²This circumstance is important, for example, in the case of $X^0(960) \rightarrow \rho\gamma$ decay, where the decay data do not permit reliable determination of the mixing parameter of $M1$ and $E2$ transition amplitudes for $J^P(X^0) = 2^-$. Analysis of the Dalitz plot for $A_1 \rightarrow \rho\pi$ decay does not at present enable us to determine the anomalous magnetic moment δ of A_1 — a parameter which is important for different models of current algebra, etc.

³The ^4He nucleus has no low-lying excited levels. This fact is important for determining meson spins from the behavior of the differential cross section and for distinguishing a coherent process experimentally.

⁴The distributions corresponding to these moments are

$$W(\theta) = 1/2 \left[1 + \sum_n (2n+1) \langle P_n \rangle^{(\epsilon)} P_n(x) \right], \quad n=2, 4, \dots; \quad x = \cos \theta.$$

⁵Agreement with experiment for resonance pair production in $\pi^+ p \rightarrow \Delta^{++} \rho^0$ required $\delta \approx 1$.^[13]

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Translated by I. Emin

Gersten ambiguities in the phase-shift analysis of πN scattering and isotopic invariance

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(Submitted May 12, 1975)

Yad. Fiz. 23, 1093-1098 (May 1976)

We investigate what measurements are required in order to exclude all the theoretical ambiguities in the phase-shift analysis of elastic πN scattering. It is shown that it is almost always sufficient to know the total cross section, the differential cross section and the polarization in the elastic channels, the sign of $\text{Re } f_+(0)$ or $\text{Re } f_-(0)$, and the charge-exchange differential cross section. Cases are pointed out which require measurements of the charge-exchange polarization and the spin rotation parameters.

PACS numbers: 13.70.Gf, 11.80.Et

1. THE GERSTEN AMBIGUITIES

Several years ago Gerstein^[1] discovered a large class of ambiguities in the phase-shift analysis of the elastic scattering of spin-0 and spin- $\frac{1}{2}$ particles. All the amplitudes belonging to this class give precisely the same differential cross section $\sigma = |f|^2 + |g|^2$, polarization $P = 2 \text{Re } f g^* / \sigma$, and imaginary part of the forward scattering amplitude $f(0)$, where $f(\theta)$ and $g(\theta)$ are the

non-spin-flip and spin-flip amplitudes, respectively. We shall make use of the conventional partial-wave expansions of the amplitudes f and g , with a limited number of partial waves; we denote the maximum value of the orbital angular momentum by L , and we write the partial-wave amplitudes in the form

$$f_{l\pm} = \xi_{l\pm} - 1 = \eta_{l\pm} \exp(2i\delta_{l\pm}) - 1, \quad (1)$$

$$0 \leq \eta_{l\pm} \leq 1 \quad (\text{unitarity}).$$