Heretic Splitter

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We describe a simple splitter design, where a defocusing quad helps the dipole to deflect the trajectory further away and a second focusing quadrupole makes the trajectory parallel again.

Figure 1 illustrates the geometry where the dipole is modeled as a steering magnet in the center of the dipole that causes an energy-dependent kick on the trajectory, shown as a dashed line, which can be understood to be proportional to the dispersion. This interpretation helps to resolve the ambiguity for dipolar fields; if they are large, they are treated dipoles that affect the reference trajectory; if they are small they are treated as steering magnets.

As the beam comes out of the dipole with an angle it separates from the reference trajectory and, after a drift of length L_1 meets a defocusing quad with focal length $-f_1$, that helps the dipole to deflect the beam further away from the axis. After a second drift space of length L_2 it is deflected back towards the axis by a focusing quadrupole with focal length f_2. We now need to calculate the focal length f_2 that makes the beam parallel.

To do so we calculate the transfer matrix R from the steerer to just after the focusing quadrupole. The transfer-matrix element R_{12} describes the parallel displacement of the beam and the R_{22} the angle after the second quadrupole. In order for the beam to be parallel we thus require $R_{22} = 0$ which provides us with

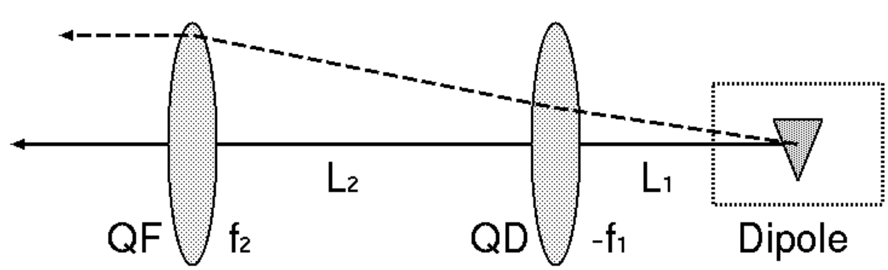

Figure 1: Schematics.

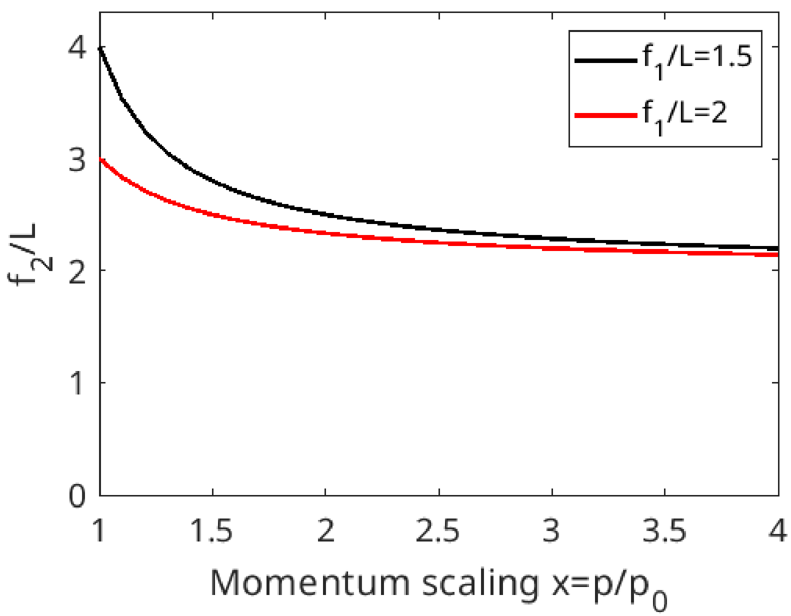

Figure 2: f_2/L as a function of the momentum scaling $x = p/p_0$ for $f_1/L = 1.5$ (black) and $f_1/L = 2$ (red).

a condition from which we determine f_2. We thus obtain

$$
\begin{aligned}
R &= \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & L_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/f_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & L_1 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 + \frac{L_2}{f_1} & L_1 + L_2\left(1 + \frac{L_1}{f_1}\right) \\ -\frac{1}{f_1} + \frac{1}{f_1}\left(1 - \frac{L_2}{f_2}\right) & -\frac{L_1}{f_1} + \left(1 - \frac{L_2}{f_2}\right)\left(1 + \frac{L_1}{f_1}\right) \end{pmatrix} . \qquad (1)
\end{aligned}
$$

The offset is thus proportional to

$$R_{12} = L_1 + L_2 + \frac{L_1 L_2}{f_1} . \qquad (2)$$

The condition for f_2 comes from the parallelity. We therefore require $R_{22} = 0$ and multiply out all terms and giving all terms the same denominator, which leads to

$$0 = \frac{f_1 f_2 - L_2 f_1 - L_1 f_2 - L_1 L_2 - L_1 f_1}{f_1 f_2} . \qquad (3)$$

Solving for f_2 then gives us

$$f_2 = \frac{(L_1 + L_2) f_1 + L_1 L_2}{f_1 - L_1} . \qquad (4)$$

In order to reduce the complexity of the equation we assume that both drift spaces are equal $L = L_1 = L_2$

$$f_2 = \frac{2Lf_1 + L^2}{f_1 - L} = L\frac{2f_1/L - 1}{f_1/L - 1} \tag{5}$$

Unfortunately the equalities only hold for one momentum. If we introduce a momentum scaling variable $x = p/p_0$ we know hat the focal lengths scale with x: $f \to xf$. Thus we plot f_2/L as a function of x for $f_1 = 2L$ and for $f_1 = 1.5L$ and show the curves in Figure 2. We find that f_2 varies and that will cause the parallelity condition to be violated. In other words, the beam does not exit parallel for all energies. On the other hand we find that the variation of f_2 is only about 30 % for the weaker-quad configuration $f_1 = 2L$. But this needs to be explored further.

We point out that this simple thin-lens model can only serve as a guideline for choosing real finite-length quads, but a starting point can be choosing them according to

$$\frac{1}{f} = k_1 l \qquad \text{with} \qquad k_1 = \frac{e\partial B_y/\partial x}{p_0} = \frac{\partial B_y/\partial x}{(B\rho)} \tag{6}$$

and the length of the quad l.