Calculating Errors for Unnormalized Moments

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Since a moments analysis consists of calculating the average of angular quantities for the data, the spread of this average is just the average of the standard deviation.

$$\delta h = \sigma_{\bar{h}} = \frac{\sigma_h}{\sqrt{N}} = \frac{1}{\sqrt{N}} \sqrt{\frac{1}{\sqrt{N}} \sum_i^N (H_i - \overline{H})^2} \qquad \text{(Eq.1)}$$

where $h = \frac{1}{N} \sum_i^N H_i$ and H_i is the value of the $D^J_{M,0}(\Omega_i)$ for event *i* .

Note that *h* is the experimental average and not the unnormalized moment. The connection is

$$H(L,M) = N \cdot h(L,M)\,. \qquad \text{(Eq.2)}$$

The errors on *H(L,M)* are determined via straight forward calculation.

$$(\delta H)^2 = (\frac{\partial H}{\partial N} \cdot \delta N)^2 + (\frac{\partial H}{\partial h} \cdot \delta h)^2 = (h \bullet \delta N)^2 + (N \bullet \delta h)^2 \qquad \text{(Eq.3)}$$

The error on the observed *N* events is $\delta N = \sqrt{N}$ and we find that

$$N^2 (\delta h)^2 = \sum_i^N (H_i - \overline{H})^2 = \sum_i^N (H_i - \frac{1}{N} \sum_i^N H_i)^2 = \sum_i^N (H_i^2 - \frac{2H_i}{N} \sum_i^N H_i + \frac{1}{N^2} (\sum_i^N H_i)^2)^2$$

$$= \sum_i^N H_i^2 - \frac{2}{N} \sum_i^N H_i \sum_i^N H_i + \frac{1}{N^2} (\sum_i^N H_i)^2 N = \sum_i^N H_i^2 - \frac{1}{N} (\sum_i^N H_i)^2$$

Therefore we find the relationship for the unnormalized moment errors.

$$(\delta H)^2 = h^2 N + \sum_i^N H_i^2 - \frac{1}{N} (hN)^2 = \sum_i^N H_i^2$$

$$\delta H(L,M) = \sqrt{\sum_i^N H_i (L,M)^2} \qquad \text{(Eq.4)}$$

A quick check shows that for $H(0,0) = 1$ the error $\delta H(0,0) = \sqrt{N}$ as expected.