Extractions of $\boldsymbol{\Sigma}$ and G asymmetries for $\boldsymbol{\pi}^{-}$ p channel with Linear pol data
Study with Maximum Log-Likelihood method

T. Kageya @ g14 meeting, Jun. 24th 2021

Maximum Log-Likelihood method (No.1)

$L_i = C_i\,[1 + P_b\,\Sigma\,\cos(2\phi_i) - P_T\,P_b\,G\,\sin(2\phi_i)\,]\,Ai$ (1) C_i : Flux A_i : Acceptance

$L_T = \prod L_i$ (2) $\prod$: product operator

$\log L_T = \log(\prod L_i) = \sum \log\{C_i\,[1 + P_b\,\Sigma\,\cos(2\phi_i) - P_T\,P_b\,G\,\sin(2\phi_i)\,]\,Ai\}$ (3) $\log(xy) = \log x + \log y$

$\log\{C_i\,[1 + P_b\,\Sigma\,\cos(2\phi_i) - P_T\,P_b\,G\,\sin(2\phi_i)\,]\,Ai\} = \log C_i + \log\,[1 + P_b\,\Sigma\,\cos(2\phi_i) - P_T\,P_b\,G\,\sin(2\phi_i)\,] + \log Ai$ (4)

$\log L_T = \log \prod C_i + \sum \log\,[1 + P_b\,\Sigma\,\cos(2\phi_i) - P_T\,P_b\,G\,\sin(2\phi_i)\,] + \log \prod A_i$ (5)

Eq. (5) can be differentiated to find the maximum with respect to Σ or G

Constant terms: $\log \prod C_i$ and $\log \prod A_i$ don't have to be considered

Maximum of the term $\sum \log\,[1 + P_b\,\Sigma\,\cos(2\phi_i) - P_T\,P_b\,G\,\sin(2\phi_i)\,]$ ➔ obtain Σ and G

Maximum Log-Likelihood method (No.2)

From each event, input P_b , ϕ_i and P_T to log $[1 + P_b \Sigma \cos(2\phi_i) - P_T P_b G \sin(2\phi_i)]$ for PERP
and to log $[1 - P_b \Sigma \cos(2\phi_k) + P_T P_b G \sin(2\phi_k)]$ for PARA

Use Minuit to minimize by multiplying -1
to log $L_T = \sum \log [1 + P_b \Sigma \cos(2\phi_i) - P_T P_b G \sin(2\phi_i)] + \sum \log [1 - P_b \Sigma \cos(2\phi_k) + P_T P_b G \sin(2\phi_k)]$

Minuit gets to minimum → gets maximum of log L_T

Maximum Log-Likelihood method (No.3)

Followings are expected;

(1) Errors are reduced to ~ 70 % of the Least Square method (~30 % smaller error)

(2) Final W bins -> can compare ours with g13b results
may cover 6 W bins

(3) Better performances for low statistic data than Least Square method

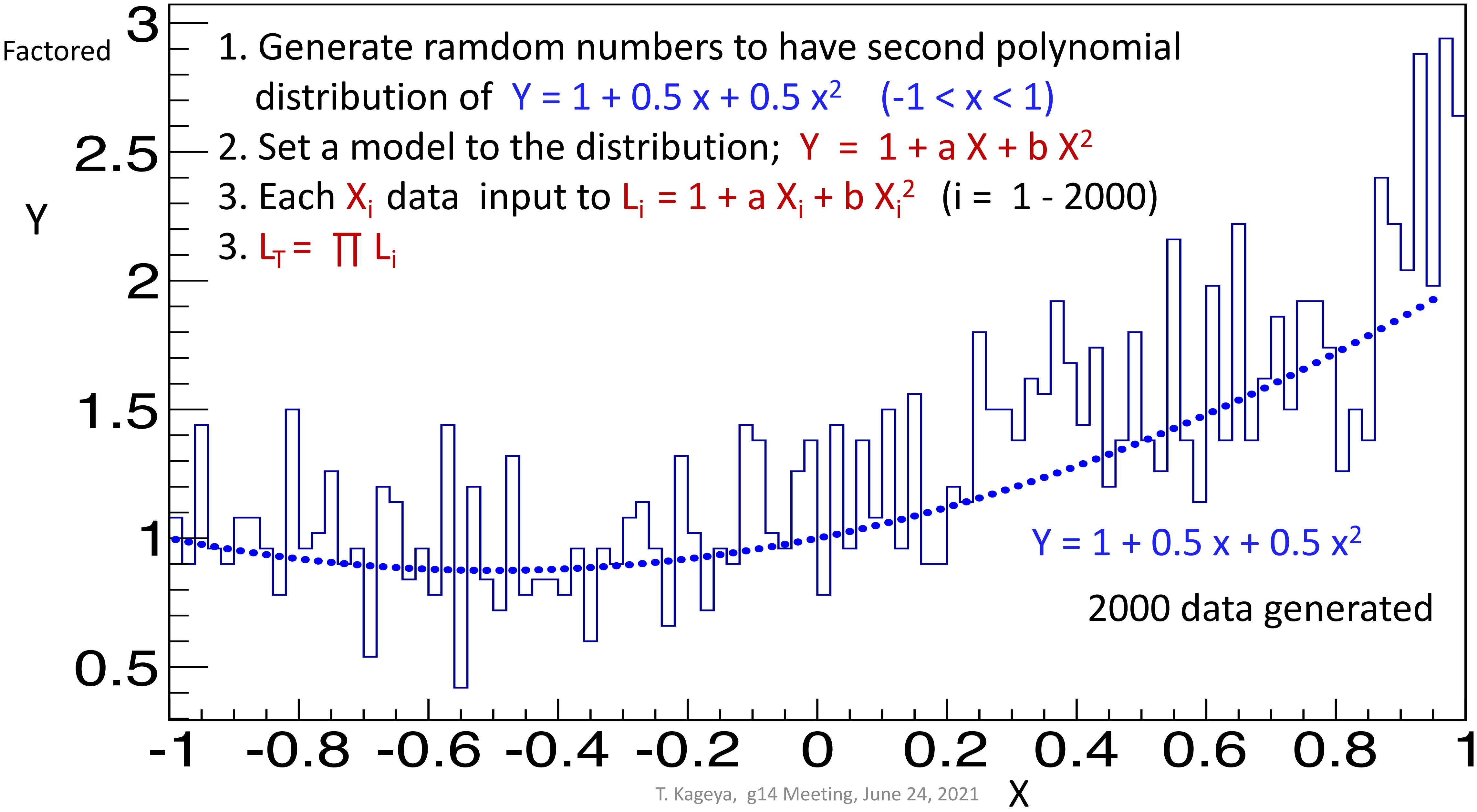
Maximum Log-Likelihood method (No.4)

a) Example 1: generate the second polynomial distribution with random numbers
   Apply the maximum log likelihood to the distribution

b) Apply MLL to g14 data to extract Σ and G asymmetries

a) Example 1: generate the second polynomial distribution with random numbers

New !



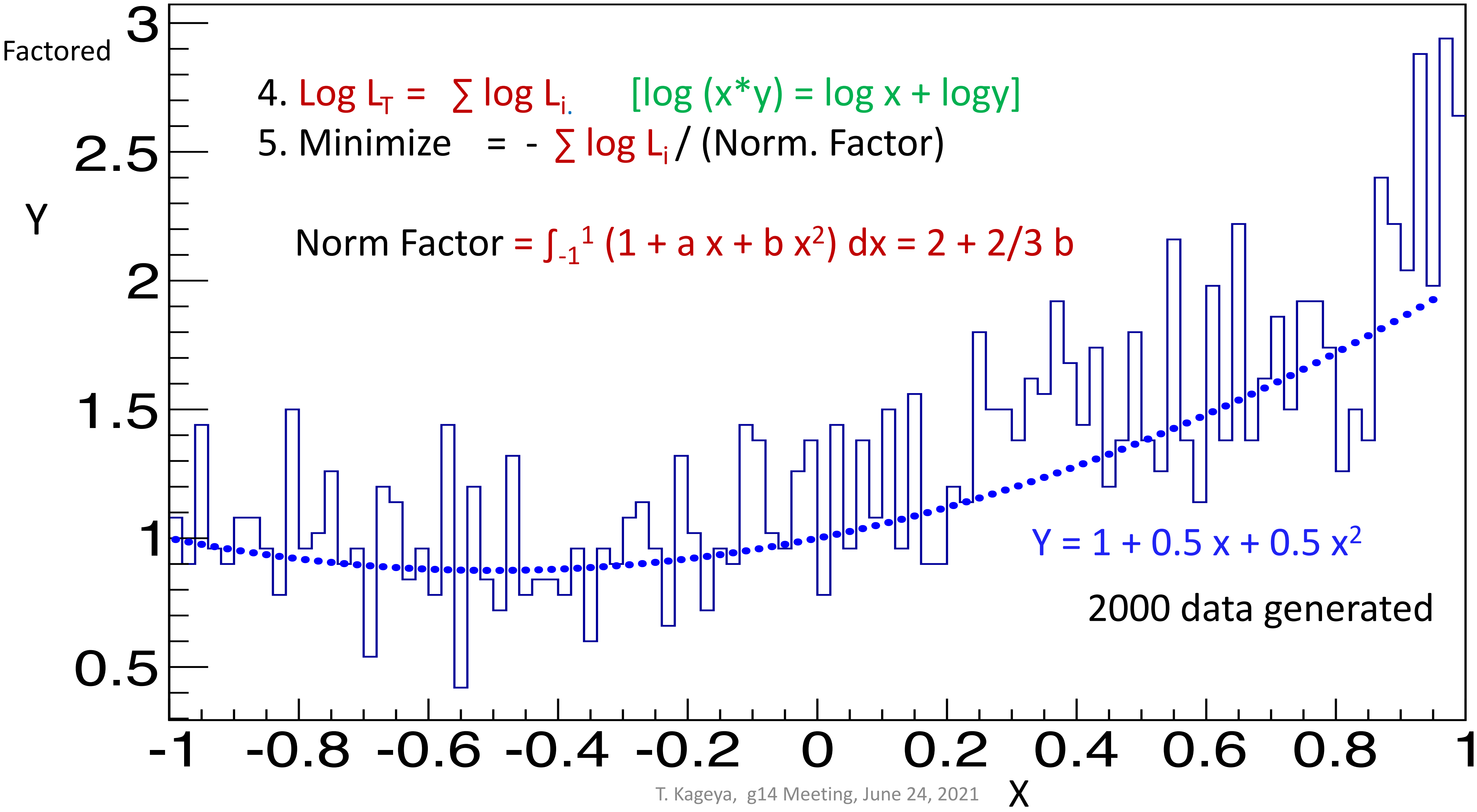
a) Example 1: generate the second polynomial distribution with random numbers

New !

4. $\text{Log } L_T = \sum \log L_i$ [log (x*y) = log x + logy]

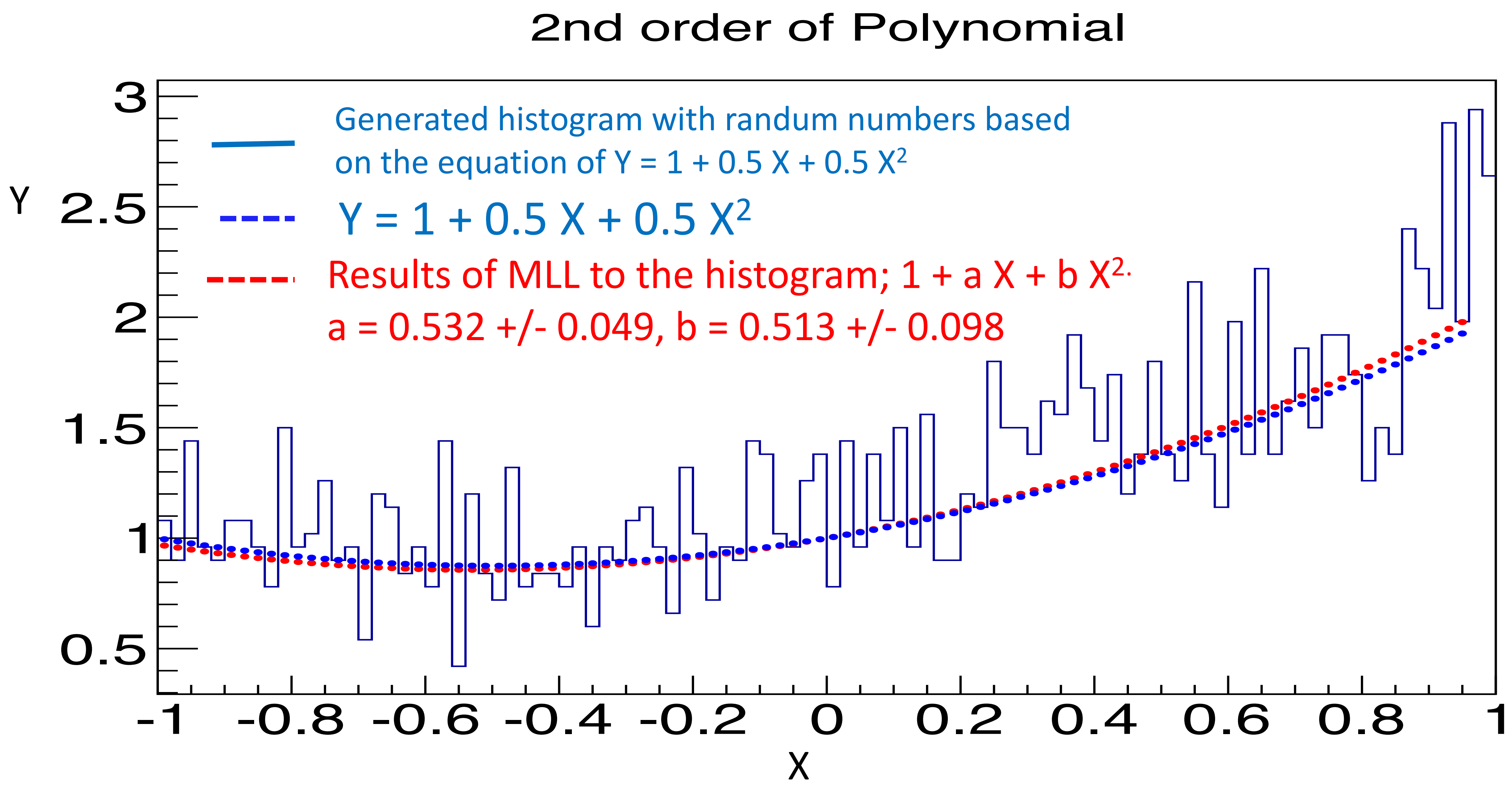
5. Minimize = $- \sum \log L_i$ / (Norm. Factor)

Norm Factor $= \int_{-1}^{1} (1 + a\,x + b\,x^2)\,dx = 2 + 2/3\ b$



a) Example 1: generate the second polynomial distribution with random numbers

New !



b) Apply MLL to g14 data to extract Σ and G asymmetries

Data analysis

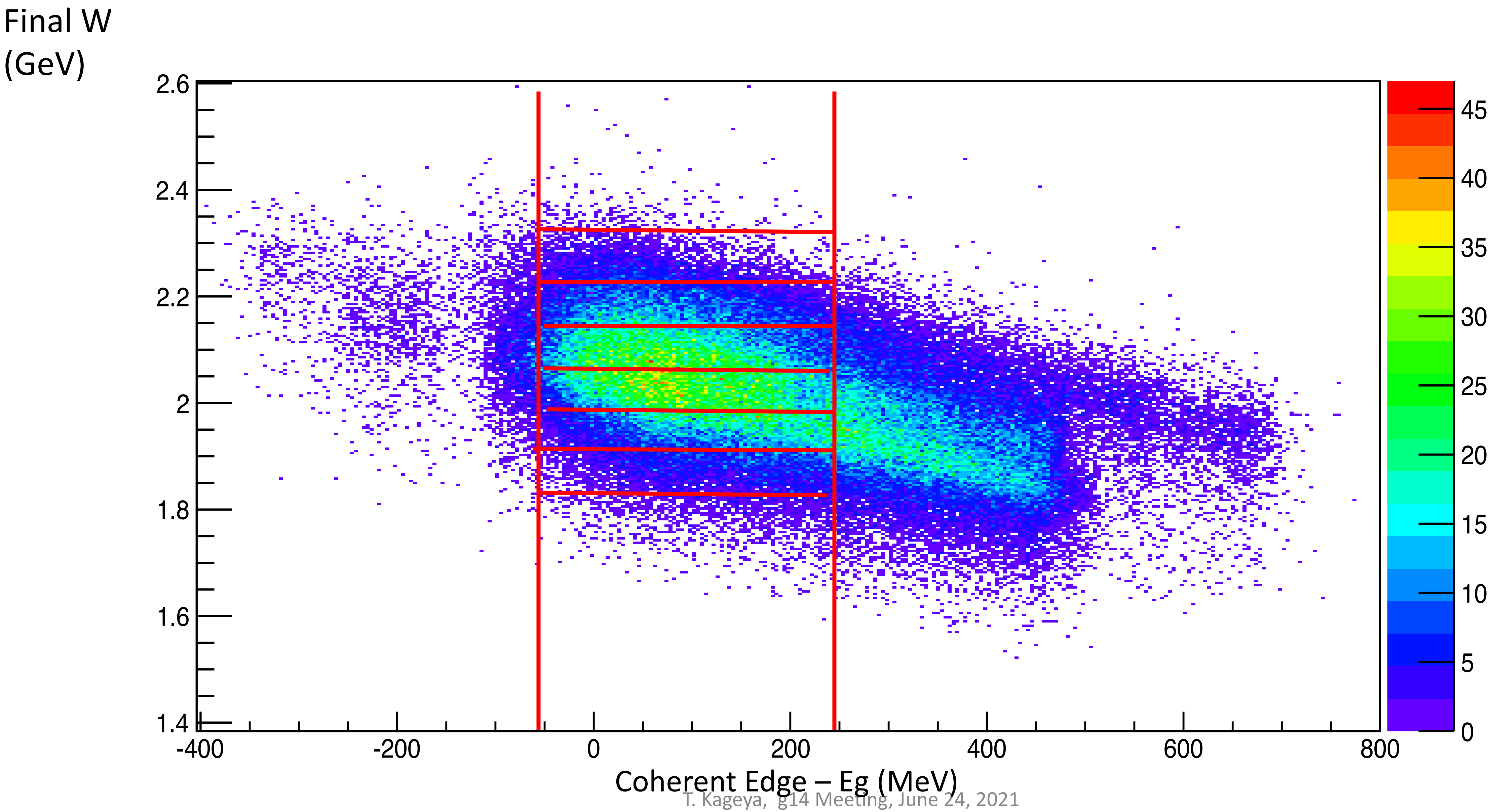
(1) Use data from (PERP, + Target) and (PARA, + Target); correspond to $(1 + P_{\perp}^{+} \Sigma \cos(2\phi) - P_{+z} P_{\perp}^{+} G \sin(2\phi))$ and $1 - P_{||}^{+} \Sigma \cos(2\phi) + P_{+z} P_{||}^{+} G \sin(2\phi)$.
All cuts are applied including vertex cut

(2) Apply cut of -50 < Coherent Edge – Eg (MeV) < 250 MeV to select events with relatively well defined beam polarization.
6 final W bins as shown in the figure at the next page.

(3) Empty target subtractions are not applied; instead about 7 % of corrections of dilution factors are applied.

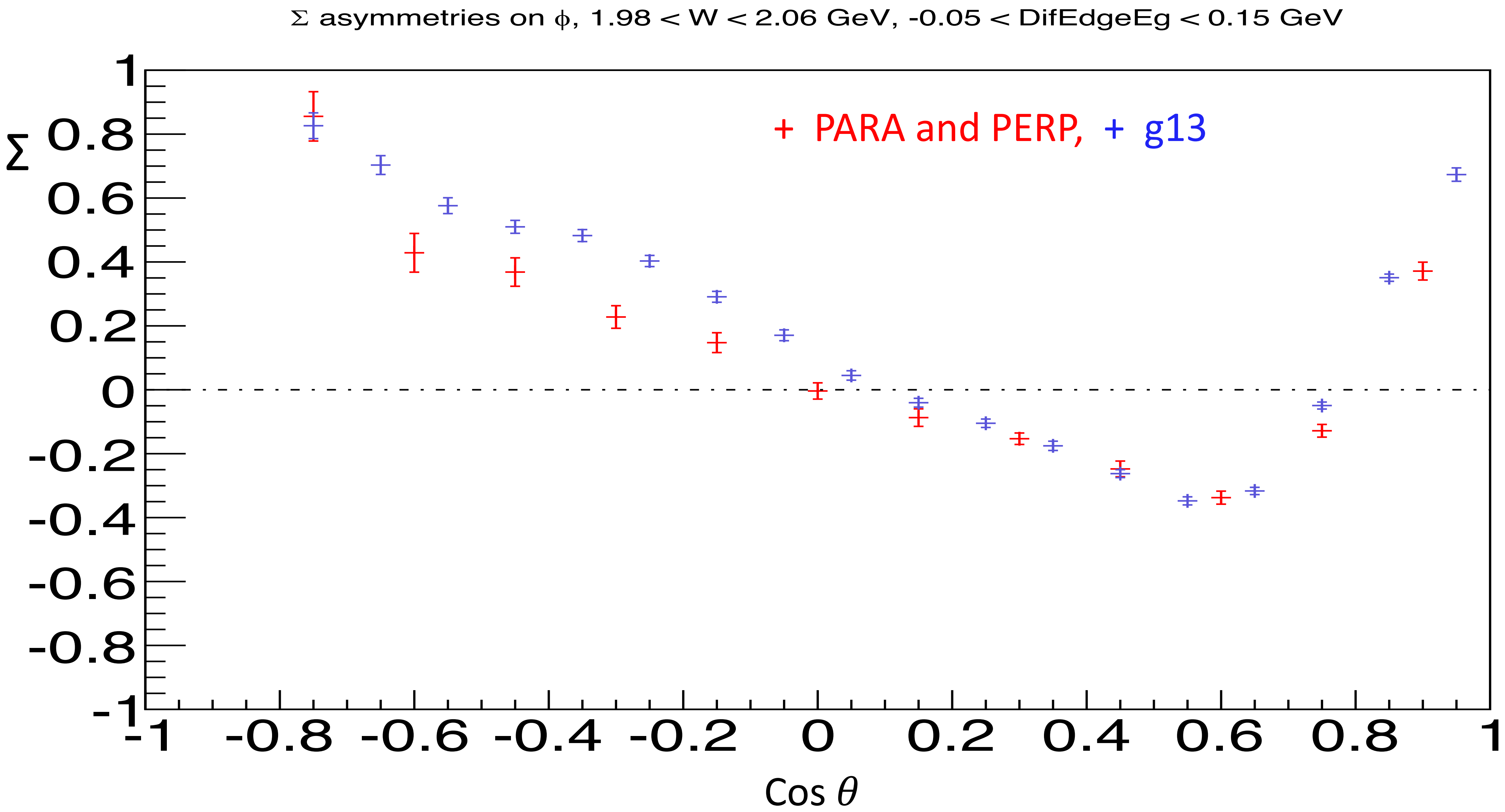
Final W distributions and bins

New !



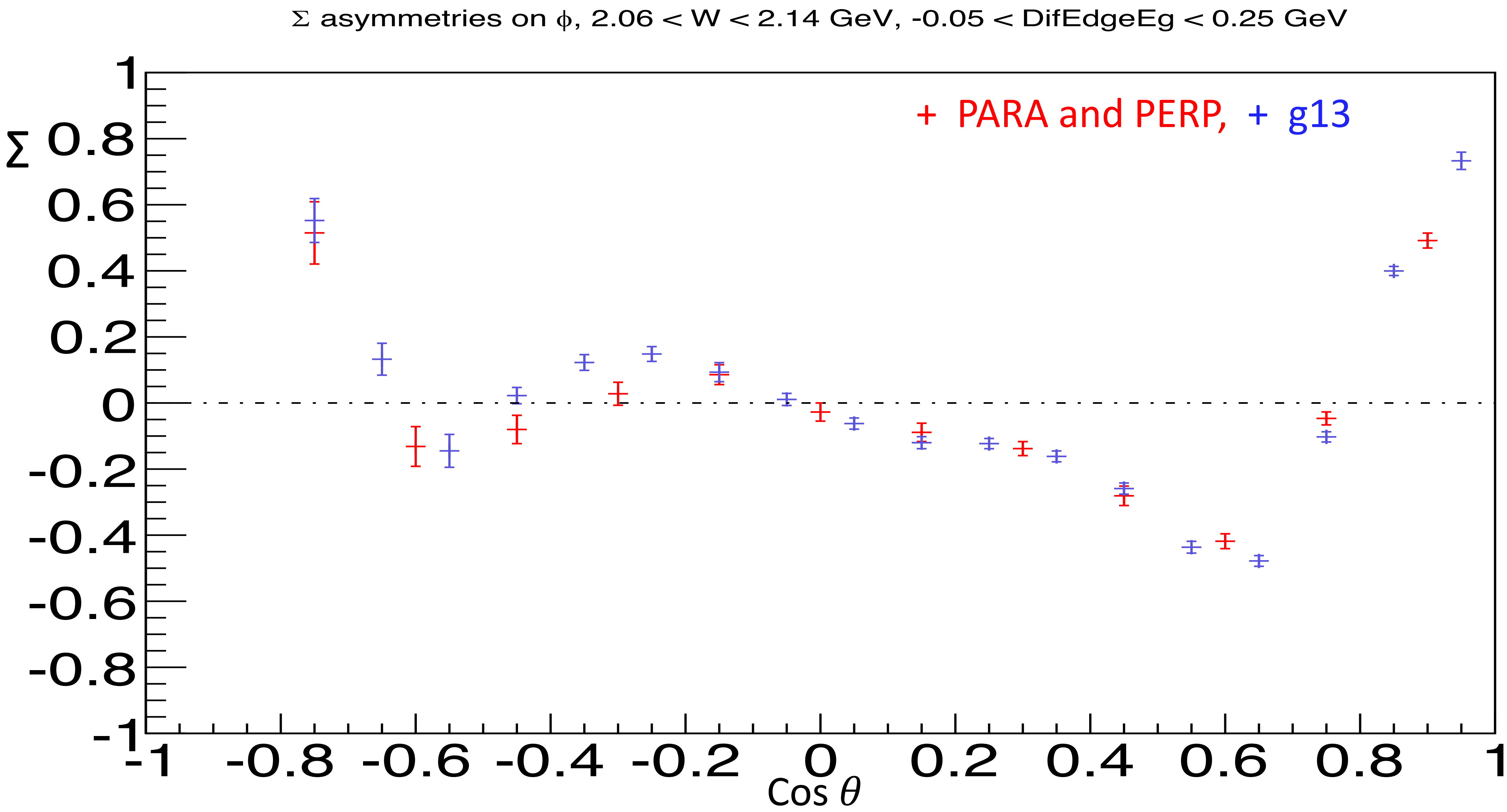
Σ asymmetries for 1.98 < fW < 2.06 Gev

New !



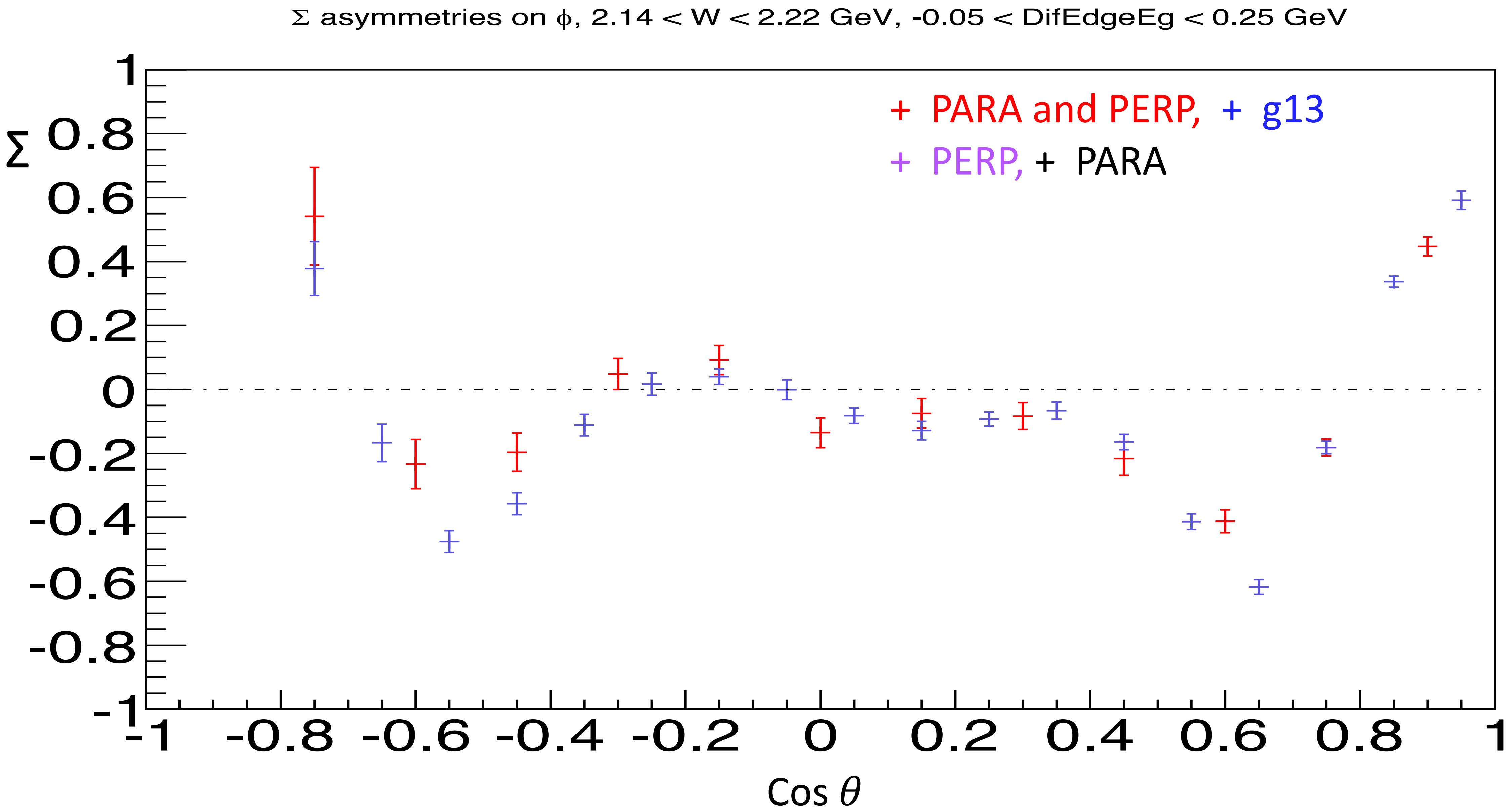
Σ asymmetries for 2.06 < fW < 2.14 Gev

New !



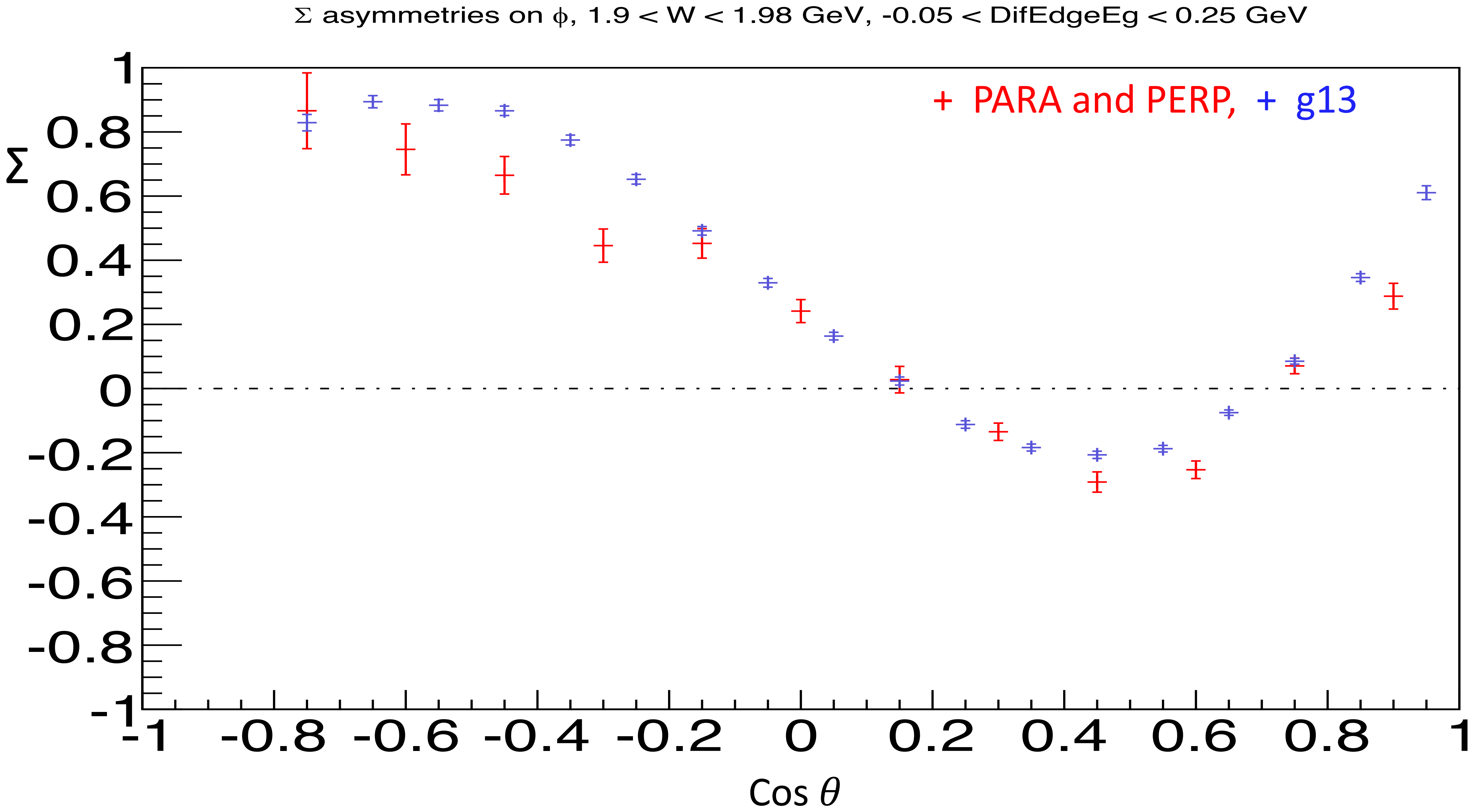
Σ asymmetries for 2.14 < fW < 2.22 Gev

New !



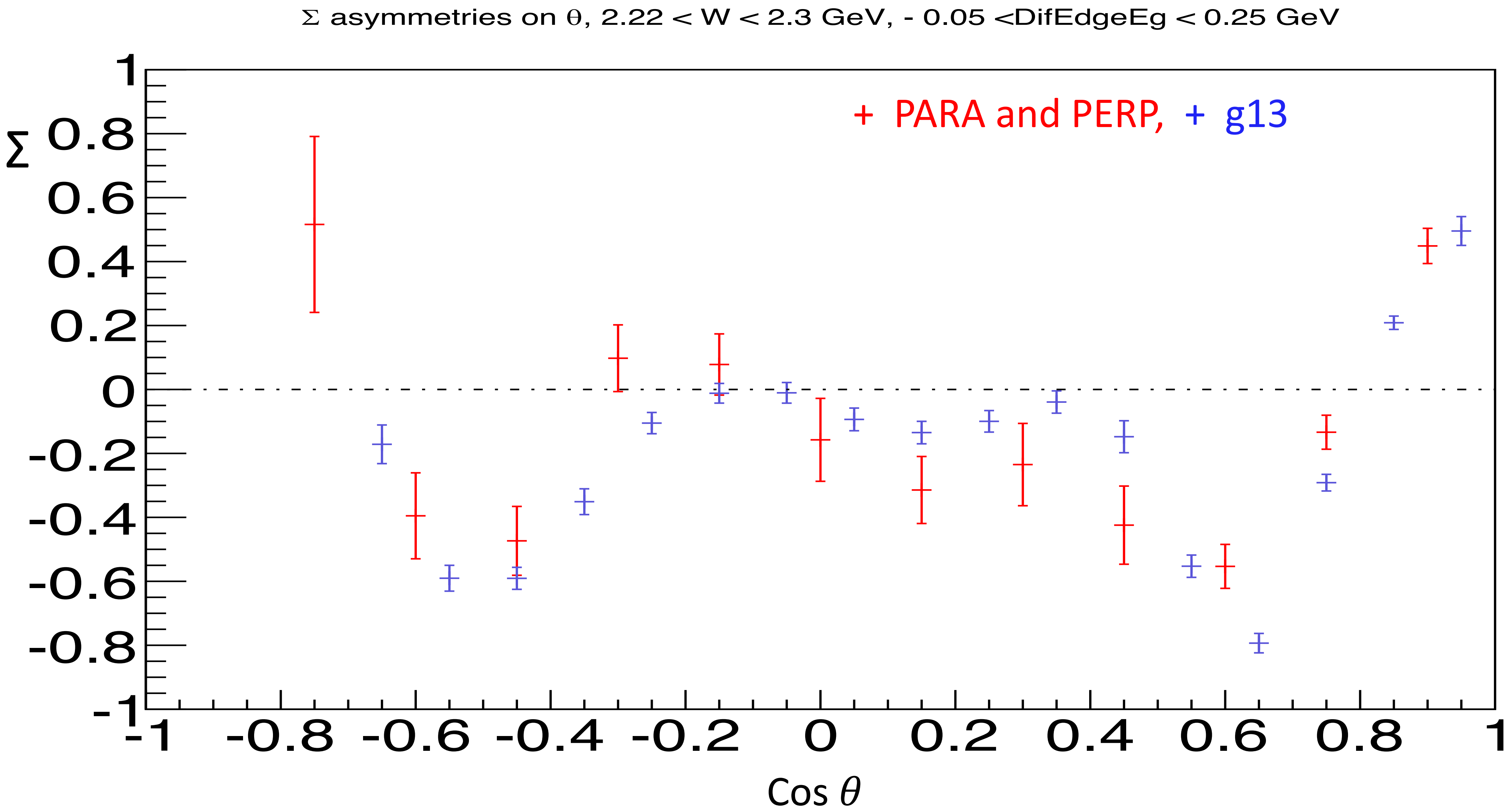
Σ asymmetries for 1.9 < fW < 1.98 Gev

New !



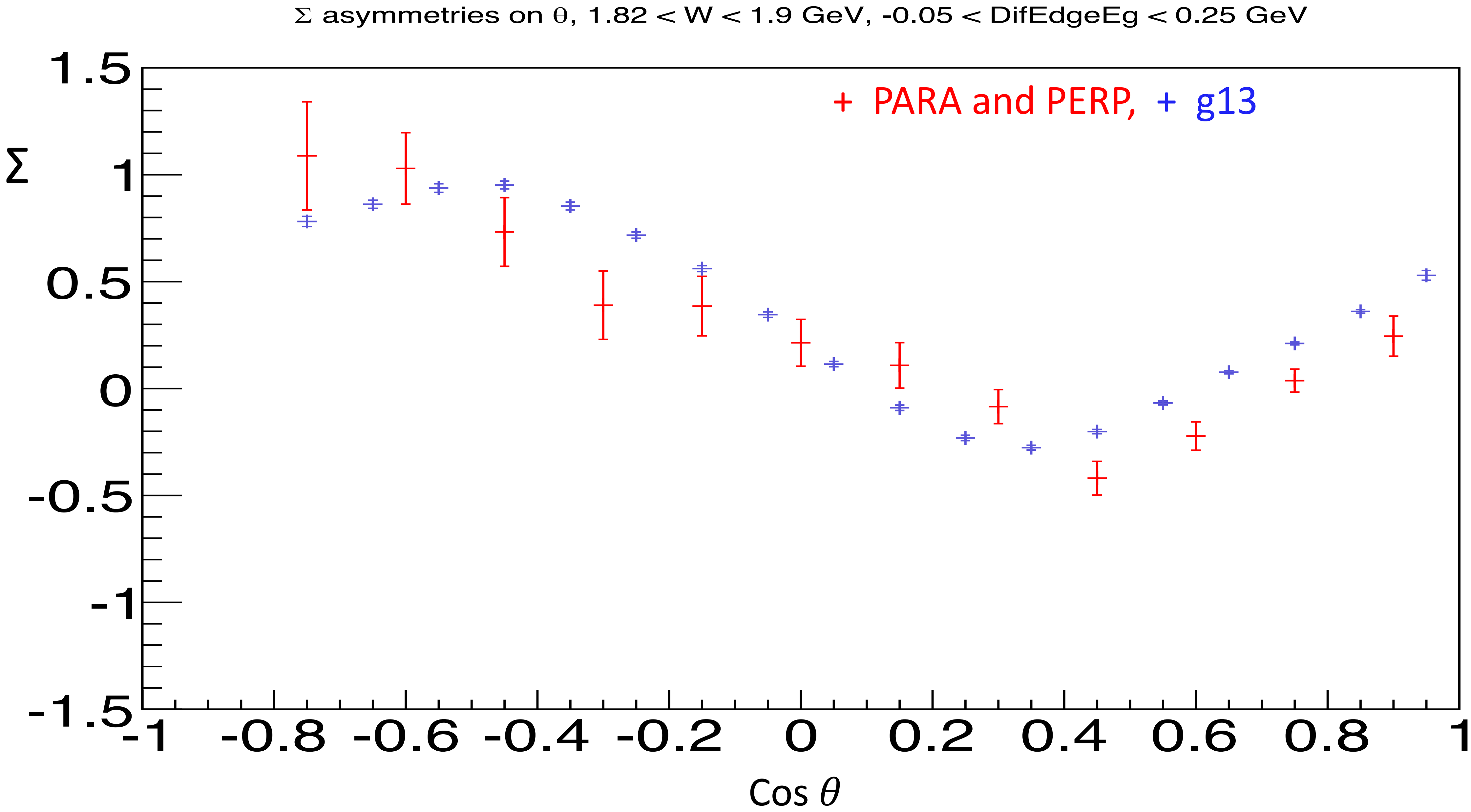
Σ asymmetries for 2.22 < fW < 2.3 Gev

New !



Σ asymmetries for 1.82 < fW < 1.9 Gev

New !



Maximum Log-Likelihood method (more understanding)

Separately analyze PERP and PARA;

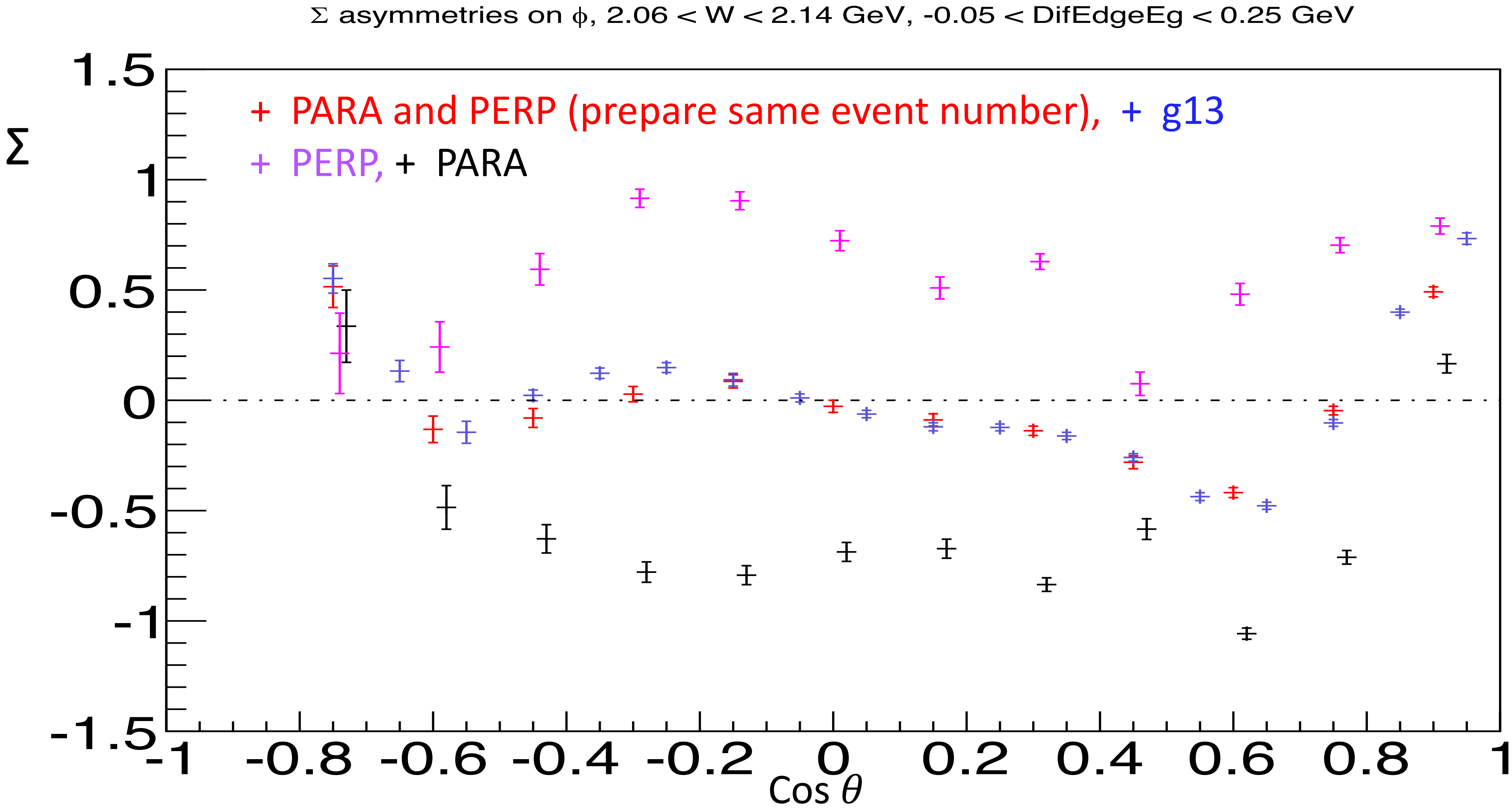
$\log L_T = \sum \log [1 + P_b \Sigma \cos(2\phi_i) - P_T P_b G \sin(2\phi_i)]$ for PERP data

$\log L_T = \sum \log [1 - P_b \Sigma \cos(2\phi_k) + P_T P_b G \sin(2\phi_k)]$ for PARA data

Minuit gets to minimum → gets maximum of log L_T , separately

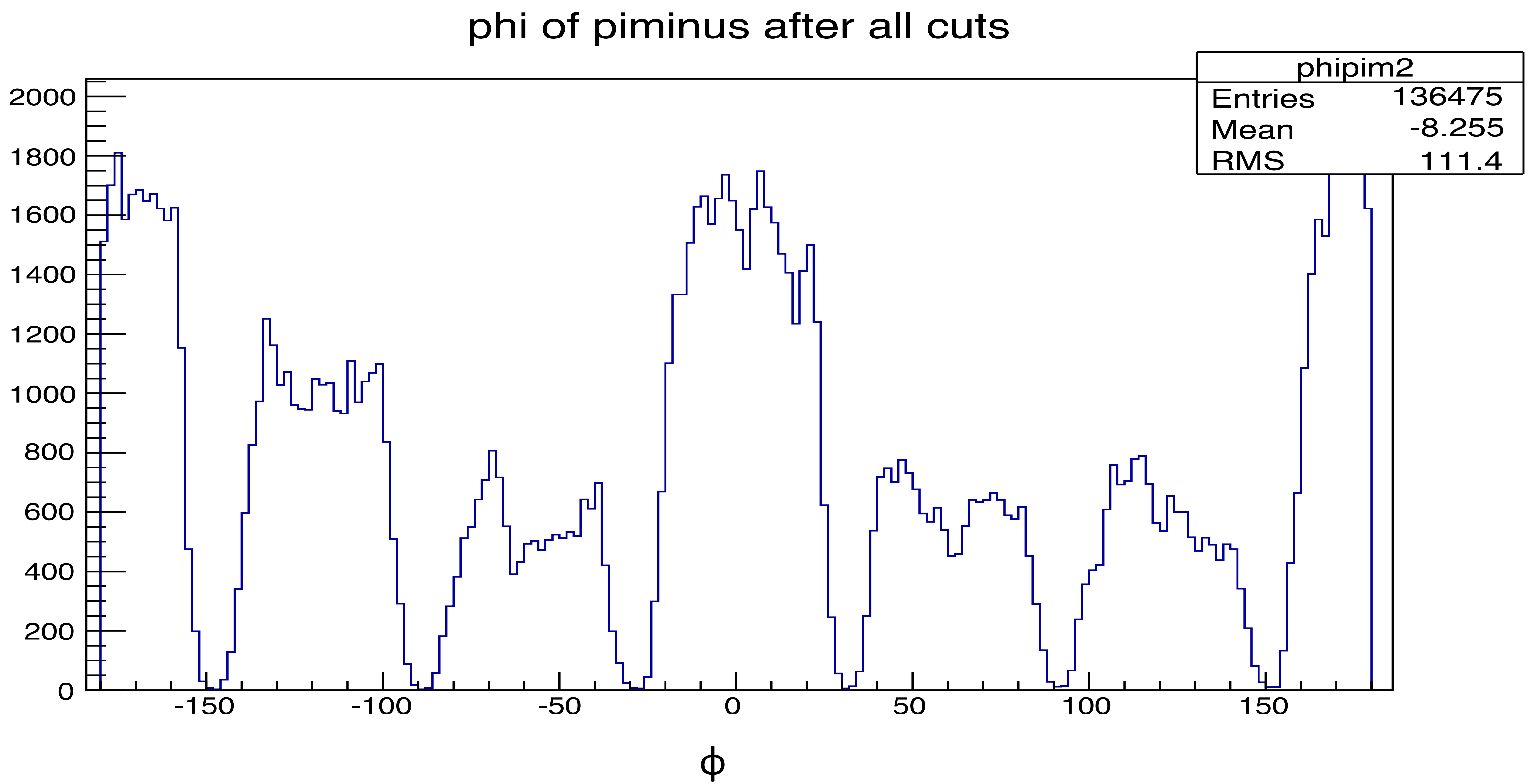
Σ asymmetries for 2.06 < fW < 2.14 Gev (analyzed PERP and PARA separately)

New !



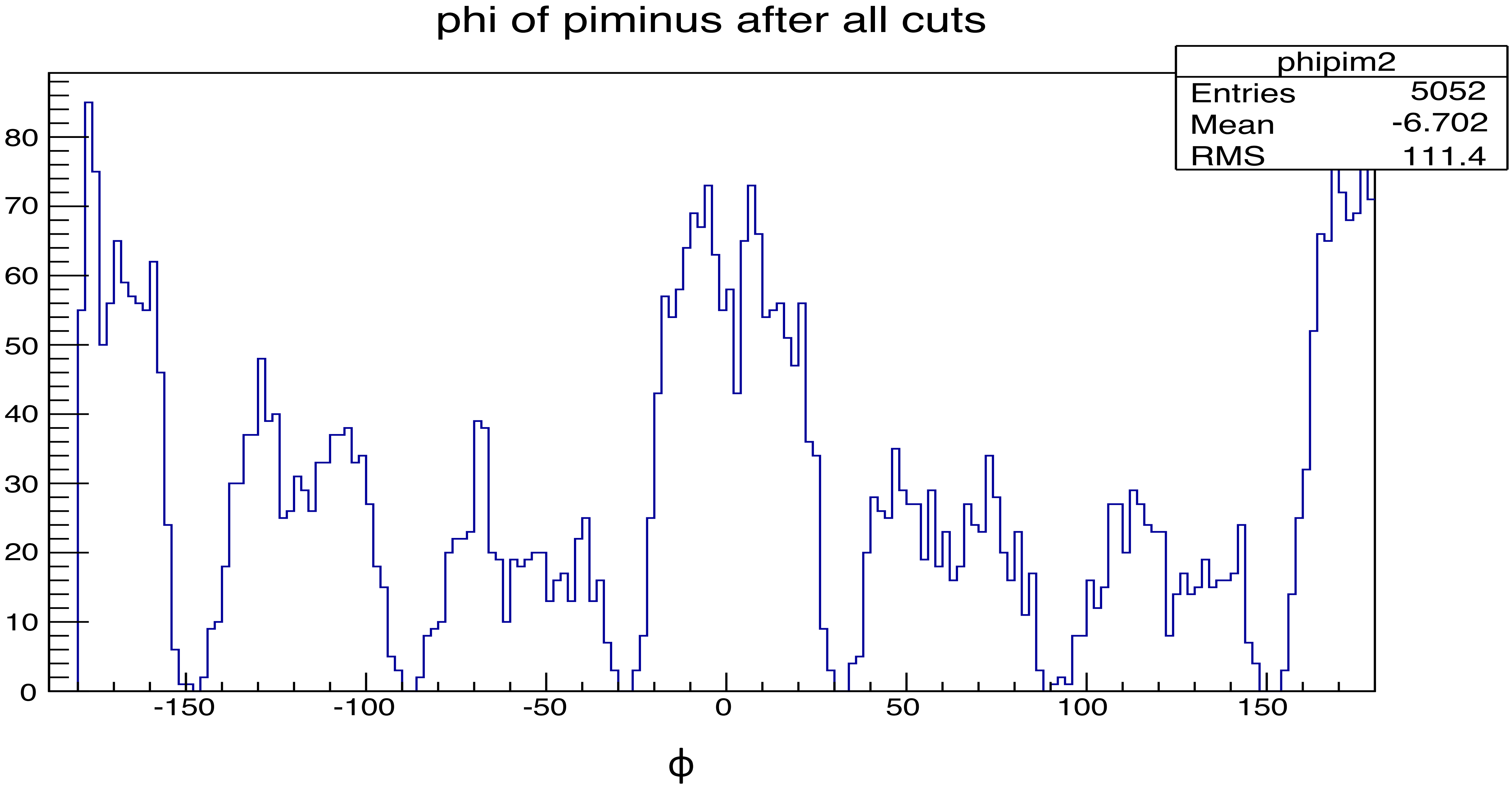
Φ distributions for PARA after all cuts applied

New !



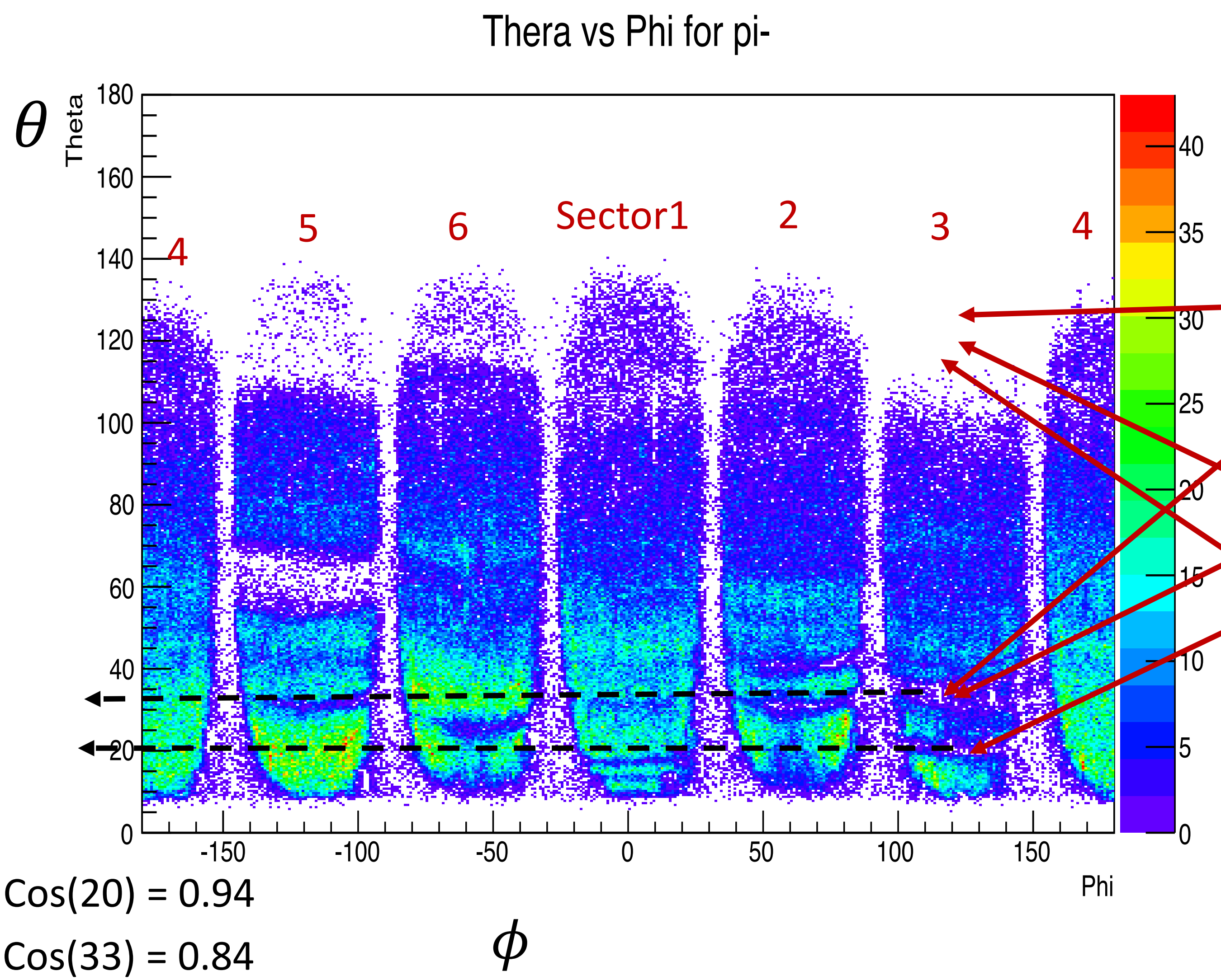
Φ distributions for AMO after all cuts applied

New !



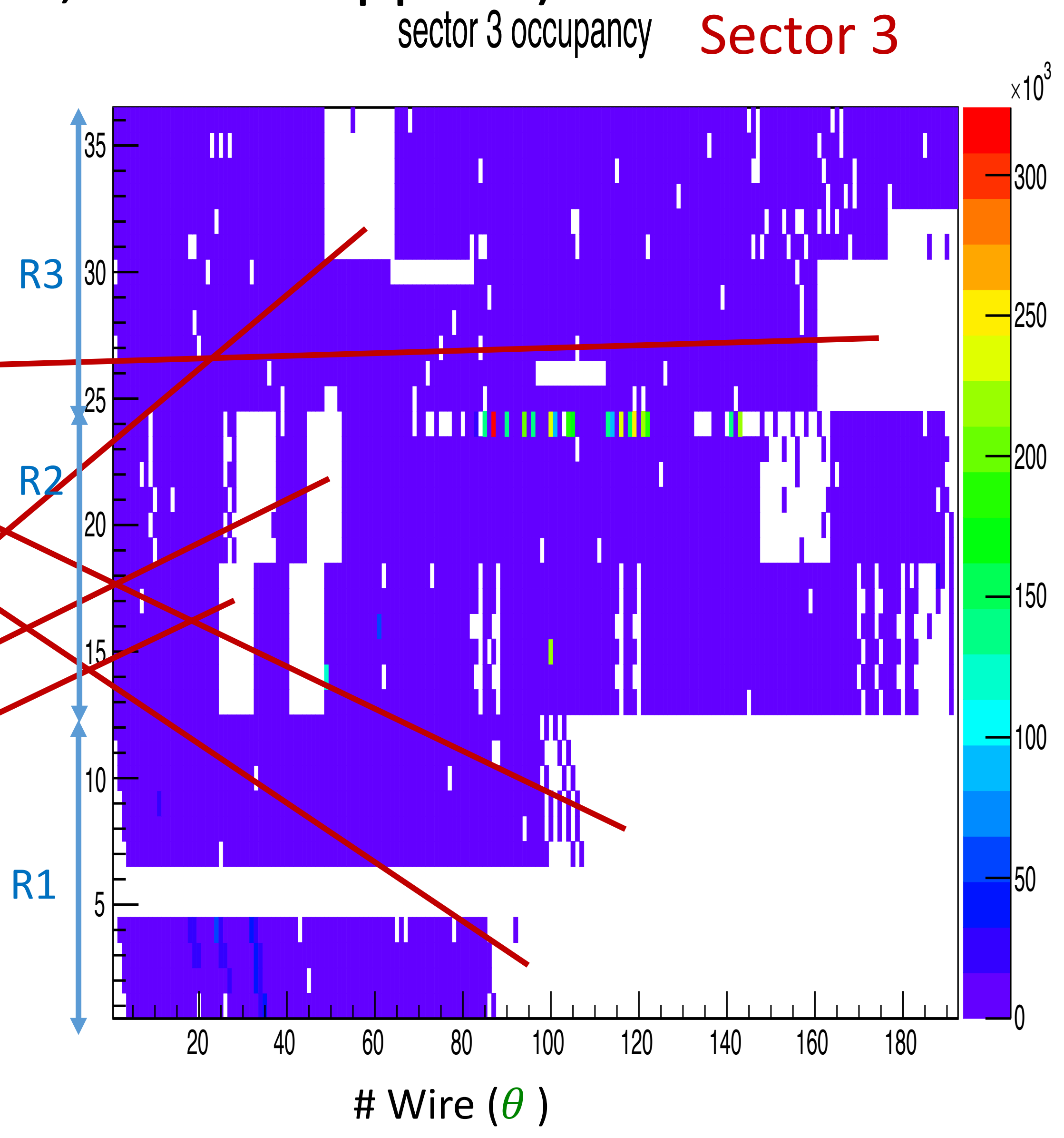
Fiducial distributions for π^- (Last 4 data, PERP, no cuts applied)

New !



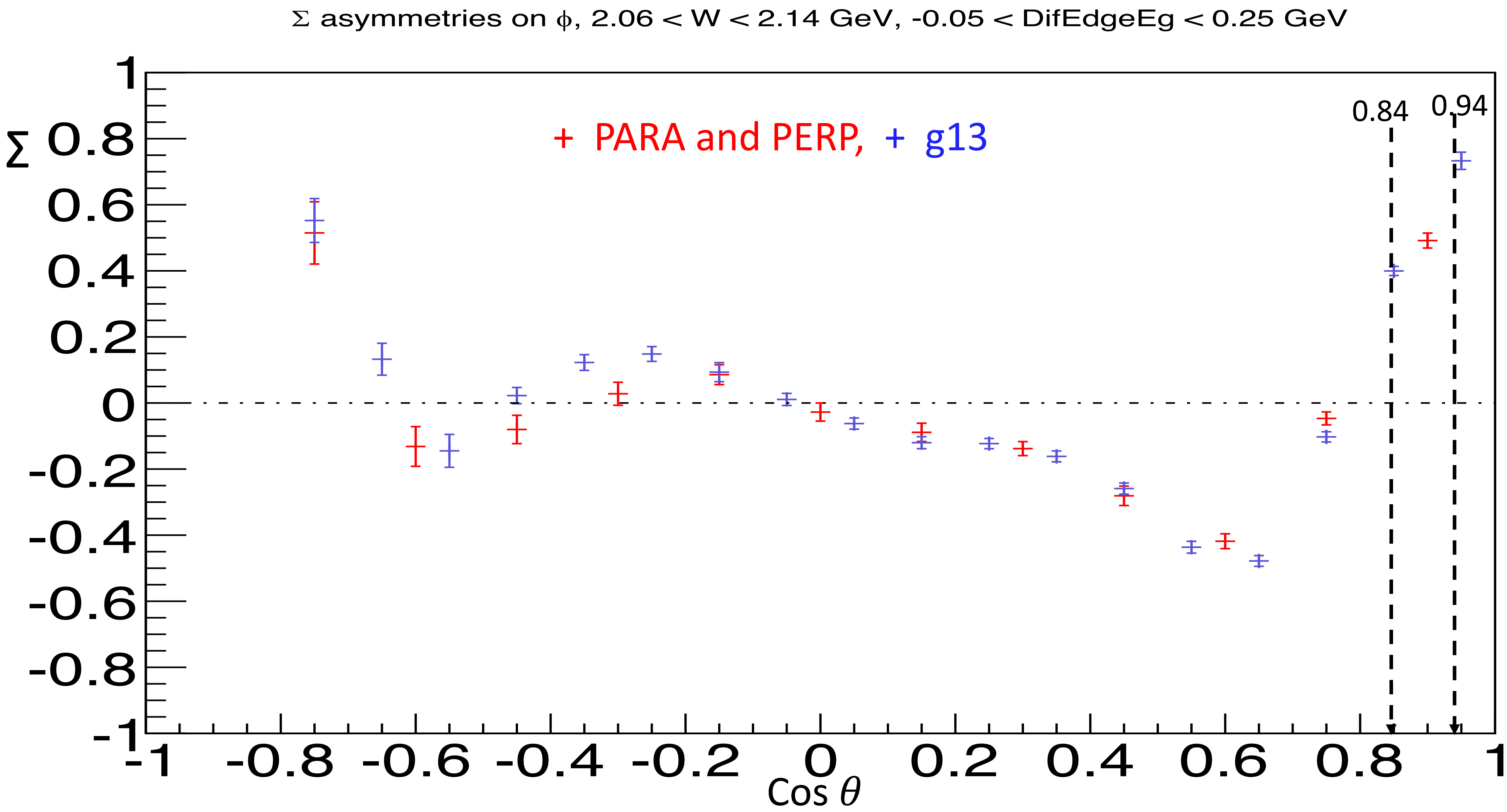
Cos(20) = 0.94

Cos(33) = 0.84



Σ asymmetries for 2.06 < fW < 2.14 Gev

New !



Application of Maximum Log-Likelihood method to g14 data

(1) Analyze same numbers of events from PERP and PARA (shown above)

Using

$\log L_T = \sum \log [1 + P_b \Sigma \cos(2\phi_i) - P_T P_b G \sin(2\phi_i)] + \sum \log [1 - P_b \Sigma \cos(2\phi_k) + P_T P_b G \sin(2\phi_k)]$

Use Monte Carlo (simply using random numbers) to check (I have a simple and Nick has more advanced (generate for ϕ, Σ and G).

(2) Separately analyze data,

for example PERP,

$\log L_T = \sum \log [1 + P_b \Sigma \cos(2\phi_i) - P_T P_b G \sin(2\phi_i)] / \int_{-\pi}^{\pi} (1 + P_b \Sigma \cos(2\phi) - P_T P_b G \sin(2\phi))\, d\phi$

the term of $\int_{-\pi}^{\pi} (1 + P_b \Sigma \cos(2\phi) - P_T P_b G \sin(2\phi))\, d\phi$ coveres the acceptance issues

Example : generate hist like g14 and use MLL

$Y = C(1 + 0.5*\cos(2\phi) - 0.5*\sin(2\phi))$ generated shown in histgram

New !