Selection on Φ_{HEL} angle

Parity check of solutions of Roberts' system of equations

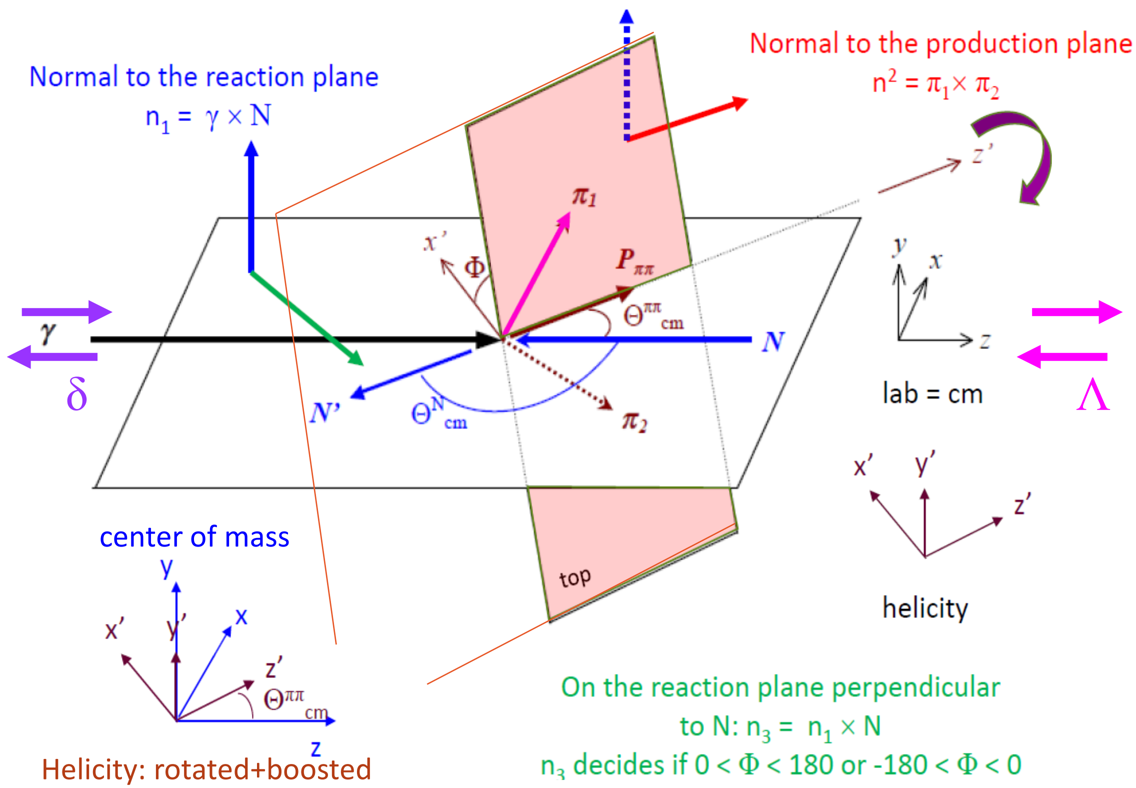
- Let's go back to the expressions for the solutions of the system of four equations coming from different helicities/spin target polarizations
- The only way to provide the correct negative parity to P_z is to explicit a dependence on φ_{hel} angle: $P_z \longrightarrow -P_z$ for negative φ_{hel}
- $I^{\odot}$ is ok as it is: there is direct dependence only on beam helicity
  - If helicities are swapped, the sign of δ is reversed as well: no change
- $P_z \longrightarrow -P_z$ for negative φ_{hel} only if $\Lambda_z \longrightarrow -\Lambda_z$
  - Can an explanation be found?

$$I_{\odot} = \frac{\dfrac{N_1^{\rightarrow\Rightarrow} - N_1^{\leftarrow\Rightarrow}}{\delta_{\odot 1}} + \dfrac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \dfrac{\mathscr{L}_{eff1}}{\mathscr{L}_{eff2}} \cdot \dfrac{N_2^{\rightarrow\Leftarrow} - N_2^{\leftarrow\Leftarrow}}{\delta_{\odot 2}}}{(N_1^{\rightarrow\Rightarrow} + N_1^{\leftarrow\Rightarrow}) + \dfrac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \dfrac{\mathscr{L}_{eff1}}{\mathscr{L}_{eff2}} \, (N_2^{\rightarrow\Leftarrow} + N_2^{\leftarrow\Leftarrow})}$$

$$P_z^{\odot} = \frac{1}{\Lambda_{z2}} \cdot \frac{\dfrac{N_1^{\rightarrow\Rightarrow} - N_1^{\leftarrow\Rightarrow}}{\delta_{\odot 1}} - \dfrac{\mathscr{L}_{eff1}}{\mathscr{L}_{eff2}} \cdot \dfrac{N_2^{\rightarrow\Leftarrow} - N_2^{\leftarrow\Leftarrow}}{\delta_{\odot 2}}}{(N_1^{\rightarrow\Rightarrow} + N_1^{\leftarrow\Rightarrow}) + \dfrac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \dfrac{\mathscr{L}_{eff1}}{\mathscr{L}_{eff2}} \, (N_2^{\rightarrow\Leftarrow} + N_2^{\leftarrow\Leftarrow})}$$

$$P_z = \frac{1}{\Lambda_{z2}} \cdot \frac{(N_1^{\rightarrow\Rightarrow} + N_1^{\leftarrow\Rightarrow}) - \dfrac{\mathscr{L}_{eff1}}{\mathscr{L}_{eff2}} \cdot (N_2^{\rightarrow\Leftarrow} + N_2^{\leftarrow\Leftarrow})}{(N_1^{\rightarrow\Rightarrow} + N_1^{\leftarrow\Rightarrow}) + \dfrac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \dfrac{\mathscr{L}_{eff1}}{\mathscr{L}_{eff2}} \, (N_2^{\rightarrow\Leftarrow} + N_2^{\leftarrow\Leftarrow})}$$

Over and again on the event topology



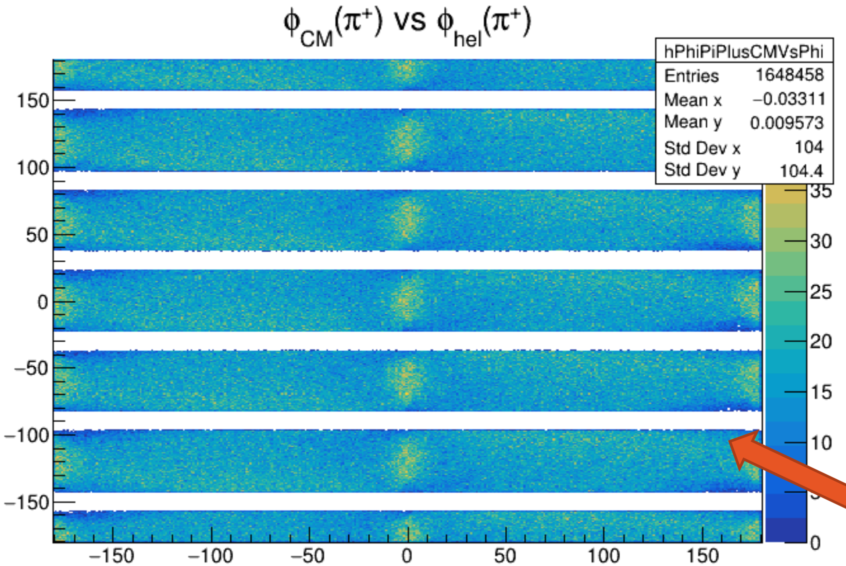
$$\cos\Phi = \frac{(\vec{\gamma}\times\vec{N})\cdot(\vec{\pi_1}\times\vec{\pi_2})}{|\vec{\gamma}\times\vec{N}||\vec{\pi_1}\times\vec{\pi_2}|}$$

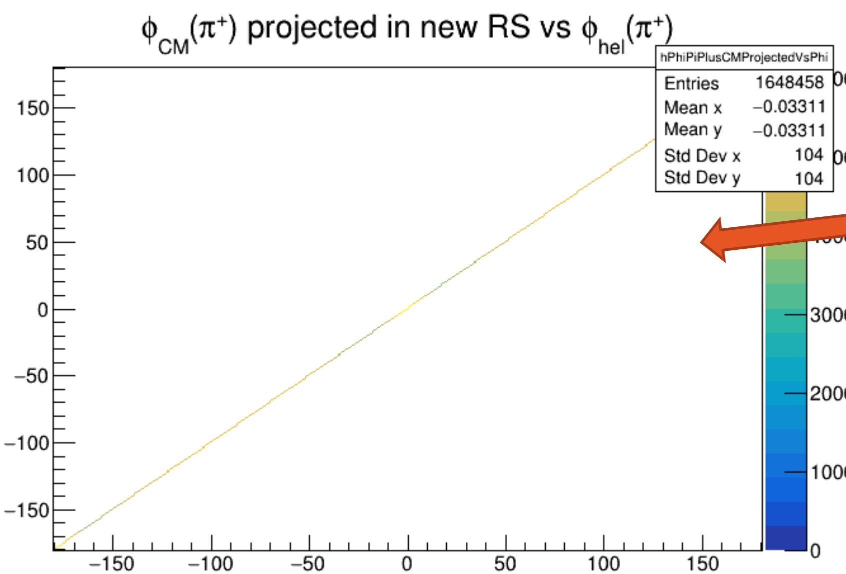
$$\sin\Phi = \frac{\left((\vec{\gamma}\times\vec{N})\times\vec{N}\right)\cdot(\vec{\pi_1}\times\vec{\pi_2})}{|(\vec{\gamma}\times\vec{N})\times\vec{N}||\vec{\pi_1}\times\vec{\pi_2}|}$$

- If $\sin\Phi > 0$: Φ is positive → $\Phi = \arccos(\cos\Phi)$
- If $\sin\Phi < 0$: Φ is negative → $\Phi = -\arccos(\cos\Phi)$

- Consider the reaction c.m. as Roberts does
- The pink plane contains the three vectors π_1, π_2 and $P_{\pi\pi}$ ($-P_{N'}$)
- Φ is the angle between the white and the pink plane
- If the pions are boosted in the dipion rest frame, they are back-to-back but still lie on the pink plane, so φ_{hel} defined in the helicity frame is the same as Φ defined in c.m. frame, in the new x'y'z' axes system
  - y and y' coincide in the two RS
- The Λ vector, as a spin, is not subject to Lorentz transformations from lab to c.m.
  - Opposite/same verse to the direction of the proton in the center of mass

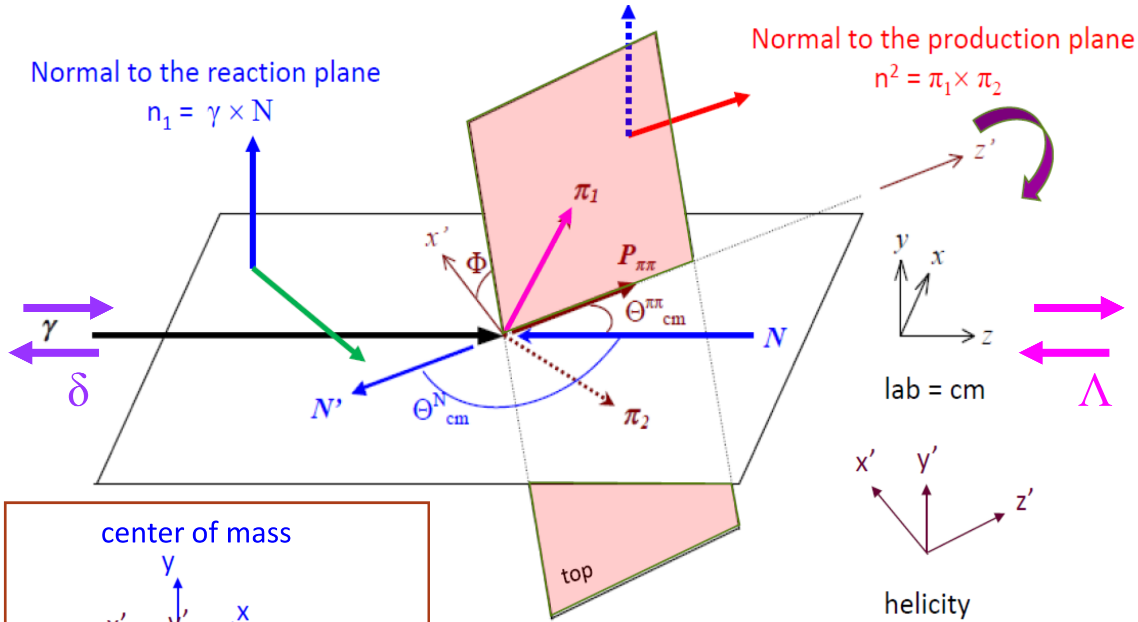
THE C.M. AND Φ HELICITY ANGLES ARE THE SAME

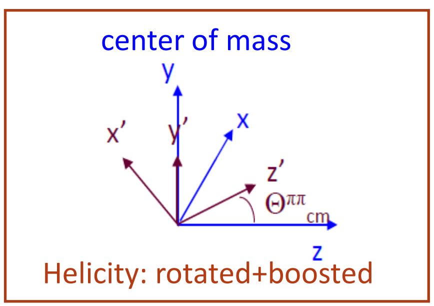


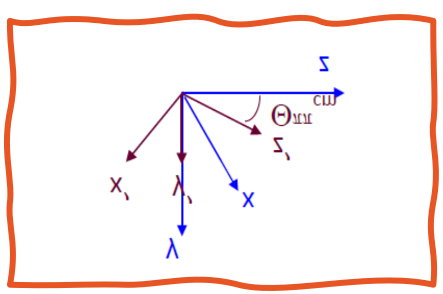


- Use of phase space montecarlo (here modulated with the apparatus acceptance)
- The φ angle of the π^+ momentum is compared as defined in different reference systems
  - π^+ CM momentum defined in xyz reference system vs π^+ momentum in helicity reference system x'y'z' (rotated + boosted)
  - π^+ CM momentum rotated in new x'y'z' vs π^+ momentum in helicity reference system
    - The φ angle in the same for boosted/no-boosted vectors

Over and again on the event topology







If the Φ angle is negative:

- y, y' axes are flipped
- the $\Theta_{\pi\pi}$ angle is sign-reversed
- to keep $\Theta_{\pi\pi}$ sign, z needs to be flipped

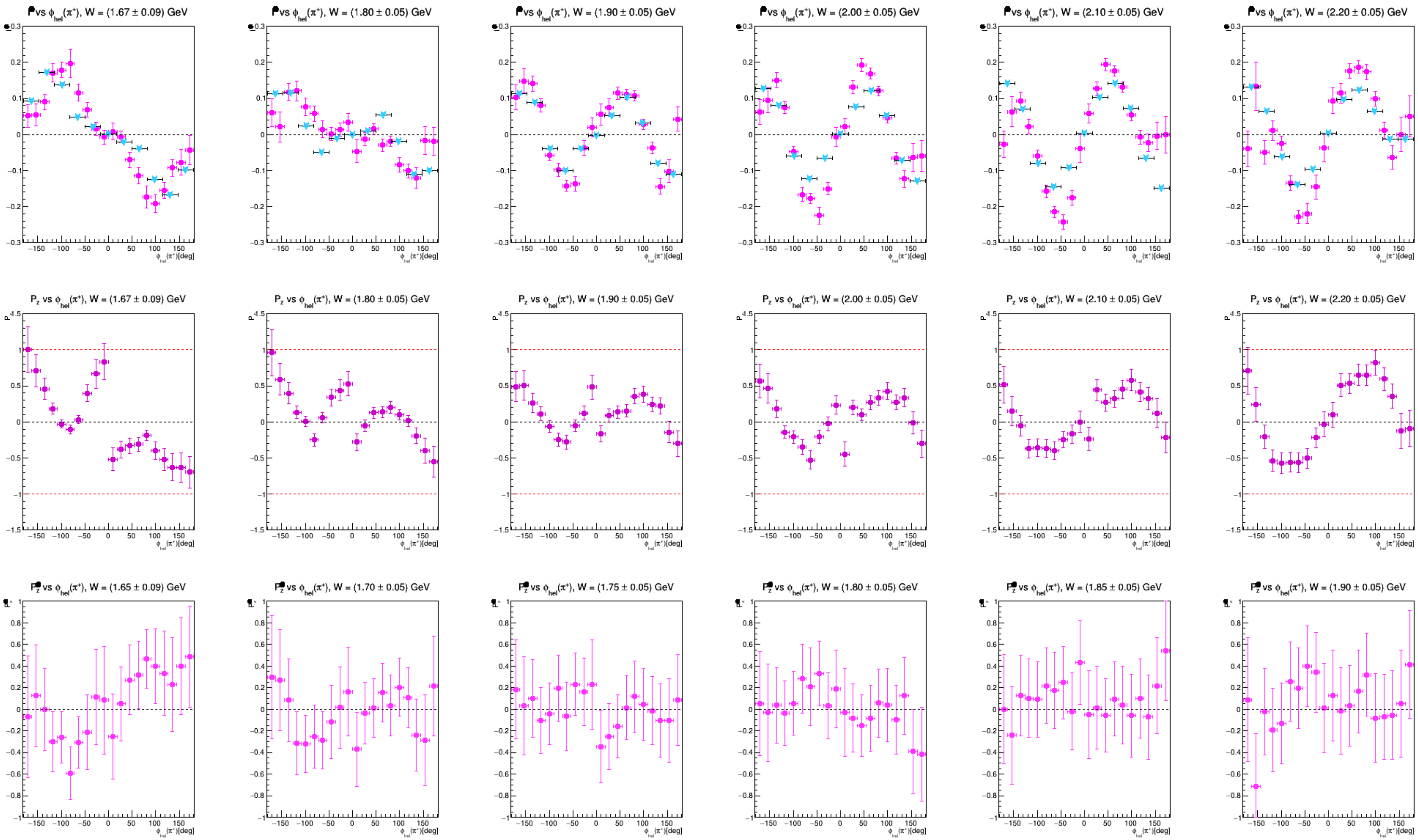
(and all vectors along z)

- **Positive Φ:**
- π_1 emitted above the reaction plane
- Positive y, y'
- Λ positive/negative (constant sign in a dataset)

- **Negative Φ:**
- π_1 emitted below the reaction plane
- Negative y, y'
- Λ negative/positive: the sign is reversed in the frame whose z is aligned along the dipion flight direction

- If this is correct, if Φ is negative one needs to flip the sign of the target polarization
  - The same holds for the beam helicities
  - This means to swap set1/set2 in the formulas (taking care of proper normalization factors)
  - P_z gets "by construction" an odd symmetry with respect to Φ

Solutions with imposed symmetry



Still to be understood

- P_z is odd but shows a discontinuity at 0 deg, which however reduces with the energy increase
  - The trend becomes very smooth in the last bin
  - Maybe it's normal? At 0 deg, P_z should be zero (since it is odd)
    - Can't we ask Roberts if this makes sense?
    - Try to stagger the angular bins to include zero as bin center? (now it is on the edge of two consecutive bins)
- $P^{\odot}{}_z$?
  - The reversal of data sets, which implies $\delta \rightarrow -\delta$ and $\Lambda_z \rightarrow -\Lambda_z$, should provide a flip sign as well
    - But the function is basically zero within the big errors, so the significance of the shape is relatively small