Polarized Gluon

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1 Gluon-Gluon

1.1 Vertex

$$
\begin{aligned}
&\frac{1}{2}G_{\mu\alpha}(x)\epsilon_{\lambda\rho\nu\beta}G^{\nu\beta}(0)\\
&\to \frac{g^2N_c\Gamma(d/2-1)}{8\pi^2(z_3^2)^{d/2-1}}\int_0^1 \mathrm{d}u\left\{\left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right)G_{\mu\alpha}(uz)\epsilon_{\lambda\rho\nu\beta}\left(x^\beta G_x{}^\nu(0)-z^\nu G_x{}^\beta(0)\right)\right.\\
&\quad\left.+\left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right)\left(z_\alpha G_{x\mu}(uz)-z_\mu G_{x\alpha}(uz)\right)\epsilon_{\lambda\rho\nu\beta}G^{\nu\beta}(0)\right\}\\
&\quad+\frac{g^2N_c\Gamma(d/2-2)}{16\pi^2(z_3^2)^{d/2-2}}\int_0^1 \mathrm{d}u 2\left[\frac{\bar{u}^{3-d}-1}{d-3}\right]_+G_{\mu\alpha}(uz)\epsilon_{\lambda\rho\nu\beta}G^{\nu\beta}(0)
\end{aligned}
$$

1.1.1 Transverse sum, $\alpha,\rho=i,i$

$$
\begin{aligned}
&\frac{1}{2}G_{\mu i}(x)\epsilon_{\lambda i\nu\beta}G^{\nu\beta}(0)\\
&\to \frac{g^2N_c\Gamma(d/2-1)}{8\pi^2(z_3^2)^{d/2-1}}\int_0^1 \mathrm{d}u\left\{\left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right)G_{\mu i}(uz)\epsilon_{\lambda i\nu\beta}\left(x^\beta G_x{}^\nu(0)-z^\nu G_x{}^\beta(0)\right)\right.\\
&\quad\left.+\left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right)\left(z_i G_{x\mu}(uz)-z_\mu G_{xi}(uz)\right)\epsilon_{\lambda i\nu\beta}G^{\nu\beta}(0)\right\}\\
&\quad+\frac{g^2N_c\Gamma(d/2-2)}{16\pi^2(z_3^2)^{d/2-2}}\int_0^1 \mathrm{d}u 2\left[\frac{\bar{u}^{3-d}-1}{d-3}\right]_+G_{\mu i}(uz)\epsilon_{\lambda i\nu\beta}G^{\nu\beta}(0)
\end{aligned}
$$

1.1.2 $\mu,\lambda=0,0$

$$
\begin{aligned}
&G_{0i}(x)\tilde{G}_{0i}(0)\\
&\to \frac{g^2N_c\Gamma(d/2-1)}{8\pi^2(z_3^2)^{d/2-1}}\int_0^1 \mathrm{d}u\left\{\left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right)G_{0i}(uz)\epsilon_{0i\nu\beta}\left(x^\beta G_x{}^\nu(0)-z^\nu G_x{}^\beta(0)\right)\right.\\
&\quad\left.+\left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right)\left(z_i G_{x0}(uz)-z_0 G_{xi}(uz)\right)\epsilon_{0i\nu\beta}G^{\nu\beta}(0)\right\}\\
&\quad+\frac{g^2N_c\Gamma(d/2-2)}{16\pi^2(z_3^2)^{d/2-2}}\int_0^1 \mathrm{d}u 2\left[\frac{\bar{u}^{3-d}-1}{d-3}\right]_+G_{0i}(uz)\epsilon_{0i\nu\beta}G^{\nu\beta}(0)\\
&=\frac{g^2N_c\Gamma(d/2-1)}{4\pi^2(z_3^2)^{d/2-2}}\int_0^1 \mathrm{d}u\left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right)G_{0i}(uz)\tilde{G}_{0i}(0)\\
&\quad+\frac{g^2N_c\Gamma(d/2-2)}{4\pi^2(z_3^2)^{d/2-2}}\int_0^1 \mathrm{d}u\left[\frac{\bar{u}^{3-d}-1}{d-3}\right]_+G_{0i}(uz)\tilde{G}_{0i}(0)
\end{aligned}
$$

1.1.3 $\mu, \lambda = 3, 3$

$$
\begin{aligned}
&G_{3i}(x)\tilde{G}_{3i}(0) \\
&\to \frac{g^2 N_c \Gamma(d/2-1)}{4\pi^2 (z_3^2)^{d/2-2}} \int_0^1 \mathrm{d}u \left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right) G_{3i}(uz)\tilde{G}_{3i}(0) \\
&\quad + \frac{g^2 N_c \Gamma(d/2-2)}{4\pi^2 (z_3^2)^{d/2-2}} \int_0^1 \mathrm{d}u \left[\frac{\bar{u}^{3-d}-1}{d-3}\right]_+ G_{3i}(uz)\tilde{G}_{3i}(0)
\end{aligned}
$$

1.2 $\mu, \lambda = j, j$

$$
\begin{aligned}
&G_{ji}(x)\tilde{G}_{ji}(0) \\
&\to \frac{g^2 N_c \Gamma(d/2-1)}{4\pi^2 (z_3^2)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ \left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right) G_{ji}(uz)\tilde{G}_{ji}(0) \right\} \\
&\quad + \frac{g^2 N_c \Gamma(d/2-2)}{4\pi^2 (z_3^2)^{d/2-2}} \int_0^1 \mathrm{d}u \left[\frac{\bar{u}^{3-d}-1}{d-3}\right]_+ G_{ji}(uz)\tilde{G}_{ji}(0)
\end{aligned}
$$

1.2.1 $\mu, \lambda = 3, 0$

$$
\begin{aligned}
&G_{3i}(x)\tilde{G}_{0i}(0) \\
&\to \frac{g^2 N_c \Gamma(d/2-1)}{2\pi^2 (z_3^2)^{d/2-2}} \int_0^1 \mathrm{d}u \left(\frac{\bar{u}^{3-d}-\bar{u}}{d-2}\right) G_{3i}(uz)\tilde{G}_{0i}(0) \\
&\quad + \frac{g^2 N_c \Gamma(d/2-2)}{4\pi^2 (z_3^2)^{d/2-2}} \int_0^1 \mathrm{d}u \left[\frac{\bar{u}^{3-d}-1}{d-3}\right]_+ G_{3i}(uz)\tilde{G}_{0i}(0)
\end{aligned}
$$

1.2.2 $\mu, \lambda = 0, 3$

$$
\begin{aligned}
&G_{0i}(x)\tilde{G}_{3i}(0) \\
&\to \frac{g^2 N_c \Gamma(d/2-2)}{4\pi^2 (z_3^2)^{d/2-2}} \int_0^1 \mathrm{d}u \left[\frac{\bar{u}^{3-d}-1}{d-3}\right]_+ G_{0i}(uz)\tilde{G}_{3i}(0)
\end{aligned}
$$

1.3 Box

$\Gamma(d/2)$

$$
(\epsilon_{\sigma\rho\mu z} z_\lambda - \epsilon_{\sigma\rho\lambda z} z_\mu) \frac{g^2 N_c \Gamma(d/2)}{2\pi^2 \left(z_3^2\right)^{d/2}} \int_0^1 \mathrm{d}u \frac{\bar{u}^3}{6} G_{x\xi}(uz) G_x{}^{\xi}(0)
$$

$\Gamma(d/2-1)$

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-1}} \int_0^1 \mathrm{d}u \Big\{ \epsilon_{\sigma\rho\mu\lambda} \frac{2\bar{u}^3}{3} G_{x\xi}(uz) G_x^{\ \xi}(0)$$
$$+\frac{\bar{u}^3}{3} \left(-\epsilon_{\sigma\rho\lambda}{}^{\nu} G_{x\nu}(uz) G_{x\mu}(0) + \epsilon_{\sigma\rho\mu}{}^{\nu} G_{x\nu}(uz) G_{x\lambda}(0)\right)$$
$$+(2u\bar{u}+\frac{\bar{u}^3}{3}) \left(-\epsilon_{\sigma\rho\lambda}{}^{\nu} G_{x\mu}(uz) G_{x\nu}(0) + \epsilon_{\sigma\rho\mu}{}^{\nu} G_{x\lambda}(uz) G_{x\nu}(0)\right)$$
$$+\bar{u}^2 \Big(\epsilon_{\sigma\rho z}{}^{\eta} G_{\lambda\eta}(uz) G_{x\mu}(0) - \epsilon_{\sigma\rho z}{}^{\eta} G_{\mu\eta}(uz) G_{x\lambda}(0) - \epsilon_{\sigma\rho}{}^{\nu\eta} z_\mu G_{x\nu}(uz) G_{\lambda\eta}(0) + \epsilon_{\sigma\rho}{}^{\nu\eta} z_\lambda G_{x\nu}(uz) G_{\mu\eta}(0) \Big)$$
$$+\bar{u}(1+u) \Big(\epsilon_{\sigma\rho z}{}^{\eta} G_{x\mu}(uz) G_{\lambda\eta}(0) - \epsilon_{\sigma\rho z}{}^{\eta} G_{x\lambda}(uz) G_{\mu\eta}(0) - \epsilon_{\sigma\rho}{}^{\nu\eta} z_\mu G_{\lambda\eta}(uz) G_{x\nu}(0) + \epsilon_{\sigma\rho}{}^{\nu\eta} z_\lambda G_{\mu\eta}(uz) G_{x\nu}(0) \Big)$$
$$+(\frac{\bar{u}^2}{2}-\frac{\bar{u}^3}{3}) \Big(\epsilon_{\sigma\rho z\lambda} G_{\mu\xi}(uz) G_x^{\ \xi}(0) - \epsilon_{\sigma\rho z\mu} G_{\lambda\xi}(uz) G_x^{\ \xi}(0) + \epsilon_{\sigma\rho z\lambda} G_{x\xi}(uz) G_\mu^{\ \xi}(0) - \epsilon_{\sigma\rho z\mu} G_{x\xi}(uz) G_\lambda^{\ \xi}(0)$$
$$-\epsilon_{\sigma\rho\lambda}{}^{\nu} z_\mu G_{\nu\xi}(uz) G_x^{\ \xi}(0) + \epsilon_{\sigma\rho\mu}{}^{\nu} z_\lambda G_{\nu\xi}(uz) G_x^{\ \xi}(0) - \epsilon_{\sigma\rho\lambda}{}^{\nu} z_\mu G_{x\xi}(uz) G_\nu^{\ \xi}(0) + \epsilon_{\sigma\rho\mu}{}^{\nu} z_\lambda G_{x\xi}(uz) G_\nu^{\ \xi}(0) \Big)$$
$$+2\bar{u}\left(\epsilon_{\sigma\rho z}{}^{\eta} z_\mu G_{\lambda\xi}(uz) G_\eta^{\ \xi}(0) - \epsilon_{\sigma\rho z}{}^{\eta} z_\lambda G_{\mu\xi}(uz) G_\eta^{\ \xi}(0)\right) - \frac{\bar{u}^3}{6}\left(\epsilon_{\sigma\rho z\lambda} z_\mu - \epsilon_{\sigma\rho z\mu} z_\lambda\right) G_{\eta\xi}(uz) G^{\eta\xi}(0) \Big\}$$

$\Gamma(d/2-2)$

$$\frac{1}{2}\epsilon_{\sigma\rho}{}^{\nu\eta} \frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \Big\{ -\bar{u}\left(G_{\lambda\eta}(uz) G_{\mu\nu}(0) - G_{\mu\eta}(uz) G_{\lambda\nu}(0) - G_{\lambda\nu}(uz) G_{\mu\eta}(0) + G_{\mu\nu}(uz) G_{\lambda\eta}(0)\right) - 2u G_{\mu\lambda}(uz) G_{\nu\eta}(0)$$
$$+\bar{u}(1-2u) G_{\nu\eta}(uz) G_{\mu\lambda}(0) + \bar{u}(1+2u) G_{\mu\lambda}(uz) G_{\nu\eta}(0)$$
$$+\frac{\bar{u}u^2}{2}\left(g_{\lambda\eta} G_{\mu\xi}(uz) G_\nu^{\ \xi}(0) - g_{\mu\eta} G_{\lambda\xi}(uz) G_\nu^{\ \xi}(0) - g_{\lambda\nu} G_{\mu\xi}(uz) G_\eta^{\ \xi}(0) + g_{\mu\nu} G_{\lambda\xi}(uz) G_\eta^{\ \xi}(0)\right)$$
$$+\frac{\bar{u}u^2}{2}\left(g_{\lambda\eta} G_{\nu\xi}(uz) G_\mu^{\ \xi}(0) - g_{\mu\eta} G_{\nu\xi}(uz) G_\lambda^{\ \xi}(0) - g_{\lambda\nu} G_{\eta\xi}(uz) G_\mu^{\ \xi}(0) + g_{\mu\nu} G_{\eta\xi}(uz) G_\lambda^{\ \xi}(0)\right)$$
$$+\bar{u}\left(g_{\mu\nu} G_{\lambda\xi}(uz) G_\eta^{\ \xi}(0) - g_{\lambda\nu} G_{\mu\xi}(uz) G_\eta^{\ \xi}(0) - g_{\mu\eta} G_{\lambda\xi}(uz) G_\nu^{\ \xi}(0) + g_{\lambda\eta} G_{\mu\xi}(uz) G_\nu^{\ \xi}(0)\right) - \left(g_{\mu\nu} g_{\lambda\eta} - g_{\nu\lambda} g_{\mu\eta}\right) \frac{\bar{u}^3}{6} G_{\zeta\xi}(uz) G^{\zeta\xi}(0) \Big\}$$

1.3.1 Transverse sum, $\lambda, \rho = i, i$

$\Gamma(d/2-1)$

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-1}} \int_0^1 \mathrm{d}u \Big\{ \frac{\bar{u}^3}{3}\left(\epsilon_{\sigma i\mu}{}^{\nu} G_{x\nu}(uz) G_{xi}(0)\right) + (2u\bar{u}+\frac{\bar{u}^3}{3})\left(\epsilon_{\sigma i\mu}{}^{\nu} G_{xi}(uz) G_{x\nu}(0)\right)$$
$$+\bar{u}^2 \Big(\epsilon_{\sigma iz}{}^{\eta} G_{i\eta}(uz) G_{x\mu}(0) - \epsilon_{\sigma iz}{}^{\eta} G_{\mu\eta}(uz) G_{xi}(0) - \epsilon_{\sigma i}{}^{\nu\eta} z_\mu G_{x\nu}(uz) G_{i\eta}(0) \Big)$$
$$+\bar{u}(1+u) \Big(\epsilon_{\sigma iz}{}^{\eta} G_{x\mu}(uz) G_{i\eta}(0) - \epsilon_{\sigma iz}{}^{\eta} G_{xi}(uz) G_{\mu\eta}(0) - \epsilon_{\sigma i}{}^{\nu\eta} z_\mu G_{i\eta}(uz) G_{x\nu}(0) \Big)$$
$$+(\frac{\bar{u}^2}{2}-\frac{\bar{u}^3}{3}) \Big(-\epsilon_{\sigma iz\mu} G_{i\xi}(uz) G_x^{\ \xi}(0) - \epsilon_{\sigma iz\mu} G_{x\xi}(uz) G_i^{\ \xi}(0) \Big) + 2\bar{u}\left(\epsilon_{\sigma iz}{}^{\eta} z_\mu G_{i\xi}(uz) G_\eta^{\ \xi}(0)\right) \Big\}$$

$\Gamma(d/2-2)$

$$\frac{1}{2}\epsilon_{\sigma i}{}^{\nu\eta} \frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \Big\{ -\bar{u}\left(G_{i\eta}(uz) G_{\mu\nu}(0) - G_{\mu\eta}(uz) G_{i\nu}(0) - G_{i\nu}(uz) G_{\mu\eta}(0) + G_{\mu\nu}(uz) G_{i\eta}(0)\right) - 2u G_{\mu i}(uz) G_{\nu\eta}(0)$$
$$+\bar{u}(1-2u) G_{\nu\eta}(uz) G_{\mu i}(0) + \bar{u}(1+2u) G_{\mu i}(uz) G_{\nu\eta}(0)$$
$$+\frac{\bar{u}u^2}{2}\left(-g_{\mu\eta} G_{i\xi}(uz) G_\nu^{\ \xi}(0) + g_{\mu\nu} G_{i\xi}(uz) G_\eta^{\ \xi}(0)\right) + \frac{\bar{u}u^2}{2}\left(-g_{\mu\eta} G_{\nu\xi}(uz) G_i^{\ \xi}(0) + g_{\mu\nu} G_{\eta\xi}(uz) G_i^{\ \xi}(0)\right)$$
$$+\bar{u}\left(g_{\mu\nu} G_{i\xi}(uz) G_\eta^{\ \xi}(0) - g_{\mu\eta} G_{i\xi}(uz) G_\nu^{\ \xi}(0)\right) \Big\}$$

1.3.2 $\mu, \sigma = 0, 0$

$\Gamma(d/2-1)$

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ \bar{u}^2 \left(G_{ij}(uz)\tilde{G}_{ij}(0) - G_{0j}(uz)\tilde{G}_{0j}(0) \right) + \bar{u}(1+u) \left(-2G_{30}(uz)\tilde{G}_{30}(0) + G_{3i}(uz)\tilde{G}_{3i}(0) \right) \right\}$$

$\Gamma(d/2-2)$

$$\frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ -\bar{u}\tfrac{1}{2}\epsilon_{0i}{}^{\nu\eta} \left(G_{i\eta}(uz)G_{0\nu}(0) - G_{0\eta}(uz)G_{i\nu}(0) - G_{i\nu}(uz)G_{0\eta}(0) + G_{0\nu}(uz)G_{i\eta}(0) \right) - 2uG_{0i}(uz)\tilde{G}_{0i}(0) \right.$$
$$\left. -\bar{u}(1-2u)G_{3i}(uz)\tilde{G}_{3i}(0) + \bar{u}(1+2u)G_{0i}(uz)\tilde{G}_{0i}(0) \right\}$$
$$= \frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ \bar{u} \left(-2G_{3i}(uz)\tilde{G}_{3i}(0) + 2G_{0i}(uz)\tilde{G}_{0i}(0) - G_{ij}(uz)\tilde{G}_{ij}(0) + 2G_{30}(uz)\tilde{G}_{30}(0) \right) \right.$$
$$\left. -2uG_{0i}(uz)\tilde{G}_{0i}(0) + 2\bar{u}uG_{0i}(uz)\tilde{G}_{0i}(0) + 2\bar{u}uG_{3i}(uz)\tilde{G}_{3i}(0) \right\}$$

1.3.3 $\mu, \sigma = 3, 3$

$\Gamma(d/2-1)$

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ \bar{u}^2 \left(2G_{30}(uz)\tilde{G}_{30}(0) - G_{3j}(uz)\tilde{G}_{3j}(0) \right) + \bar{u}(1+u) \left(G_{0i}(uz)\tilde{G}_{0i}(0) - G_{ij}(uz)\tilde{G}_{ij}(0) \right) \right\}$$

$\Gamma(d/2-2)$

$$\frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ \bar{u} \left(2G_{3i}(uz)\tilde{G}_{3i}(0) - 2G_{0i}(uz)\tilde{G}_{0i}(0) + G_{ij}(uz)\tilde{G}_{ij}(0) - 2G_{30}(uz)\tilde{G}_{30}(0) \right) \right.$$
$$\left. -2uG_{3i}(uz)\tilde{G}_{3i}(0) + 2\bar{u}uG_{0i}(uz)\tilde{G}_{0i}(0) + 2\bar{u}uG_{3i}(uz)\tilde{G}_{3i}(0) \right\}$$

1.4 $\mu, \sigma = j, j$

$\Gamma(d/2-1)$

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ 2\bar{u}^2 G_{0i}(uz)\tilde{G}_{0i}(0) - 2\bar{u}(1+u)G_{3i}(uz)\tilde{G}_{3i}(0) \right\}$$

$\Gamma(d/2-2)$

$$-\frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ 2\bar{u} \left(G_{0i}(uz)\tilde{G}_{0i}(0) - G_{3i}(uz)\tilde{G}_{3i}(0) \right) \right.$$
$$\left. + 2\bar{u}(1-2u)G_{30}(uz)\tilde{G}_{30}(0) + (\bar{u}-2)(1-2u)G_{ij}(uz)\tilde{G}_{ij}(0) \right\}$$

1.4.1 00+jj

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ \bar{u}^2 G_{0i}(uz)\tilde{G}_{0i}(0) - \bar{u}(1+u)G_{3i}(uz)\tilde{G}_{3i}(0) + \bar{u}^2 G_{ij}(uz)\tilde{G}_{ij}(0) - 2\bar{u}(1+u)G_{30}(uz)\tilde{G}_{30}(0) \right\}$$
$$+ \frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ (2\bar{u}u - 2u)G_{0i}(uz)\tilde{G}_{0i}(0) + 2\bar{u}uG_{3i}(uz)\tilde{G}_{3i}(0) - 2u^2 G_{ij}(uz)\tilde{G}_{ij}(0) + 4\bar{u}uG_{30}(uz)\tilde{G}_{30}(0) \right\}$$

1.4.2 00+33

$\Gamma(d/2-1)$

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u 2\bar{u}u \left(G_{0j}(uz)\tilde{G}_{0j}(0) + G_{3j}(uz)\tilde{G}_{3j}(0) - 2G_{30}(uz)\tilde{G}_{30}(0) - G_{ij}(uz)\tilde{G}_{ij}(0)\right)$$

$\Gamma(d/2-2)$

$$\frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left(4\bar{u}u - 2u \right) \left(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{3i}(uz)\tilde{G}_{3i}(0)\right)$$

1.4.3 $\mu, \sigma = 3, 0$

$\Gamma(d/2-1)$

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left(2\bar{u}G_{i0}(uz)\tilde{G}_{i3}(0) - 2\bar{u}^2 G_{3i}(uz)\tilde{G}_{0i}(0)\right)$$

$\Gamma(d/2-2)$

$$\frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ \left(4\bar{u}u - 2u\right) G_{3i}(uz)\tilde{G}_{0i}(0) - \bar{u}G_{0i}(uz)\tilde{G}_{3i}(0) + \bar{u}G_{3i}(uz)\tilde{G}_{0i}(0) \right\}$$

1.4.4 $\mu, \sigma = 0, 3$

$\Gamma(d/2-1)$

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u 2u\bar{u}G_{3i}(uz)\tilde{G}_{0i}(0)$$

$\Gamma(d/2-2)$

$$\frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ \left(4\bar{u}u - 2u\right) G_{0i}(uz)\tilde{G}_{3i}(0) + \bar{u}G_{0i}(uz)\tilde{G}_{3i}(0) - \bar{u}G_{3i}(uz)\tilde{G}_{0i}(0) \right\}$$

1.5 Link

$$\begin{aligned} &\frac{g^2 N_c \Gamma(d/2-1)}{4\pi^2 (z_3^2)^{d/2-2}} \frac{-1}{(d-3)(d-4)} G_{\mu\alpha}(z) G_{\nu\beta}(0) \\ &= \frac{g^2 N_c}{8\pi^2} \left\{ \frac{1}{\epsilon_{\mathrm{UV}}} + \log\left(z_3^2 e^{\gamma}\right) + 2 \right\} G_{\mu\alpha}(z)\tilde{G}_{\nu\beta}(0) \end{aligned}$$

1.6 Self energy

$$-\frac{g^2 N_c}{8\pi^2} \left(\frac{1}{\epsilon_{UV}} + \log\left(z_3^2 e^{\gamma}\right) + \frac{1}{\epsilon_{IR}} - \log\left(z_3^2 e^{\gamma}\right) \right) \frac{1}{6} G_{\mu\alpha}(z)\tilde{G}_{\lambda\beta}(0)$$

1.7 Kernel

Using:

$$\begin{aligned} \int_0^1 \mathrm{d}u \int_0^u \mathrm{d}v f(u,v) \left\langle F(ux)F(vx) \right\rangle &= \int_0^1 \mathrm{d}v \int_v^1 \mathrm{d}u f(u,v) \left\langle F((u-v)x)F(0) \right\rangle \\ &= \int_0^1 \mathrm{d}u \left\langle F(ux)F(0) \right\rangle \int_0^{\bar{u}} \mathrm{d}v f(u+v, v) \\ &= \int_0^1 \mathrm{d}u \left\langle F(ux)F(0) \right\rangle \int_0^{\bar{u}} \mathrm{d}v f(\bar{v}, \bar{u}-v) \end{aligned}$$

The kernel should be:

$$
\begin{aligned}
\mathcal{K}^{gg,A}\left(\bar{u},v\right)/N_c &= \Big\{4(u-v)+\left[u^2/\bar{u}\right]_+\delta(v)+\left[\bar{v}^2/v\right]_+\delta(\bar{u})\Big\}+(11/6-3)\,\delta(\bar{u})\delta(v) \\
&\to \int_0^{\bar{u}} \mathrm{d}v \Big\{4u+\left[(u+v)^2/(1-u-v)\right]_+\delta(v)+\left[\bar{v}^2/v\right]_+\delta(\bar{u}-v)\Big\}+(11/6-3)\,\delta(\bar{u}-v)\delta(v) \\
&= \Big\{4u\bar{u}+2\left[u^2/\bar{u}\right]_+\Big\}+(11/6-3)\,\delta(\bar{u}) \\
&= \left\{4u\bar{u}-2u+2\left[\frac{u}{\bar{u}}\right]_+\right\}+(11/6-2)\,\delta(\bar{u}) \\
&= 4u\bar{u}-2u+2\left[\frac{u}{\bar{u}}\right]_+-\frac{1}{6}\delta(\bar{u})
\end{aligned}
$$

1.8 Full terms

1.8.1 00

$$
\begin{aligned}
&\to \frac{g^2N_c}{8\pi^2}\left\{\frac{1}{\epsilon_{\mathrm{UV}}}+\log\left(z_3^2e^\gamma\right)\right\}\frac{8}{6}G_{0i}(z)\tilde{G}_{0i}(0) \\
&+\frac{g^2N_c}{8\pi^2}\int_0^1\mathrm{d}u\left[\left(\frac{1}{\bar{u}}-\bar{u}\right)_+-\left(\frac{4u}{\bar{u}}+\frac{4\log(1-u)}{\bar{u}}\right)_+ +2\delta(\bar{u})-\bar{u}^2\right]G_{0i}(uz)\tilde{G}_{0i}(0) \\
&+\frac{g^2N_c}{8\pi^2}\int_0^1\mathrm{d}u\bar{u}(1+u)\Big(\;G_{3i}(uz)\tilde{G}_{3i}(0)\;\Big) \\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1\mathrm{d}u\left[2\left[\frac{u^2}{\bar{u}}\right]_+-\frac{1}{2}\left(\frac{\beta_0}{N_c}+6\right)\right]G_{0i}(uz)\tilde{G}_{0i}(0) \\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1\mathrm{d}u\Big\{\;(2\bar{u}u+2\bar{u})\,G_{0i}(uz)\tilde{G}_{0i}(0)+(2\bar{u}u-2\bar{u})\,G_{3i}(uz)\tilde{G}_{3i}(0)\;\Big\} \\
&+\frac{g^2N_c}{8\pi^2}\int_0^1\mathrm{d}u\Big\{\;\bar{u}^2\Big(\;G_{ij}(uz)\tilde{G}_{ij}(0)\;\Big)+(\bar{u}^2-2\bar{u})\Big(\;2G_{30}(uz)\tilde{G}_{30}(0)\;\Big)\;\Big\} \\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1\mathrm{d}u\bar{u}\Big\{\;2G_{30}(uz)\tilde{G}_{30}(0)-G_{ij}(uz)\tilde{G}_{ij}(0)\;\Big\}
\end{aligned}
$$

1.8.2 33

$$
\begin{aligned}
&\to \frac{g^2N_c}{8\pi^2}\left\{\frac{1}{\epsilon_{\mathrm{UV}}}+\log\left(z_3^2e^\gamma\right)\right\}\frac{8}{6}G_{3i}(z)\tilde{G}_{3i}(0) \\
&+\frac{g^2N_c}{8\pi^2}\int_0^1\mathrm{d}u\left[\left(\frac{1}{\bar{u}}-\bar{u}\right)_+-\left(\frac{4u}{\bar{u}}+\frac{4\log(1-u)}{\bar{u}}\right)_+ +2\delta(\bar{u})-\bar{u}^2\right]G_{3i}(uz)\tilde{G}_{3i}(0) \\
&+\frac{g^2N_c}{8\pi^2}\int_0^1\mathrm{d}u\bar{u}(1+u)\Big(\;G_{0i}(uz)\tilde{G}_{0i}(0)\;\Big) \\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1\mathrm{d}u\left[2\left[\frac{u^2}{\bar{u}}\right]_+-\frac{1}{2}\left(\frac{\beta_0}{N_c}+6\right)\right]G_{3i}(uz)\tilde{G}_{3i}(0) \\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1\mathrm{d}u\Big\{\;(2\bar{u}u-2\bar{u})\,G_{0i}(uz)\tilde{G}_{0i}(0)+(2\bar{u}u+2\bar{u})\,G_{3i}(uz)\tilde{G}_{3i}(0)\;\Big\} \\
&+\frac{g^2N_c}{8\pi^2}\int_0^1\mathrm{d}u\Big\{\;\bar{u}^2\Big(2G_{30}(uz)\tilde{G}_{30}(0)\Big)+(\bar{u}^2-2\bar{u})\Big(G_{ij}(uz)\tilde{G}_{ij}(0)\Big)\;\Big\} \\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1\mathrm{d}u\Big\{\;\bar{u}\Big(G_{ij}(uz)\tilde{G}_{ij}(0)-2G_{30}(uz)\tilde{G}_{30}(0)\Big)\;\Big\}
\end{aligned}
$$

1.8.3 03

$$
\begin{aligned}
&\to \frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{UV}}+\log\left(z_3^2e^\gamma\right)\right)\frac{5}{6}G_{0i}(z)\tilde{G}_{3i}(0)\\
&+\frac{g^2N_c}{8\pi^2}\int_0^1 \mathrm{d}u\left[-\left(\frac{4u}{\bar{u}}+\frac{4\log(1-u)}{\bar{u}}\right)_+ +2\delta(\bar{u})\right]G_{0i}(uz)\tilde{G}_{3i}(0)\\
&+\frac{g^2N_c}{8\pi^2}\int_0^1 \mathrm{d}u 2u\bar{u}G_{3i}(uz)\tilde{G}_{0i}(0)\\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1 \mathrm{d}u\left[4\bar{u}u+2\left[\frac{u^2}{\bar{u}}\right]_+-\frac{1}{2}\left(\frac{\beta_0}{N_c}+6\right)\right]G_{0i}(uz)\tilde{G}_{3i}(0)\\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1 \mathrm{d}u\left\{\bar{u}\left(G_{0i}(uz)\tilde{G}_{3i}(0)-G_{3i}(uz)\tilde{G}_{0i}(0)\right)\right\}
\end{aligned}
$$

1.8.4 30

$$
\begin{aligned}
&\to \frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{UV}}+\log\left(z_3^2e^\gamma\right)\right)\frac{11}{6}G_{3i}(z)\tilde{G}_{0i}(0)\\
&+\frac{g^2N_c}{8\pi^2}\int_0^1 \mathrm{d}u\left[2\left(\frac{1}{\bar{u}}-\bar{u}\right)_+-\left(\frac{4u}{\bar{u}}+\frac{4\log(1-u)}{\bar{u}}\right)_+ +2\delta(\bar{u})-2\bar{u}^2\right]G_{3i}(uz)\tilde{G}_{0i}(0)\\
&+\frac{g^2N_c}{8\pi^2}\int_0^1 \mathrm{d}u 2\bar{u}G_{i0}(uz)\tilde{G}_{i3}(0)\\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1 \mathrm{d}u\left[4\bar{u}u+2\left[\frac{u^2}{\bar{u}}\right]_+-\frac{1}{2}\left(\frac{\beta_0}{N_c}+6\right)\right]G_{3i}(uz)\tilde{G}_{0i}(0)\\
&-\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1 \mathrm{d}u\left\{\bar{u}\left(G_{0i}(uz)\tilde{G}_{3i}(0)-G_{3i}(uz)\tilde{G}_{0i}(0)\right)\right\}
\end{aligned}
$$

1.8.5 jj

$$
\begin{aligned}
&\to \frac{g^2N_c}{8\pi^2}\left\{\frac{1}{\epsilon_{\mathrm{UV}}}+\log\left(z_3^2e^\gamma\right)\right\}\frac{8}{6}G_{ij}(z)\tilde{G}_{ij}(0)\\
&+\frac{g^2N_c}{8\pi^2}\int_0^1 \mathrm{d}u\left[\left(\frac{1}{\bar{u}}-\bar{u}\right)_+-\left(\frac{4u}{\bar{u}}+\frac{4\log(1-u)}{\bar{u}}\right)_+ +2\delta(\bar{u})\right]G_{ij}(uz)\tilde{G}_{ij}(0)\\
&+\frac{g^2N_c}{8\pi^2}\int_0^1 \mathrm{d}u\left\{2\bar{u}^2G_{0i}(uz)\tilde{G}_{0i}(0)-2\bar{u}(1+u)G_{3i}(uz)\tilde{G}_{3i}(0)\right\}\\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1 \mathrm{d}u\left[\bar{u}(1+2u)+2\left[\frac{u^2}{\bar{u}}\right]_+-\frac{1}{2}\left(\frac{\beta_0}{N_c}+4\right)\right]G_{ij}(uz)\tilde{G}_{ij}(0)\\
&+\frac{g^2N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^\gamma\right)\right)\int_0^1 \mathrm{d}u\left\{2\bar{u}\left(G_{3i}(uz)\tilde{G}_{3i}(0)-G_{0i}(uz)\tilde{G}_{0i}(0)\right)\right.\\
&\left.+2\bar{u}(u-\bar{u})G_{30}(uz)\tilde{G}_{30}(0)\right\}
\end{aligned}
$$

1.8.6 $00 + 33$

$$
\begin{aligned}
& G_{0i}(x)\tilde{G}_{0i}(0) + G_{3i}(x)\tilde{G}_{3i}(0) \\
& \to \frac{g^2 N_c}{8\pi^2}\left(\frac{1}{\epsilon_{\rm UV}} + \log\left(z_3^2 e^{\gamma}\right) + 2\right)\Big(G_{0i}(z)\tilde{G}_{0i}(0) + G_{3i}(z)\tilde{G}_{3i}(0)\Big) \\
& \quad - \frac{g^2 N_c}{8\pi^2}\left(\frac{1}{\epsilon_{UV}} + \log\left(z_3^2 e^{\gamma}\right)\right)\frac{1}{6}\Big(G_{0i}(z)\tilde{G}_{0i}(0) + G_{3i}(z)\tilde{G}_{3i}(0)\Big) \\
& \quad + \frac{g^2 N_c}{16\pi^2}\int_0^1 \mathrm{d}u \left(\frac{1}{\epsilon_{UV}} + \log\left(z_3^2 e^{\gamma}\right)\right)\Big(G_{0i}(z)\tilde{G}_{0i}(0) + G_{3i}(z)\tilde{G}_{3i}(0)\Big) \\
& \quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left(\frac{1}{\bar{u}} - \bar{u}\right)_+ \Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{3i}(uz)\tilde{G}_{3i}(0)\Big) \\
& \quad + \frac{g^2 N_c}{4\pi^2}\int_0^1 \mathrm{d}u \bar{u} u \Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{3i}(uz)\tilde{G}_{3i}(0) - 2G_{30}(uz)\tilde{G}_{30}(0) - G_{ij}(uz)\tilde{G}_{ij}(0)\Big) \\
& \quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left[\left(\frac{1}{\epsilon_{IR}} - \log\left(z_3^2 e^{\gamma}\right)\right)\left(4\bar{u}u - 2u + 2\left(\frac{u}{\bar{u}}\right)_+ - \frac{1}{6}\delta(\bar{u})\right) - \left(\frac{4u}{\bar{u}} + \frac{4\log(1-u)}{\bar{u}}\right)_+\right]\Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{3i}(uz)\tilde{G}_{3i}(0)\Big) \\
& = \frac{g^2 N_c}{8\pi^2} 2\Big(G_{0i}(z)\tilde{G}_{0i}(0) + G_{3i}(z)\tilde{G}_{3i}(0)\Big) \\
& \quad + \frac{g^2 N_c}{8\pi^2}\left(\frac{1}{\epsilon_{UV}} + \log\left(z_3^2 e^{\gamma}\right)\right)\frac{8}{6}\Big(G_{0i}(z)\tilde{G}_{0i}(0) + G_{3i}(z)\tilde{G}_{3i}(0)\Big) \\
& \quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left(\frac{1}{\bar{u}} - \bar{u}\right)_+ \Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{3i}(uz)\tilde{G}_{3i}(0)\Big) \\
& \quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u 2\bar{u} u \Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{3i}(uz)\tilde{G}_{3i}(0) - 2G_{30}(uz)\tilde{G}_{30}(0) - G_{ij}(uz)\tilde{G}_{ij}(0)\Big) \\
& \quad - \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left[\frac{4u}{\bar{u}} + \frac{4\log(1-u)}{\bar{u}}\right]_+ \Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{3i}(uz)\tilde{G}_{3i}(0)\Big) \\
& \quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left(\frac{1}{\epsilon_{IR}} - \log\left(z_3^2 e^{\gamma}\right)\right)\left[\left\{4u\bar{u} + 2\left[u^2/\bar{u}\right]_+\right\} - \frac{1}{2}\left(\frac{\beta_0}{N_c} + 6\right)\delta(\bar{u})\right]\Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{3i}(uz)\tilde{G}_{3i}(0)\Big)
\end{aligned}
$$

1.8.7 $00 + ii$

$$
\begin{aligned}
& G_{0i}(z)\tilde{G}_{0i}(0) + G_{ij}(z)\tilde{G}_{ij}(0) \\
& \to \frac{g^2 N_c}{8\pi^2}\left[\frac{8}{6}\left(\frac{1}{\epsilon_{\rm UV}} + \log\left(z_3^2 e^{\gamma}\right)\right) + 2\right]\Big(G_{0i}(z)\tilde{G}_{0i}(0) + G_{3i}(z)\tilde{G}_{3i}(0)\Big) \\
& \quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left(\frac{1}{\bar{u}} - \bar{u}\right)_+ \Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{3i}(uz)\tilde{G}_{3i}(0)\Big) \\
& \quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left\{ \bar{u}^2\Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{ij}(uz)\tilde{G}_{ij}(0)\Big) - \bar{u}(1+u)\Big(G_{3i}(uz)\tilde{G}_{3i}(0) + 2G_{30}(uz)\tilde{G}_{30}(0)\Big) \right\} \\
& \quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left(\left(\frac{1}{\epsilon_{IR}} - \log\left(z_3^2 e^{\gamma}\right)\right)\left(2\bar{u}u - 2u + 2\left[\frac{u}{\bar{u}}\right]_+ - \frac{1}{6}\delta(\bar{u})\right) - \left[\frac{4u}{\bar{u}} + \frac{4\log(1-u)}{\bar{u}}\right]_+\right)\Big(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{ij}(uz)\tilde{G}_{ij}(0)\Big) \\
& \quad + \frac{g^2 N_c}{8\pi^2}\left(\frac{1}{\epsilon_{IR}} - \log\left(z_3^2 e^{\gamma}\right)\right)\int_0^1 \mathrm{d}u 2\bar{u} u \Big(G_{3i}(uz)\tilde{G}_{3i}(0) + 2G_{30}(uz)\tilde{G}_{30}(0)\Big)
\end{aligned}
$$

1.8.8 03

$$
\begin{aligned}
& G_{0i}(x)\tilde{G}_{3i}(0) \\
& \to \frac{g^2 N_c \Gamma(d/2-2)}{4\pi^2 (z_3^2)^{d/2-2}}\int_0^1 \mathrm{d}u \left[\frac{\bar{u}^{3-d} - 1}{d-3}\right]_+ G_{0i}(uz)\tilde{G}_{3i}(0)
\end{aligned}
$$

$\Gamma(d/2-1)$

$$\frac{g^2 N_c \Gamma(d/2-1)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u 2u\bar{u} G_{3i}(uz) \tilde{G}_{0i}(0)$$

$\Gamma(d/2-2)$

$$\frac{g^2 N_c \Gamma(d/2-2)}{8\pi^2 \left(z_3^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{ \left(4\bar{u}u - 2u\right) G_{0i}(uz) \tilde{G}_{3i}(0) + \bar{u} G_{0i}(uz) \tilde{G}_{3i}(0) - \bar{u} G_{3i}(uz) \tilde{G}_{0i}(0) \right\}$$

1.9 Structures

$$
\begin{aligned}
&M_{\mu\alpha;\lambda\beta}(z,p) = \\
&\left(g_{\mu\lambda}s_\alpha p_\beta - g_{\mu\beta}s_\alpha p_\lambda - g_{\alpha\lambda}s_\mu p_\beta + g_{\alpha\beta}s_\mu p_\lambda\right)\mathcal{M}_{sp} \\
&+\left(g_{\mu\lambda}p_\alpha s_\beta - g_{\mu\beta}p_\alpha s_\lambda - g_{\alpha\lambda}p_\mu s_\beta + g_{\alpha\beta}p_\mu s_\lambda\right)\mathcal{M}_{ps} \\
&+\left(g_{\mu\lambda}s_\alpha z_\beta - g_{\mu\beta}s_\alpha z_\lambda - g_{\alpha\lambda}s_\mu z_\beta + g_{\alpha\beta}s_\mu z_\lambda\right)\mathcal{M}_{sz} \\
&+\left(g_{\mu\lambda}z_\alpha s_\beta - g_{\mu\beta}z_\alpha s_\lambda - g_{\alpha\lambda}z_\mu s_\beta + g_{\alpha\beta}z_\mu s_\lambda\right)\mathcal{M}_{zs} \\
&+\left(p_\mu s_\alpha p_\lambda z_\beta - p_\alpha s_\mu p_\lambda z_\beta - p_\mu s_\alpha p_\beta z_\lambda + p_\alpha s_\mu p_\beta z_\lambda\right) \\
&\quad\times\mathcal{M}_{pspz} \\
&+\left(p_\mu z_\alpha p_\lambda s_\beta - p_\alpha z_\mu p_\lambda s_\beta - p_\mu z_\alpha p_\beta s_\lambda + p_\alpha z_\mu p_\beta s_\lambda\right) \\
&\quad\times\mathcal{M}_{pzps} \\
&+\left(s_\mu z_\alpha p_\lambda z_\beta - s_\alpha z_\mu p_\lambda z_\beta - s_\mu z_\alpha p_\beta z_\lambda + s_\alpha z_\mu p_\beta z_\lambda\right) \\
&\quad\times\mathcal{M}_{szpz} \\
&+\left(p_\mu z_\alpha s_\lambda z_\beta - p_\alpha z_\mu s_\lambda z_\beta - p_\mu z_\alpha s_\beta z_\lambda + p_\alpha z_\mu s_\beta z_\lambda\right) \\
&\quad\times\mathcal{M}_{pzsz} \\
&+(sz)\left[\left(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\right)\mathcal{M}_g + \left(g_{\mu\lambda}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\lambda - g_{\alpha\lambda}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\lambda\right)\mathcal{M}_{pp}\right. \\
&+\left(g_{\mu\lambda}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\lambda - g_{\alpha\lambda}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\lambda\right)\mathcal{M}_{zz} \\
&+\left(g_{\mu\lambda}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\lambda - g_{\alpha\lambda}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\lambda\right)\mathcal{M}_{zp} \\
&+\left(g_{\mu\lambda}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\lambda - g_{\alpha\lambda}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\lambda\right)\mathcal{M}_{pz} \\
&\left.+\left(p_\mu z_\alpha p_\lambda z_\beta - p_\alpha z_\mu p_\lambda z_\beta - p_\mu z_\alpha p_\beta z_\lambda + p_\alpha z_\mu p_\beta z_\lambda\right)\mathcal{M}_{ppzz}\right]
\end{aligned}
$$

$$
G_{\mu\alpha}(z)\tilde{G}_{\lambda\beta}(0) = G_{\mu\alpha}(0)\tilde{G}_{\lambda\beta}(-z)
$$

Relevant projections (with summation over i) in the basis of the $\mathcal{M}$ structures are:

$$
\begin{aligned}
M_{0i;0i} =& -2s_0p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps}) - (sz)\left(2\mathcal{M}_g + 2p_0^2\mathcal{M}_{pp}\right) \\
M_{3i;3i} =& -2p_3s_3(\mathcal{M}_{sp}+\mathcal{M}_{ps}) - 2z_3s_3(\mathcal{M}_{zs}+\mathcal{M}_{sz}) \\
&-(sz)\left(-2\mathcal{M}_g + 2p_3^2\mathcal{M}_{pp} + 2z_3^2\mathcal{M}_{zz} + 2z_3p_3\left(\mathcal{M}_{zp}+\mathcal{M}_{pz}\right)\right) \\
M_{0i;3i} =& -2\left(s_0p_3\mathcal{M}_{sp} + s_3p_0\mathcal{M}_{ps}\right) - 2s_0z_3\mathcal{M}_{sz} - 2(sz)\left(p_0p_3\mathcal{M}_{pp} + p_0z_3\mathcal{M}_{pz}\right) \\
M_{3i;0i} =& -2\left(s_3p_0\mathcal{M}_{sp} + s_0p_3\mathcal{M}_{ps}\right) - 2s_0z_3\mathcal{M}_{zs} - 2(sz)\left(p_3p_0\mathcal{M}_{pp} + z_3p_0\mathcal{M}_{zp}\right) \\
M_{ij;ij} =&\ 2(sz)\mathcal{M}_g \\
M_{30;30} =&\, s_3z_3\left(\mathcal{M}_{sz}+\mathcal{M}_{zs}\right) \\
&+\left(p_0s_3p_0z_3 - p_3s_0p_0z_3\right)\left(\mathcal{M}_{pspz}+\mathcal{M}_{pzps}\right) + s_0z_3p_0z_3\left(\mathcal{M}_{szpz}+\mathcal{M}_{pzsz}\right) \\
&+(sz)\left(-\mathcal{M}_g - m^2\mathcal{M}_{pp} + z_3^2\mathcal{M}_{zz} + z_3p_3\left(\mathcal{M}_{zp}+\mathcal{M}_{pz}\right) + p_0^2z_3^2\mathcal{M}_{ppzz}\right)
\end{aligned}
$$

where we've used the requirement that $(sp) = 0$. Additionally, the requirement that $s^2 = -1$ allows us to define: $s^\mu = \left(p^3, 0, 0, p^0\right)/m$, and so we can write:

$$
\begin{aligned}
M_{0i;0i} =& -2p_3p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps})/m + p_0z_3\left(2\mathcal{M}_g + 2p_0^2\mathcal{M}_{pp}\right)/m \\
M_{3i;3i} =& -2p_3p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps})/m - 2z_3p_0(\mathcal{M}_{zs}+\mathcal{M}_{sz})/m + p_0z_3\left(-2\mathcal{M}_g + 2p_3^2\mathcal{M}_{pp} + 2z_3^2\mathcal{M}_{zz} + 2z_3p_3\left(\mathcal{M}_{zp}+\mathcal{M}_{pz}\right)\right)/m \\
M_{0i;3i} =& -2\left(p_3^2\mathcal{M}_{sp} + p_0^2\mathcal{M}_{ps}\right)/m - 2p_3z_3\mathcal{M}_{sz}/m + 2p_0z_3\left(p_0p_3\mathcal{M}_{pp} + p_0z_3\mathcal{M}_{pz}\right)/m \\
M_{3i;0i} =& -2\left(p_0^2\mathcal{M}_{sp} + p_3^2\mathcal{M}_{ps}\right)/m - 2p_3z_3\mathcal{M}_{zs}/m + 2p_0z_3\left(p_3p_0\mathcal{M}_{pp} + z_3p_0\mathcal{M}_{zp}\right)/m \\
M_{ij;ij} =& -2p_0z_3\mathcal{M}_g/m \\
M_{30;30} =&\, p_0z_3\left(\mathcal{M}_{sz}+\mathcal{M}_{zs}\right)/m \\
&+p_0z_3\left(\mathcal{M}_{pspz}+\mathcal{M}_{pzps}\right) + p_3p_0z_3^2\left(\mathcal{M}_{szpz}+\mathcal{M}_{pzsz}\right)/m \\
&-p_0z_3\left(-\mathcal{M}_g - m^2\mathcal{M}_{pp} + z_3^2\mathcal{M}_{zz} + z_3p_3\left(\mathcal{M}_{zp}+\mathcal{M}_{pz}\right) + p_0^2z_3^2\mathcal{M}_{ppzz}\right)/m
\end{aligned}
$$

We can find more information about the relationship between different structures and matrix elements by looking at the interchange of fields in the matrix element.

$$\begin{aligned}
\langle p| G_{0i}(z_3)\tilde{G}_{0i}(0)|p\rangle &= \frac{1}{2}\langle p| G_{0i}(z_3)\epsilon_{0i}{}^{3j}G_{3j}(0)|p\rangle = \frac{1}{2}\langle p| G_{0i}(0)\epsilon_{0i}{}^{3j}G_{3j}(-z_3)|p\rangle \\
&= -\frac{1}{2}\langle p| G_{3j}(-z_3)\epsilon_{3j}{}^{0i}G_{0i}(0)|p\rangle = -\langle p| G_{3i}(-z_3)\tilde{G}_{3i}(0)|p\rangle \\
\langle p| G_{0i}(z_3)\tilde{G}_{3i}(0)|p\rangle &= \frac{1}{2}\langle p| G_{0i}(z_3)\epsilon_{3i}{}^{0j}G_{0j}(0)|p\rangle = \frac{1}{2}\langle p| G_{0i}(0)\epsilon_{3i}{}^{0j}G_{0j}(-z_3)|p\rangle \\
&= -\frac{1}{2}\langle p| G_{0j}(-z_3)\epsilon_{3j}{}^{0i}G_{0i}(0)|p\rangle = -\langle p| G_{0i}(-z_3)\tilde{G}_{3i}(0)|p\rangle \\
\langle p| G_{ij}(z_3)\tilde{G}_{ij}(0)|p\rangle &= -2\langle p| G_{30}(-z_3)\tilde{G}_{30}(0)|p\rangle
\end{aligned}$$

Looking at the second relation:

$$\begin{aligned}
&-2\left(p_3^2\mathcal{M}_{sp}(\nu)+p_0^2\mathcal{M}_{ps}(\nu)\right)/m - 2p_3z_3\mathcal{M}_{sz}(\nu)/m + 2p_0z_3\left(p_0p_3\mathcal{M}_{pp}(\nu)+p_0z_3\mathcal{M}_{pz}(\nu)\right)/m \\
&= 2\left(p_3^2\mathcal{M}_{sp}(-\nu)+p_0^2\mathcal{M}_{ps}(-\nu)\right)/m - 2p_3z_3\mathcal{M}_{sz}(-\nu)/m + 2p_0z_3\left(p_0p_3\mathcal{M}_{pp}(-\nu)-p_0z_3\mathcal{M}_{pz}(-\nu)\right)/m
\end{aligned}$$

which, along with a similar relation for $M_{3i;0i}$, gives us:

$$\begin{aligned}
\mathcal{M}_{sp}(\nu) &= -\mathcal{M}_{sp}(-\nu) \\
\mathcal{M}_{ps}(\nu) &= -\mathcal{M}_{ps}(-\nu) \\
\mathcal{M}_{sz}(\nu) &= \mathcal{M}_{sz}(-\nu) \\
\mathcal{M}_{zs}(\nu) &= \mathcal{M}_{zs}(-\nu) \\
\mathcal{M}_{pp}(\nu) &= \mathcal{M}_{pp}(-\nu) \\
\mathcal{M}_{pz}(\nu) &= -\mathcal{M}_{pz}(-\nu) \\
\mathcal{M}_{zp}(\nu) &= -\mathcal{M}_{zp}(-\nu)
\end{aligned}$$

Combinations:

$$M_{0i;0i}+M_{3i;3i} = -4p_3p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps})/m - 2z_3p_0(\mathcal{M}_{zs}+\mathcal{M}_{sz})/m - 2p_0z_3\left(p_0^2+p_3^2\right)\mathcal{M}_{pp}/m + p_0\left(2z_3^3\mathcal{M}_{zz}+2p_3z_3^2\left(\mathcal{M}_{zp}+\mathcal{M}_{pz}\right)\right)$$

$$\begin{aligned}
M_{0i;0i}+M_{ij;ij} &= -2p_3p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps})/m + 2p_0z_3p_0^2\mathcal{M}_{pp}/m \\
M_{3i;3i}+2M_{30;30} &= -2p_3p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps})/m + 2p_0z_3p_0^2\mathcal{M}_{pp}/m \\
&\quad +2p_0z_3\left(\mathcal{M}_{pspz}+\mathcal{M}_{pzps}\right) + 2p_3p_0z_3^2\left(\mathcal{M}_{szpz}+\mathcal{M}_{pzsz}\right)/m \\
&\quad -2p_0z_3\left(p_0^2z_3^2\mathcal{M}_{ppzz}\right)/m
\end{aligned}$$

So for $M_{0i;0i}+M_{ij;ij}$ we have:

$$\begin{aligned}
&-2p_3p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps})/m + 2p_0z_3p_0^2\mathcal{M}_{pp}/m \\
&\to \frac{g^2N_c}{8\pi^2}\frac{8}{6}\left(\frac{1}{\epsilon_{\text{UV}}}+\log\left(z_3^2e^{\gamma}\right)\right)\left(-2p_3p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps})/m + 2p_0z_3p_0^2\mathcal{M}_{pp}/m\right) \\
&\quad + \frac{g^2N_c}{8\pi^2}\int_0^1 \mathrm{d}u\left\{2\left(1-\bar{u}u\right)+\left(\frac{1}{\bar{u}}-\bar{u}\right)_+ - \left[\frac{4u}{\bar{u}}+\frac{4\log(1-u)}{\bar{u}}\right]_+\right\}\left(-2p_3p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps})/m + 2p_0z_3p_0^2\mathcal{M}_{pp}/m\right) \\
&\quad + \frac{g^2N_c}{8\pi^2}\int_0^1 \mathrm{d}u\left(\frac{1}{\epsilon_{IR}}-\log\left(z_3^2e^{\gamma}\right)\right)\left[\left\{4u\bar{u}+2\left[u^2/\bar{u}\right]_+\right\}-\frac{1}{2}\left(\frac{\beta_0}{N_c}+6\right)\delta(\bar{u})\right]\left(-2p_3p_0(\mathcal{M}_{sp}+\mathcal{M}_{ps})/m + 2p_0z_3p_0^2\mathcal{M}_{pp}/m\right) \\
&\quad + \mathcal{O}\left(\text{twist } 3\right)
\end{aligned}$$