Parton Distribution Functions and Amplitudes...

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1. *Why do you request to run your projects on three different platforms?*

   First we note that the work proposed on the RTX "gamer"-GPU nodes can equally be performed on the KNLs (or SKX) nodes, as noted in the text; the reason for requesting these nodes was simply to note that this part of the calculation involved minimal global sums and therefore would be suitable for gamer-type cards. As regards the SKX vs KNL, the rest of the calculation can indeed use either resource, and indeed for the *corrs* in Tables 2 and 3, the SKX is probably somewhat more efficient in their respective core-hours, by a factor of at least 2.

2. *You have 4 lattice ensembles to estimate the volume and discretization corrections. How do you assess the higher twist effect in $1/P_z$? How many hadron momenta will you use? How large are they?*

   The basic input to lattice calculations in the pseudo-PDF approach is the same as that of the quasi-PDF approach, namely a quark- and anti-quark connected by a Wilson line $U(z, 0)$:

   $$M^{\alpha}(p, z) = \langle p \mid \bar{\psi}(z)\gamma^{\alpha}U(z; 0)\psi(0) \mid p\rangle$$

   where z is the separation, which in a lattice calculation we take to be along the z axis, and p is the momentum of the hadron. The difference lies in the treatment of the ultra-violet divergences, and, importantly for the question posed above, in how the matrix elements computed on the lattice are related to the PDF. In the pseudo-PDF approach, the data are expressed in terms of two Lorentz invariants, namely the z^2 and the $\nu = p \cdot z$, the Ioffe time generalized to spatial separations, or pseudo Ioffe time. The former provides a short-distance scale, and higher-twist effects are expressed as terms of $\mathcal{O}(z^2\Lambda^2)$, not $\mathcal{O}(\Lambda^2/p_z^2)$ which is manifestly not Lorentz invariant. The *Good Lattice Cross Section* (LCS) approach is likewise expressed in terms of the Lorentz invariants ν and z^2, with once again z^2 providing a short-distance scale and higher-twist contributions manifest in a like manner. Note it is the requirement that z be short distance that drives the need for fine lattice spacings.

The resulting lattice calculation yields the Ioffe-time distribution $Q(\nu)$ that is the Fourier transform of the PDF $f(x)$, viz

$$Q(\nu) = \int_{-1}^{1} dx\, e^{i\nu x} f(x),$$

where details of the matching to a scale μ are suppressed. The need for high momenta, and indeed fine graining of the momenta, is now clear: it is needed to address the inverse problem represented by the above, and is not a measure of higher-twist effects. We discuss this further in response to question (4) below. As regards the momenta that we will use in our calculation, the use of boosted distillation suggests we will be able to reach $ap_z = (0, 0, \frac{2\pi.6}{L})$ where L is the spatial extent, as indicated in Figure 4.

3. *Do you have a plan to calculate the renormalization and mixing for the glue distribution?*

   One advantage of the reduced pseudo-PDF approach is the cancellation of all UV divergences in the reduced Ioffe time distribution. Renormalisation of this object is therefore unnecessary. In 1809.10836, it was noted that the lattice gluon operator is multiplicative renormalizable, and the matching coefficient that relates this object to the light-cone PDF in the MS-bar scheme has been calculated at one loop in 1910.13963. Mixing with the isoscalar quark distribution can be included as part of this matching procedure, and requires the perturbative matching coefficient and nonperturbative determination of the corresponding reduced Ioffe time distribution. We are working towards including the relevant disconnected contributions in our lattice calculations.

4. *How do you treat the Fourier transform to obtain the PDF and GDP?*

   The inversion of the Fourier transform is a paradigm for the inverse problem with incomplete data, which requires additional input to solve. In the case of our calculation of PDFs, we have followed the approach adopted by the global-fitting community by providing that additional input in the form of a parametrization, for example $f(x) = Nx^a(1-x)^b(1+c\sqrt{x})$, though we try also two- and four-parameter fits.

   An analogous situation arises in the case of GPDs, where for simplicity we now specialize to the case of the pion, as presented in *A.Radyushkin,* *Phys. Rev. D 100, 116011 (2019)*. After matching to a scale μ, we obtain from the lattice calculation the Generalized Ioffe-Time distribution (GITD) $\tilde{\mathcal{I}}(\nu, \xi, t, \mu^2)$ which is related to the *Fourier Transform* of the GPD through:

$$\tilde{\mathcal{I}}(\nu, \xi, t, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} H(x, \xi, t; \mu^2) \tag{1}$$

where ν, ξ, t are each discretized, as we describe below. The most straightforward way of solving the inverse problem in this case is once again to assume a parametrization for $H(x, \xi, t)$, such as the double distribution introduced in *A.Radyushkin, Phys. Rev. D59,014030 (1999)*. We conclude by noting that exploring additional approaches to the inverse problem is an active area of research by several of our group, e.g. see *J.Karpie et al, JHEP 04 (2019) 057*.

5. *It is hard for experiments to go away from the $\xi = 0$ limit. Do you have a plan to calculate GPD with non-zero ξ? How many Q^2 do you plan to calculate for GPD?*

   As noted in the responses to questions (2) and (4) above, in the pseudo-PDF approach quantities are expressed in terms of the Lorentz invariants $\nu = p.z$ and z^2. Extending this to the calculation of GPDs, other Lorentz invariants are introduced: $\nu_1 = p_1.z, \nu_2 = p_2.z$ and t, the momentum transfer, where p_1 and p_2 are the momenta of the incoming and outgoing hadron, respectively. The Ioffe time ν and skewness ξ of eqn. 1 are related through:

$$\begin{aligned} \nu &= \frac{\nu_1 + \nu_2}{2} \\ \xi &= \frac{\nu_1 - \nu_2}{\nu_1 + \nu_2}. \end{aligned}$$

   As noted in Table 4, we will calculate the GPD for 11 different momentum transfers.

6. *How do you plan to pursue the 3 projects if less than requested computing time amount is allocated?*

   Projects II and III are each the central topic of a graduate-student thesis, and pursuing them is therefore vital. Project I is capitalizing, in an important way, on previous work to compute the isovector PDF of the nucleon at larger volumes and close-to-physical quark masses using the framework of distillation which promises both reduced statistical uncertainties and better isolation of the ground-state nucleon. If the requested time is reduced, we would accommodate this by reducing the number of configurations employed in Project I, and seek other resources to achieve the needed statistics.