Calculation of Gluon PDF using Pseudo-PDF Technique

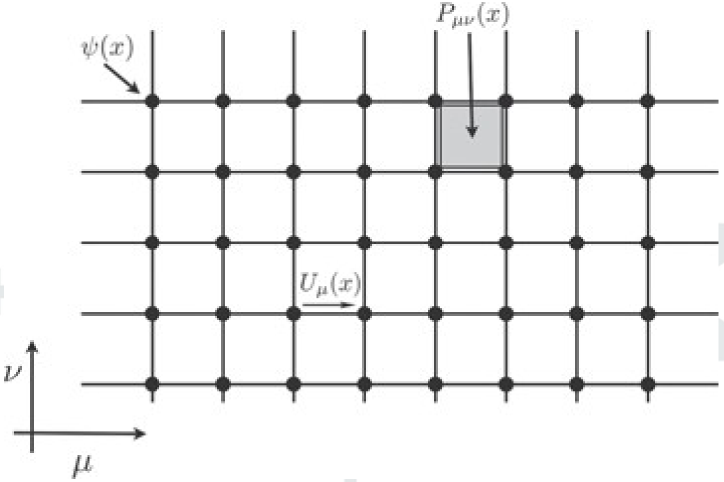
Tanjib Khan

On behalf of the

HadStruc Collaboration

Lattice QCD

- Non-perturbatively solving QCD
- Time & space discretized by lattice spacing
- Imaginary time (Euclidean space)
- Quarks at the lattice points.
- Gauge field at the links.



Gluon PDF

- **PDF:** Probability of finding partons in a hadron as a function of the hadron's longitudinal momentum fraction, x carried by the parton, probed at a factorization scale, *μ*

- **Gluon PDF:** Fourier transform of lightcone gluonic correlator in the hadron

[J. Collins et al., Nuc Phy B, 194(3):445–492]

$$f_g(x,\mu^2) = \int \frac{\mathrm{d}\xi^-}{2\pi x P^+}\, e^{-ix\xi^- P^+} \left\langle P \middle| \left[F^{+\rho}(\xi^-)\, U(\xi^-,0)\, F^{+\rho}(0)\right](\mu^2) \middle| P \right\rangle$$

(on lattice)

$$\mathrm{Tr}[F_{\rho\sigma}(z)U(z,0)F^{\mu\nu}(0)U(0,z)]$$

(Gluon Operator)

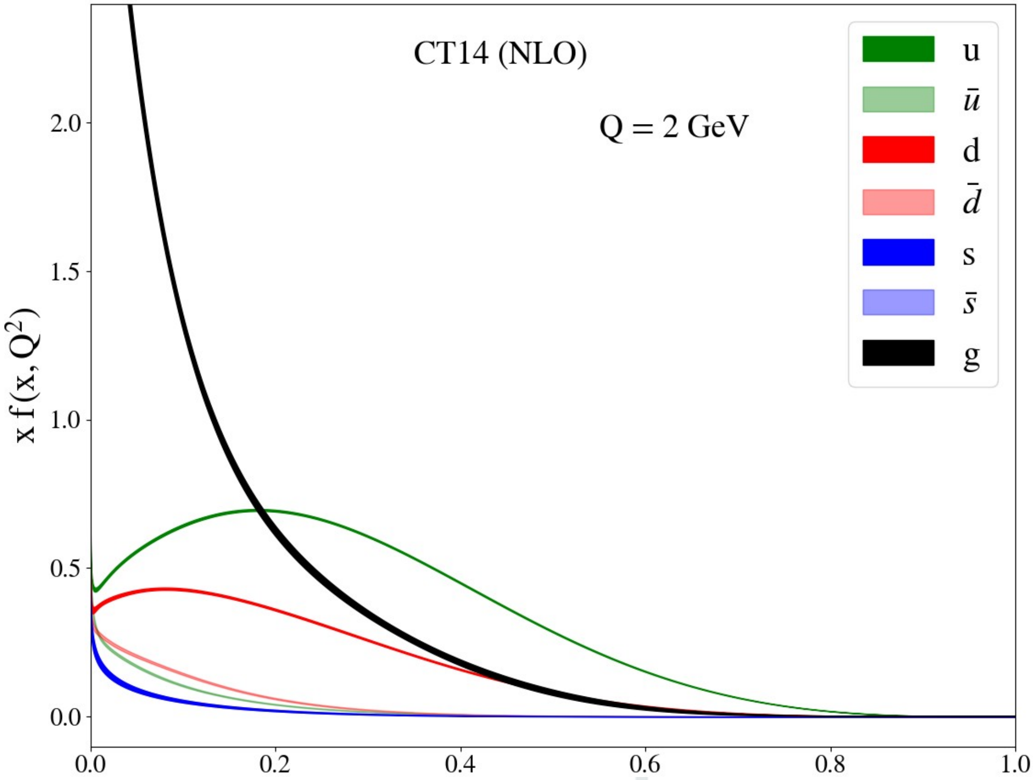
U = gauge link (Wilson line).

z = separation along spatial direction.

$\xi^{\pm}$ = space-time co-ordinate along lightcone direction.

Gluon PDF (Contd.)

- Present status:
  - Phenomenological data at moderate x-region very small.
  - Lattice calculation can provide complementary information of gluon PDF in medium to large x region.
  - Lattice needs a lot of statistics.



- Pseudo-PDF method:

[A. Radyushkin, Phys. Rev. D, 96(3): 034025, 2017]

$$f_g(x, \mu^2) = \lim_{z \to 0} f_g(x, \mu^2; z)$$

Gluonic Operator Calculation

- Lattice Gluonic Operator: (multiplicatively renormalizable [1, 2])

$$\left[F_{ti}(z)U(z,0)F^{it}(0)U(0,z)\right];$$

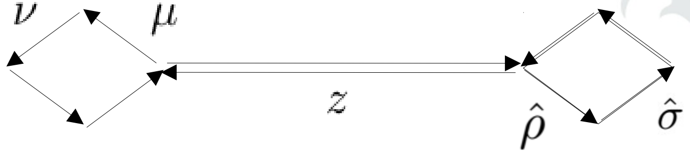
$$\left[F_{ji}(z)U(z,0)F^{ij}(0)U(0,z)\right]; \quad i,j = x,y$$

1. J. Zhang et al., Phys. Rev. Lett. 122, 142001
2. Z. Li et al., Phys. Rev. Lett. 122, 062002

- Lattice field-strength tensor: [Annals of Physics, Vol 304, Issue 1, 2003, Pages 1-21]

$$\frac{-i}{2}\left[W_{\mu\nu}^{(1\times1)} - W_{\mu\nu}^{(1\times1)\dagger} - \frac{1}{3}\mathrm{Tr}\left(W_{\mu\nu}^{(1\times1)} - W_{\mu\nu}^{(1\times1)\dagger}\right)\right] = ga^2\left[F_{\mu\nu}(x_0) + \mathcal{O}(a^2) + \mathcal{O}(g^2a^2)\right]$$

- Plaquette operator: $W_{\mu\nu}^{(1\times1)} = U_\mu(x)\,U_\nu(x+a\hat{\mu})\,U_\mu^\dagger(x+a\hat{\nu})\,U_\nu^\dagger(x)$



Nucleon 2-pt Correlator

- **Distillation:** Quark smearing technique. [M. Peardon et al., Phys Rev D, 80:054506, 2009]
- **Distillation operator:** Consists of eigenvectors of lattice Laplacian, second-order three-dimensional differential operator.

$$\Box_{xy}(t) = \sum_{k=1}^{N_D} \nu_x^{(k)}(t)\, \nu_y^{(k)\dagger}(t)$$

- Annihilation operator after distillation:

$$\chi_B(t) = \epsilon^{abc}\, S_{\alpha_1\alpha_2\alpha_3} \Big(\mathcal{D}_1 \Box\, d\Big)^a_{\alpha_1} \Big(\mathcal{D}_2 \Box\, u\Big)^b_{\alpha_2} \Big(\mathcal{D}_3 \Box\, u\Big)^c_{\alpha_3}(t)$$

- Two-point correlator:

$$C_B(t', t) = -\Big\langle \chi_B(t')\, \bar{\chi}_B(t) \Big\rangle$$

- This method imposes momentum projection at both source and sink, providing a more complete sampling of each gauge configuration.

Gluon 3-pt Correlator

- Gluon 3pt Correlator: $C^i_{3pt}(t, t_g) = \left(C^i_{2pt}(t) - \langle C_{2pt}(t)\rangle\right)\left(O^i_g(t_g) - \langle O_g(t_g)\rangle\right)$
- sGEVP Method: [J. Bulava et al., JHEP, 01:140, 2012]

$$C^s_{3pt}(t_{src}, t_{snk}) = \sum_{t_g=(t_{src}+1)}^{(t_{snk}-1)} C^i_{3pt}(t_{src}, t_{snk}, t_g)$$

$$C_{2pt}(t)\, u_n(t, t_0) = \lambda_n(t, t_0)\, C_{2pt}(t_0)\, u_n(t, t_0)$$

$$\mathcal{M}^{\mathrm{eff},s}_{nn}(t, t_0) = -\partial_t \left\{ \frac{\left|\left(u_n,\ \left[C^s_{3pt}(t)[\lambda_n(t, t_0)]^{-1} - C^s_{3pt}(t_0)\right] u_n\right)\right|}{\left(u_n,\ C_{2pt}(t_0) u_n\right)} \right\}$$

$$\mathcal{M}^{\mathrm{eff},s}_{nn}(t, t_0) = \mathcal{M}_{nn} + O\big(t \exp(-\Delta_{N+1,n}\, t)\big)$$

Matrix Element Calculation

- Wilson flow analysis: [M. Luscher, JHEP 08:071, 2010]
  - Flow of a gauge field, $B_\mu(\tau, x_\mu)$ so that $B_\mu\Big|_{\tau=0} = A_\mu$
- Calculating the matrix elements for each flow-time:

[I. Balitsky et al., Phys. Lett. B, 808:135621, 2020]

$$\mathcal{M}_{ti;it} + \mathcal{M}_{ji;ij} = 2p_0^2\,\mathcal{M}_{pp}$$

- Reduced Matrix Elements: [B. Joo et al., JHEP, 12:081, 2019]

$$\mathfrak{M}(\nu, z^2, \tau) = \left(\frac{\mathcal{M}(\nu, z^2, \tau)}{\mathcal{M}(\nu, 0, \tau)|_{z=0}}\right) / \left(\frac{\mathcal{M}(0, z^2, \tau)|_{p=0}}{\mathcal{M}(0, 0, \tau)|_{p=0,z=0}}\right)$$

- The ratio cancels the ultra-violet divergence, reduces higher twist corrections and suppresses the effect of τ at moderate τ

Position-space Matching

- Calculating pseudo-ITD from reduced matrix elements by taking $\tau \to 0$ limit.
- Taking leading order of α_s and ignoring quark-gluon mixing, the normalized ITD is calculated.

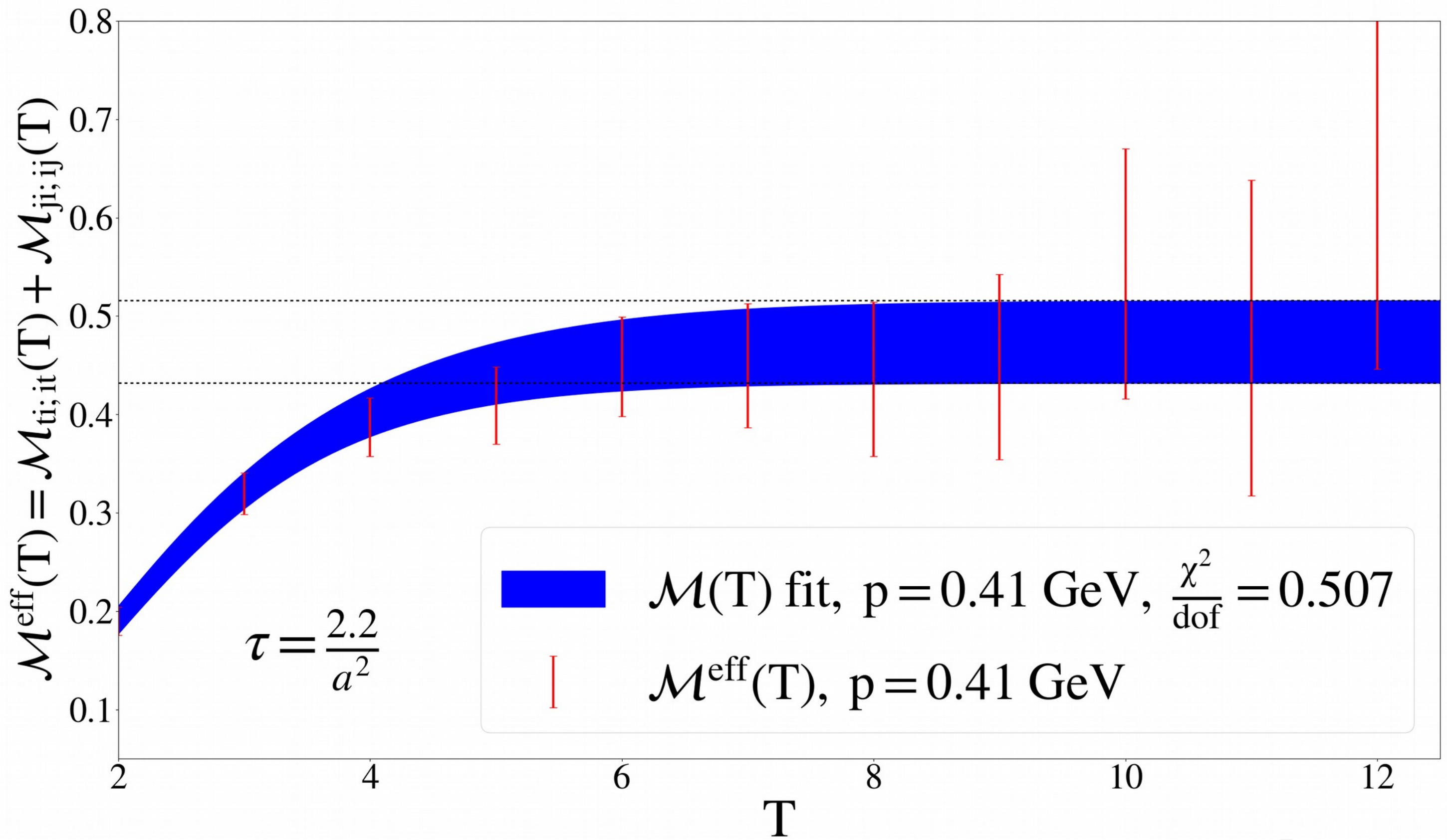
$$\frac{\mathcal{I}_g(\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} = \mathfrak{M}(\nu, z_3^2) + \frac{\alpha_s N_c}{2\pi}\int_0^1 du\, \mathfrak{M}(u\nu, z_3^2)\left\{\ln\left(\frac{z_3^2\mu^2 e^{2\gamma_E}}{4}\right) B_{gg}(u) + 4\left[\frac{u + ln(\bar{u})}{\bar{u}}\right]_+ + \frac{2}{3}\left[1 + 6u - 6u^2 - u^3\right]_+\right\}$$

- Altarelli-Parisi kernel: $B_{gg}(u) = 2\left[\frac{(1-u\bar{u})^2}{1-u}\right]_+$

Lattice Specification

- Isotropic lattice: 32^3 X 64
- 2+1 flavor Clover-Wilson fermion
- Pion mass: 358 MeV
- $\beta = 6.3$
- Lattice spacing = 0.094 fm
- Gauge conf. = 351

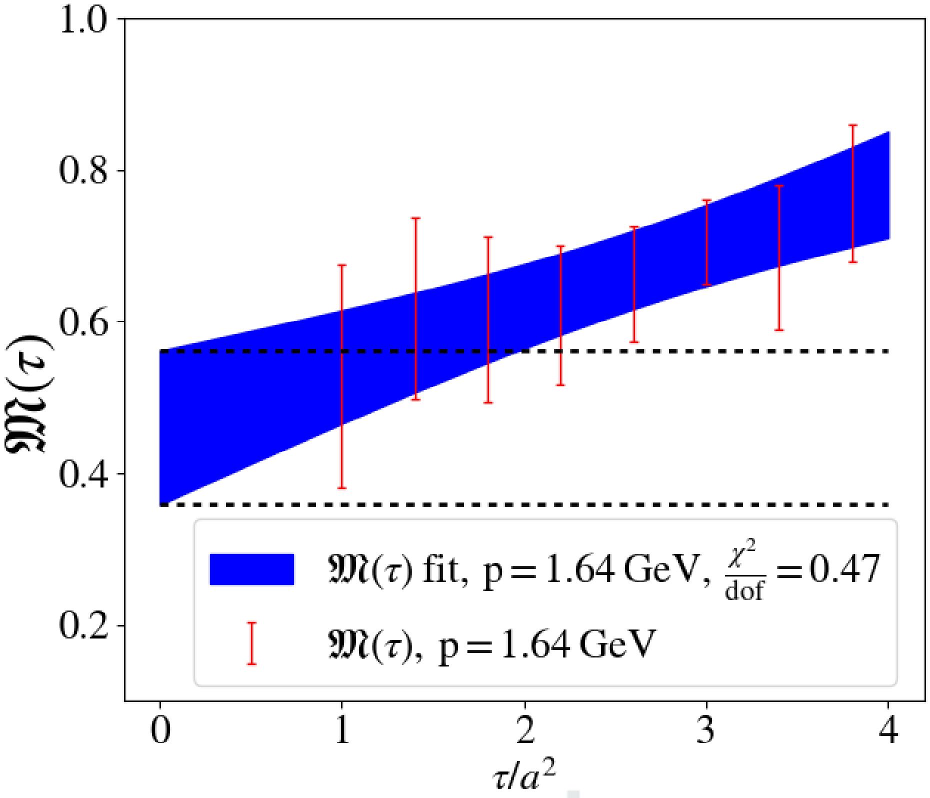
Matrix Element Extraction



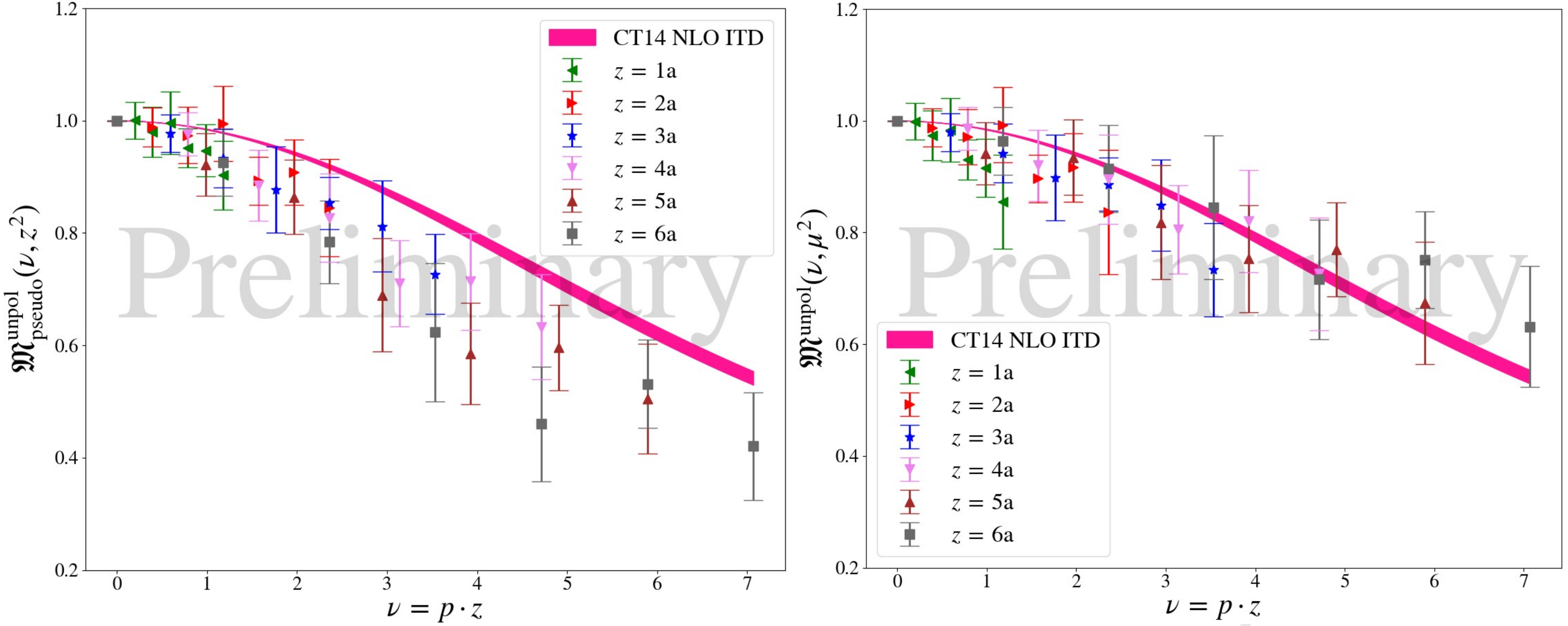
Flow-time Independent Extrapolation

- Fit-expression:

$$\mathfrak{M}(\tau) = \mathrm{A} + \tau\mathrm{B}$$



Ioffe-Time Distribution



Future Work

- Increasing statistics.
- Extracting gluon PDF from lattice ITD.
- Singlet quark ITD calculation to incorporate the quark-gluon mixing.