Outline

- **Highlight data**
  - Positivity and phenomenological global analyses
- **Matrix Element Fits**
  - Linear Fits
- **Amplitudes**
  - Pseudo-inverse (SVD) extraction
  - Form Factor Results
- **Shrink the Covariance**
  - For when you have many observables and not enough samples

What Momenta exist?

- /work/JAM/jkarpie/data/pseudoGPD/unpol_quark_matelems/cl21_32_64_b6p3_m0p2350_m0p2050/unphased

```
Apptainer> ls
allSDBs.list                snk0.1.1_src0.0.0      snk1.0.-2_src0.0.-2   snk1.1.1_src1.0.2      snk2.0.0_src2.0.-2    snk2.1.0_src1.0.2      snk2.1.-1_src2.1.-2
key.xml                     snk0.1.-1_src0.0.-1    snk1.0.2_src0.0.2     snk1.1.-1_src1.1.0     snk2.0.0_src2.0.2     snk2.1.0_src2.0.-1     snk2.1.1_src2.1.2
make_3pt_key.sh             snk0.1.1_src0.0.1      snk1.0.-2_src1.0.0    snk1.1.1_src1.1.0      snk2.0.-1_src0.0.0    snk2.1.0_src2.0.1      snk2.1.-2_src0.1.0
momsWithProj.list           snk0.1.-1_src0.0.-2    snk1.0.2_src1.0.0     snk1.1.-1_src1.1.-2    snk2.0.1_src0.0.0     snk2.1.0_src2.0.-2     snk2.1.2_src0.1.0
newBuild                    snk0.1.1_src0.0.2      snk1.0.-2_src1.0.-1   snk1.1.1_src1.1.2      snk2.0.-1_src0.0.-2   snk2.1.0_src2.0.2      snk2.1.-2_src0.1.-1
open_edb_notsubduced.sh     snk0.1.-1_src0.1.0     snk1.0.2_src1.0.1     snk1.1.-2_src0.0.0     snk2.0.1_src0.0.2     snk2.1.0_src2.1.-1     snk2.1.2_src0.1.1
snk0.0.0_src0.0.-1          snk0.1.1_src0.1.0      snk1.1.0_src0.0.-1    snk1.1.2_src0.0.0      snk2.0.-1_src1.0.-1   snk2.1.0_src2.1.1      snk2.1.-2_src1.0.0
snk0.0.0_src0.0.1           snk0.1.-1_src0.1.-2    snk1.1.0_src0.0.1     snk1.1.-2_src0.0.-1    snk2.0.1_src1.0.1     snk2.1.0_src2.1.-2     snk2.1.2_src1.0.0
snk0.0.0_src0.0.-2          snk0.1.1_src0.1.2      snk1.1.0_src0.0.-2    snk1.1.2_src0.0.1      snk2.0.-1_src2.0.0    snk2.1.0_src2.1.2      snk2.1.-2_src1.0.-1
snk0.0.0_src0.0.2           snk0.1.-2_src0.0.0     snk1.1.0_src0.0.2     snk1.1.-2_src0.0.-2    snk2.0.1_src2.0.0     snk2.1.-1_src0.1.0     snk2.1.2_src1.0.1
snk0.0.-1_src0.0.0          snk0.1.2_src0.0.0      snk1.1.0_src1.0.-1    snk1.1.2_src0.0.2      snk2.0.-1_src2.0.-2   snk2.1.1_src0.1.0      snk2.1.-2_src1.0.-2
snk0.0.1_src0.0.0           snk0.1.-2_src0.0.-1    snk1.1.0_src1.0.1     snk1.1.-2_src0.1.-2    snk2.0.1_src2.0.2     snk2.1.-1_src0.1.-2    snk2.1.2_src1.0.2
snk0.0.1_src0.0.-1          snk0.1.2_src0.0.1      snk1.1.0_src1.0.-2    snk1.1.2_src0.1.2      snk2.0.-2_src0.0.0    snk2.1.1_src0.1.2      snk2.1.-2_src1.1.-2
snk0.0.1_src0.0.1           snk0.1.-2_src0.0.-2    snk1.1.0_src1.0.2     snk1.1.-2_src1.0.0     snk2.0.2_src0.0.0     snk2.1.-1_src1.0.0     snk2.1.2_src1.1.2
snk0.0.-1_src0.0.-2         snk0.1.2_src0.0.2      snk1.1.0_src1.1.-1    snk1.1.2_src1.0.0      snk2.0.-2_src0.0.-1   snk2.1.1_src1.0.0      snk2.1.-2_src2.0.0
snk0.0.1_src0.0.2           snk0.1.-2_src0.1.0     snk1.1.0_src1.1.1     snk1.1.-2_src1.0.-1    snk2.0.2_src0.0.1     snk2.1.-1_src1.0.-1    snk2.1.2_src2.0.0
snk0.0.-2_src0.0.0          snk0.1.2_src0.1.0      snk1.1.0_src1.1.-2    snk1.1.2_src1.0.1      snk2.0.-2_src1.0.-2   snk2.1.1_src1.0.1      snk2.1.-2_src2.0.-1
snk0.0.2_src0.0.0           snk0.1.-2_src0.1.-1    snk1.1.0_src1.1.2     snk1.1.-2_src1.1.0     snk2.0.2_src1.0.2     snk2.1.-1_src1.0.-2    snk2.1.2_src2.0.1
snk0.0.-2_src0.0.-1         snk0.1.2_src0.1.1      snk1.1.-1_src0.0.0    snk1.1.2_src1.1.0      snk2.0.-2_src2.0.0    snk2.1.1_src1.0.2      snk2.1.-2_src2.0.-2
snk0.0.2_src0.0.1           snk1.0.0_src1.0.-1     snk1.1.1_src0.0.0     snk1.1.-2_src1.1.-1    snk2.0.2_src2.0.0     snk2.1.-1_src1.1.-1    snk2.1.2_src2.0.2
snk0.1.0_src0.0.-1          snk1.0.0_src1.0.1      snk1.1.-1_src0.0.-1   snk1.1.2_src1.1.1      snk2.0.-2_src2.0.-1   snk2.1.1_src1.1.1      snk2.1.-2_src2.1.0
snk0.1.0_src0.0.1           snk1.0.0_src1.0.-2     snk1.1.1_src0.0.1     snk1.1.-3_src0.0.-3    snk2.0.2_src2.0.1     snk2.1.-1_src2.0.0     snk2.1.2_src2.1.0
snk0.1.0_src0.0.-2          snk1.0.0_src1.0.2      snk1.1.-1_src0.0.-2   snk1.1.3_src0.0.3      snk2.1.0_src0.1.-1    snk2.1.1_src2.0.0      snk2.1.-2_src2.1.-1
snk0.1.0_src0.0.2           snk1.0.-1_src0.0.-1    snk1.1.1_src0.0.2     snk2.0.0_src0.0.-1     snk2.1.0_src0.1.1     snk2.1.-1_src2.0.-1    snk2.1.2_src2.1.1
snk0.1.0_src0.1.-1          snk1.0.1_src0.0.1      snk1.1.-1_src0.1.-1   snk2.0.0_src0.0.1      snk2.1.0_src0.1.-2    snk2.1.1_src2.0.1
snk0.1.0_src0.1.1           snk1.0.-1_src1.0.0     snk1.1.1_src0.1.1     snk2.0.0_src0.0.-2     snk2.1.0_src0.1.2     snk2.1.-1_src2.0.-2
snk0.1.0_src0.1.-2          snk1.0.1_src1.0.0      snk1.1.-1_src1.0.0    snk2.0.0_src0.0.2      snk2.1.0_src1.0.-1    snk2.1.1_src2.0.2
snk0.1.0_src0.1.2           snk1.0.-1_src1.0.-2    snk1.1.1_src1.0.0     snk2.0.0_src2.0.-1     snk2.1.0_src1.0.1     snk2.1.-1_src2.1.0
snk0.1.-1_src0.0.0          snk1.0.1_src1.0.2      snk1.1.-1_src1.0.-2   snk2.0.0_src2.0.1      snk2.1.0_src1.0.-2    snk2.1.1_src2.1.0
```

- cl21_32_64_b6p3_m0p2350_m0p2050.nuc.pGITD_pf011_pi001.n64.t0_avg_tsnk6.Z-8.Z8.insertionsNotSubduced.edb
- Contains DA operator results after Colin had undone the subductions
- Skewness=0 has no C4 type momentum transfers.

What Momenta exist?

- /work/JAM/jkarpie/data/pseudoGPD/unpol_quark_matelems/cl21_32_64_b6p3_m0p2350_m0p2050/phased/d001_2.00/

```
Apptainer> ls
open_all.sh        snk0.1.6_src0.0.6  snk1.0.6_src0.0.6  snk1.1.4_src1.0.4  snk1.1.5_src1.0.5  snk1.1.6_src1.0.6  snk2.0.6_src1.0.6
snk0.1.4_src0.0.4  snk1.0.4_src0.0.4  snk1.1.4_src0.0.4  snk1.1.5_src0.0.5  snk1.1.6_src0.0.6  snk2.0.4_src1.0.4
snk0.1.5_src0.0.5  snk1.0.5_src0.0.5  snk1.1.4_src0.1.4  snk1.1.5_src0.1.5  snk1.1.6_src0.1.6  snk2.0.5_src1.0.5
Apptainer>
```

- Only skewness=0 combinations

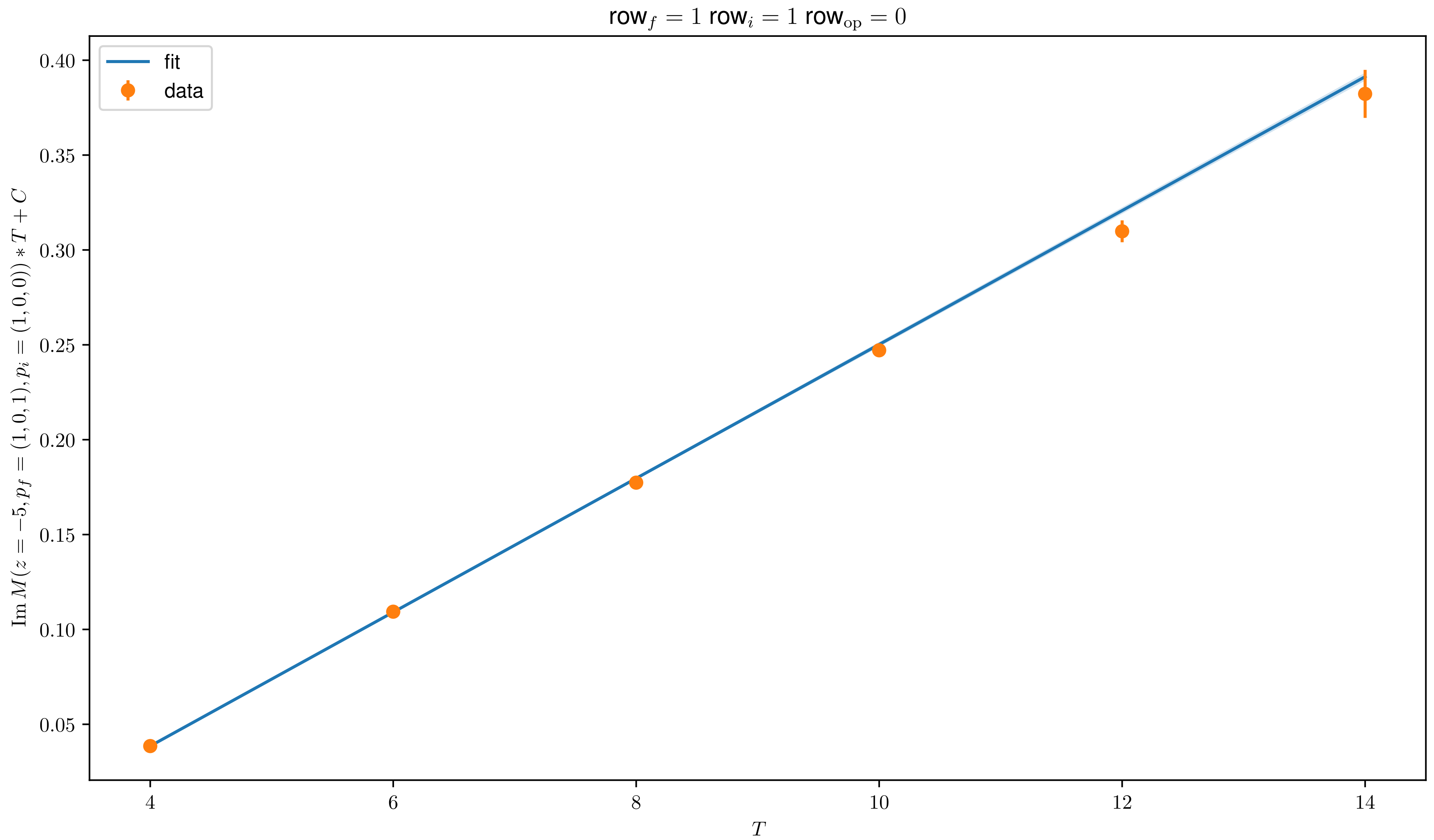
- cl21_32_64_b6p3_m0p2350_m0p2050.nuc.pGITD_pf014_pi004.n64.t0_0_tsnk5.Z-8.Z8.edb

- Contains DX operators

Linear fits

- Summed ratio of Nucleon D0J0 operators only

$$R(T) = \sum_{t=2}^{T-1} \frac{C_3(t,T)}{C_{snk}(T)} \sqrt{\frac{C_{src}(T-t)C_{snk}(t)C_{snk}(T)}{C_{snk}(T-t)C_{src}(t)C_{src}(T)}} \approx MT + C$$



Matrix Elements, Lorentz Invariant Amplitudes, and the Pseudo-Inverse

- Matrix Elements to Amplitudes

$$M^{\mu,\lambda_i,\lambda_f} = \langle p_f, \lambda_f | O^\mu(-\frac{z}{2}, \frac{z}{2}) | p_i, \lambda_i \rangle = \sum_{i=1}^{8} K_i^{\mu,\lambda_i,\lambda_f}(q, P, z) A_i(\nu, \xi, t)$$

- $K^{\mu,\lambda_i,\lambda_f}$ is a function of spinor bilinears $\bar{u}(p_f, \lambda_f)\Gamma u(p_i, \lambda_i)$ and the vectors in the problem

- Calculate for more matrix elements than there are amplitudes to create a matrix relationship $M = KA$

Penrose, Roger (1956). “On best approximate solution of linear matrix equations”
https://doi.org/10.1017/S0305004100030929

- Multivariable linear regression by minimizing $\chi^2 = |M - KA|^2$

- Minimum found with pseudo-inverse $\tilde{A} = K^+ M$

- Evaluated $K_i^{\mu,\lambda_i,\lambda_f}$ with code from https://github.com/CEgerer93/analysis/blob/master/eval_kin_mat.cc

- Stored at /work/JAM/jkarpie/kin_mat_files and kin_mat_files_no_z

Amplitudes and Double Distributions

- Choice of kinematic matrices defines which DDs appear where

$$K_1 = \bar{u}\gamma^\mu u, \quad K_2 = z^\mu \bar{u}u, \quad K_3 = i\bar{u}\sigma^{\mu z}u, \quad K_4 = \frac{i}{2m}\bar{u}\sigma^{\mu q}u$$

$$K_5 = \frac{q^\mu}{2m}\bar{u}u, \quad K_6 = \frac{i}{2m}P^\mu \bar{u}\sigma^{zq}u, \quad K_7 = \frac{i}{2m}q^\mu \bar{u}\sigma^{zq}u, \quad K_8 = \frac{i}{2m}z^\mu \bar{u}\sigma^{zq}u$$

- The following amplitudes are related to the lightcone DDs

$$A_1 = H_{DD} + 2\xi\nu Y \qquad \frac{-1}{2}A_5 = D + \nu Y$$

$$A_4 + \nu A_6 + \xi\nu A_7 = E_{DD} - 2\nu\xi Y$$

- Ioffe time to DD space

$$A(\nu, \xi, t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta} d\alpha e^{i\nu(\beta+\xi\alpha)} DD(\beta, \alpha, t)$$

$z = 0$ Form Factors

- Choice of kinematic matrices defines which DDs appear where

$$K_1 = \bar{u}\gamma^\mu u, \quad K_2 = z^\mu \bar{u}u, \quad K_3 = i\bar{u}\sigma^{\mu z}u, \quad K_4 = \frac{i}{2m}\bar{u}\sigma^{\mu q}u$$

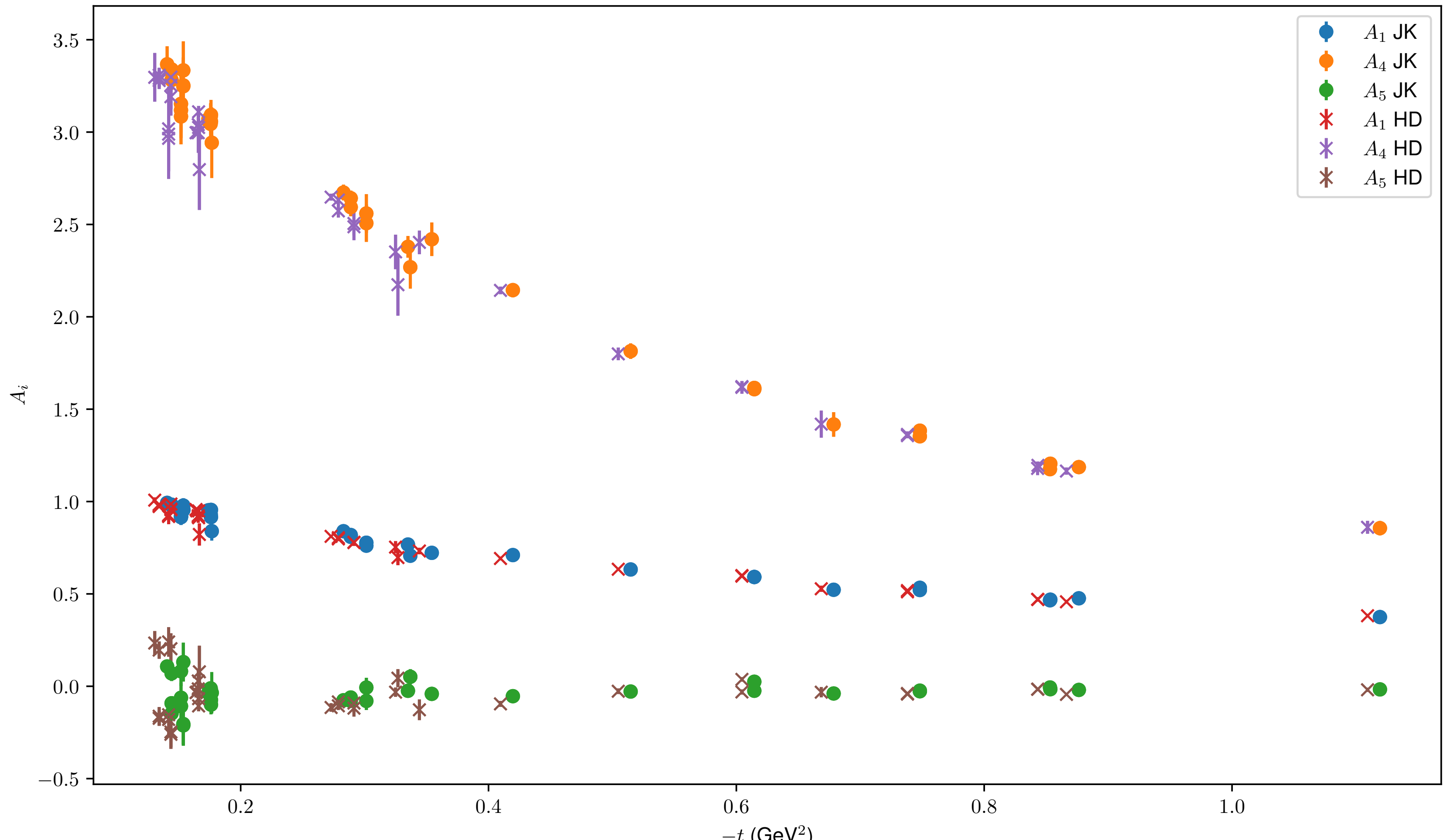
$$K_5 = \frac{q^\mu}{2m}\bar{u}u, \quad K_6 = \frac{i}{2m}P^\mu \bar{u}\sigma^{zq}u, \quad K_7 = \frac{i}{2m}q^\mu \bar{u}\sigma^{zq}u, \quad K_8 = \frac{i}{2m}z^\mu \bar{u}\sigma^{zq}u$$

- The following amplitudes are related to the lightcone DDs

$$A_1 = F_1$$
$$A_4 = F_2$$
$$A_5 = 0$$

- Compare to Herve's multistate fits

- γ^z data was needed to constrain A_5

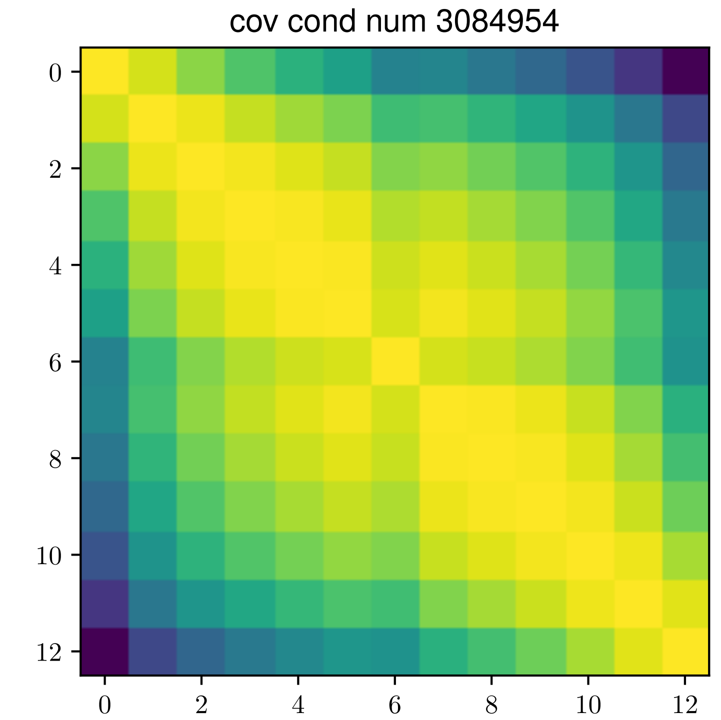


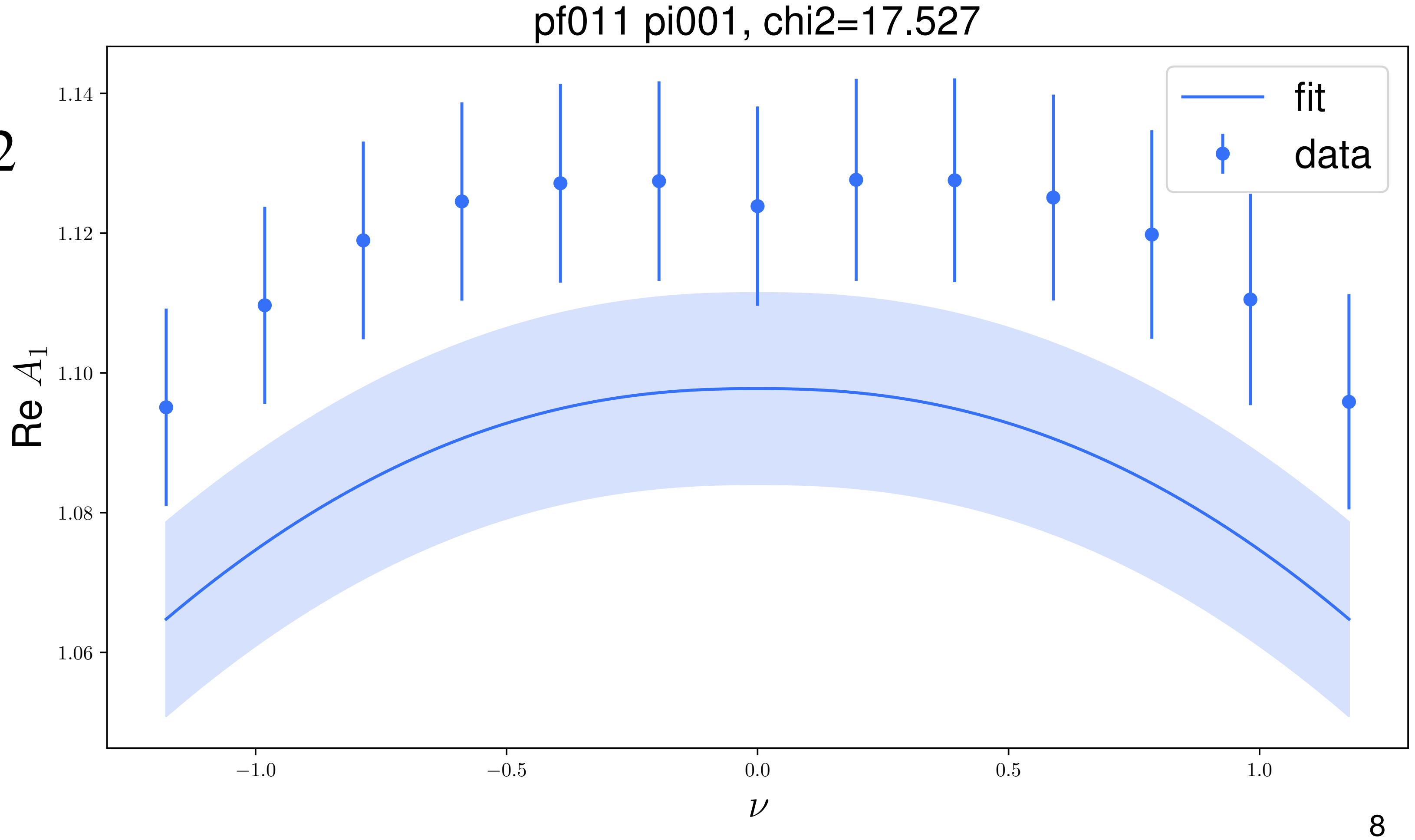
Shrinking the covariance matrix

- The covariance matrices of some of the amplitudes has a large condition numbers from poor estimation

- For p observations, there are $\dfrac{p(p-1)}{2}$ (co)variances

- Results in inaccurate χ^2 estimation





Shrinking the covariance matrix

- Quants have portfolios of vary many stocks compared the number of historical estimations available

O. Ledoit, M. Wolf. “A well-conditioned estimator for large-dimensional covariance matrices”
doi:10.1016/S0047-259X(03)00096-4

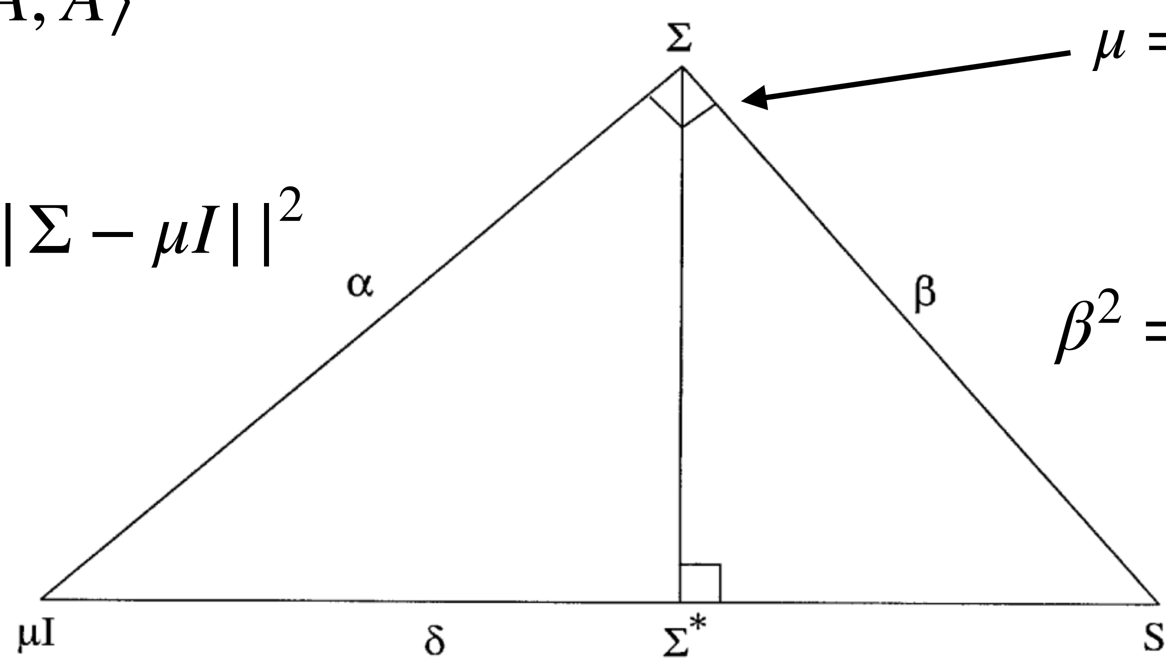
- Define a meaning of “closeness” via a dot product

$$\langle A, B \rangle = \mathrm{Tr}[A^{\dagger}, B]/p$$

$$||A||^2 = \langle A, A \rangle$$

True Covariance

$\mu = \langle \Sigma, I \rangle$ chosen to make right triangle



$$\alpha^2 = ||\Sigma - \mu I||^2$$

$$\beta^2 = E[\,||S - \Sigma||^2\,]$$

Shrinkage Target

Sample Covariance

$$\Sigma^* = \frac{\beta^2}{\delta^2}\mu I + \frac{\alpha^2}{\delta^2} S$$

Optimal point between μI and S

$$\delta^2 = E[\,||S - \mu I||^2\,]$$

Shrinking the covariance matrix

O. Ledoit, M. Wolf. “A well-conditioned estimator for large-dimensional covariance matrices”
doi:10.1016/S0047-259X(03)00096-4

You don't actually know Σ, but Ledoit and Wolf define estimators for $\alpha^2, \beta^2, \delta^2, \mu$

General asymptotic $p_n, n \to \infty, \frac{p_n}{n} \to C$

Regular Statistics limit is $C = 0$

μ use S instead of Σ

δ^2 as it looks expected

β^2 Var of covariance from sample

α^2 from a triangle relation

$$m_n = \langle S_n, I_n \rangle$$

$$d_n^2 = ||S_n - m_n I_n||^2$$

$$\bar{b}_n^2 = \frac{1}{n} || \sum_{k=1}^{n} (x_k - \bar{x})(x_k - \bar{x})^T - S_n ||^2]$$

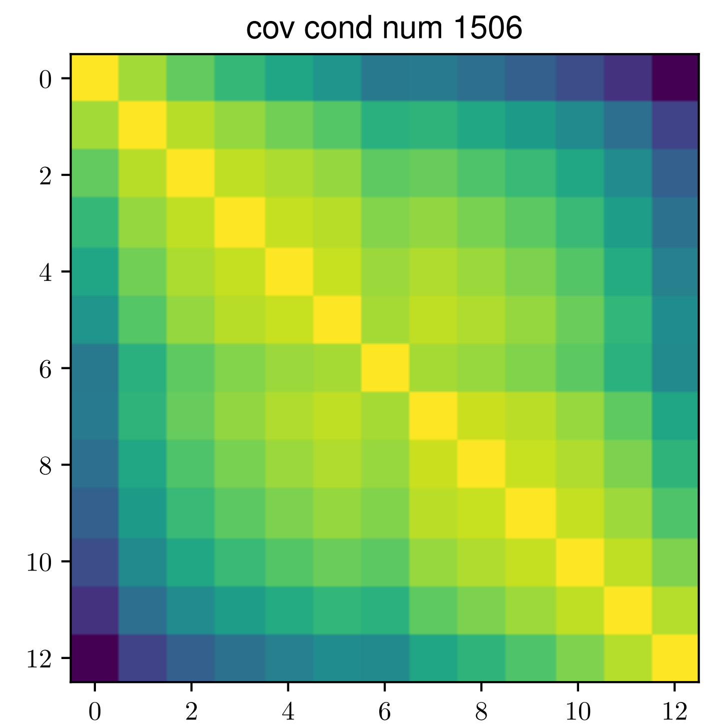
$$b_n^2 = \begin{cases} \bar{b}_n^2, & \text{if } \bar{b}_n < d_n \\ d_n, & else \end{cases}$$

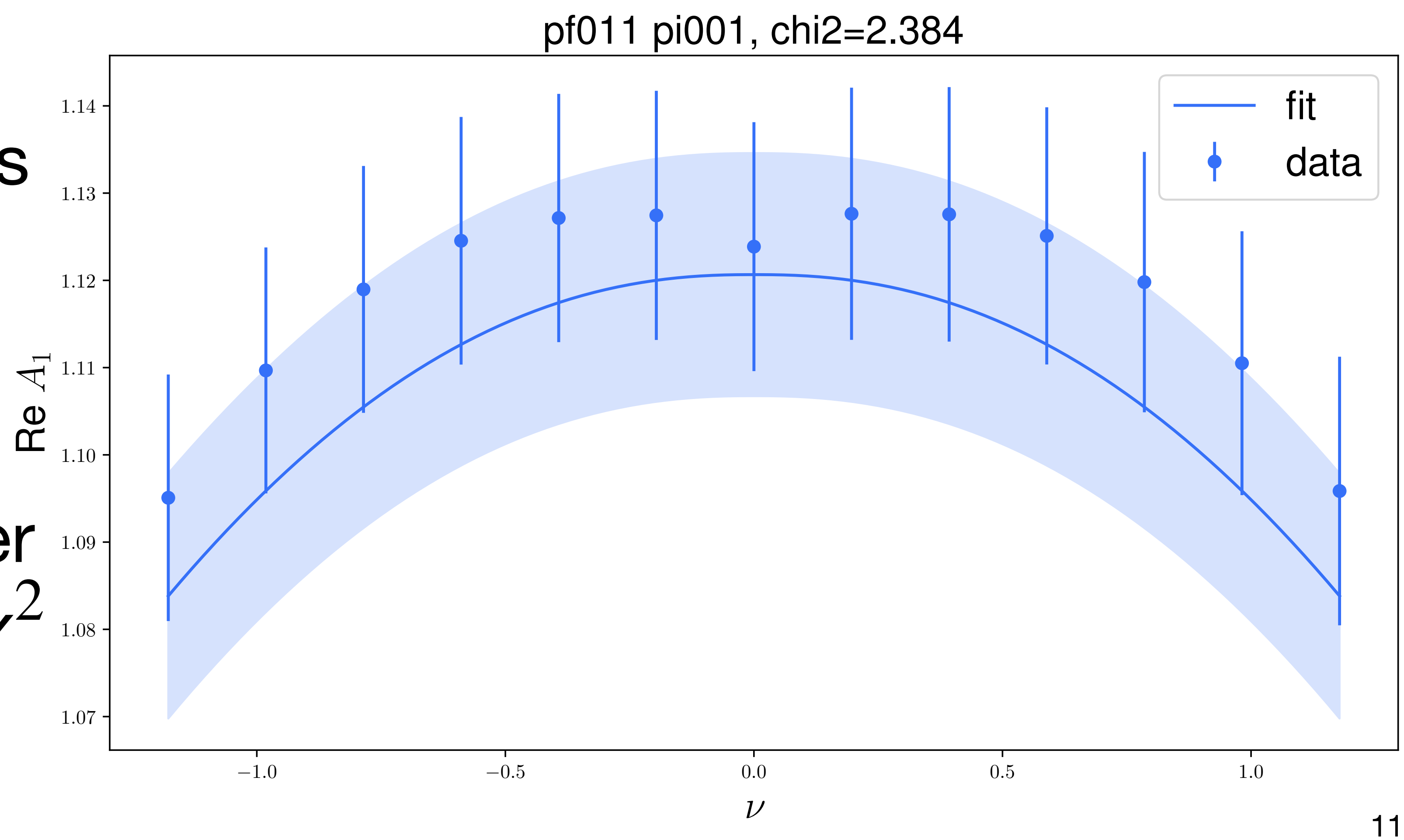
$$a_n^2 = d_n^2 - b_n^2$$

$$S_n^* = \frac{b_n^2}{d_n^2} m_n I + \frac{a_n^2}{d_n^2} S_n$$

Shrinking the covariance matrix

- The covariance matrices of some of the amplitudes has a large condition numbers from poor estimation
- For p observations, there are $\dfrac{p(p-1)}{2}$ (co)variances
- Shrunk covariance adds some amount to the diagonal the reweights
- Lower condition number means more accurate χ^2

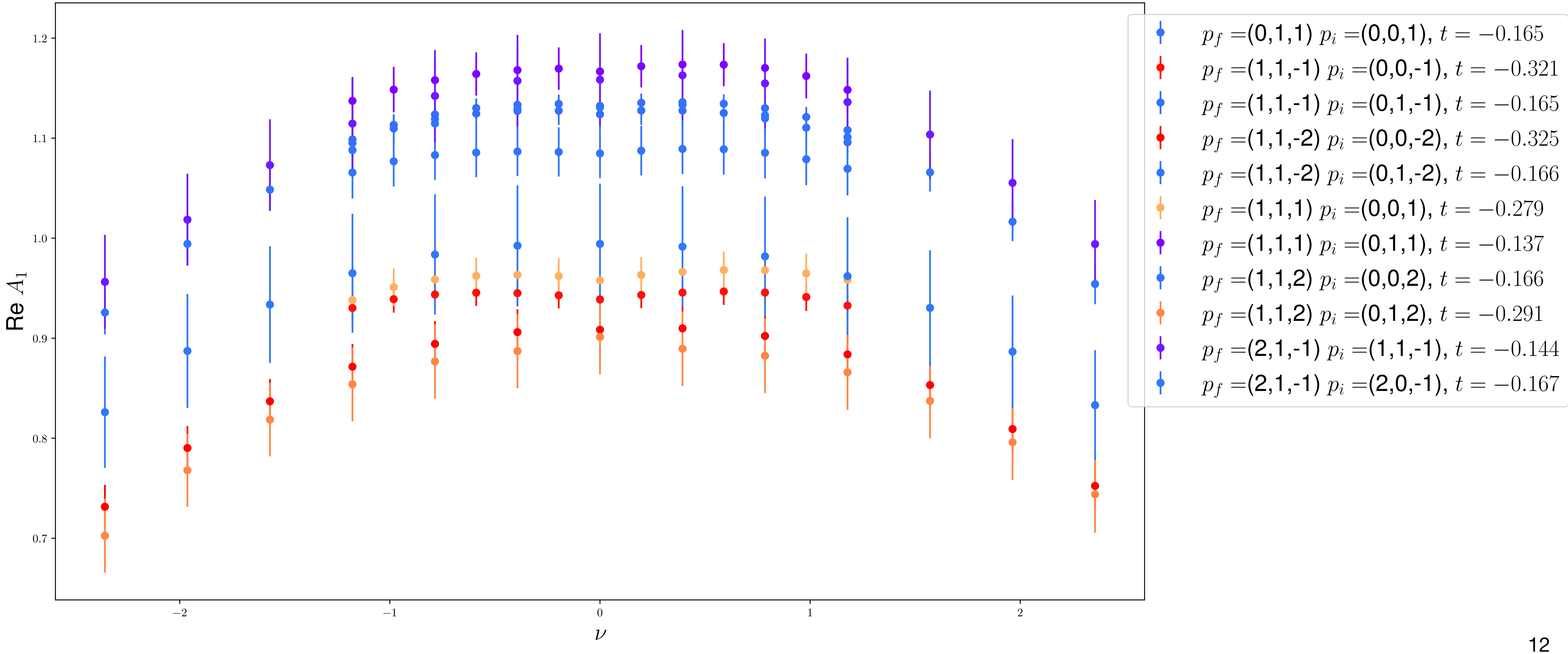




Amplitude Data

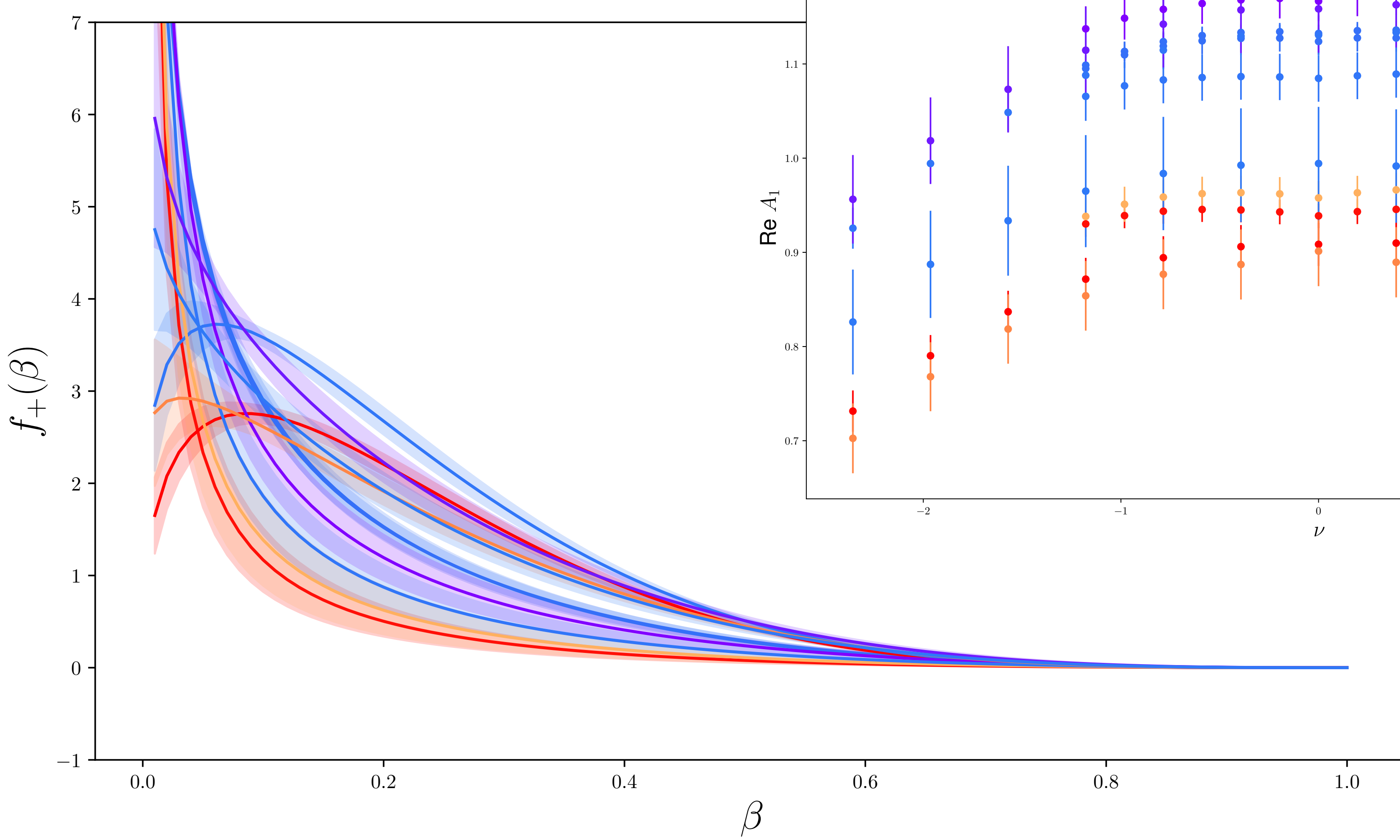
- Jackknife samples (reweight errors by $\sqrt{N-1}$) /work/JAM/jkarpie/data/pseudoGPD/unpol_quark_matelems/cl21_32_64_b6p3_m0p2350_m0p2050/unphased/snk*src*/fit_res/amplitude* next to plots/ subdirectory and other data

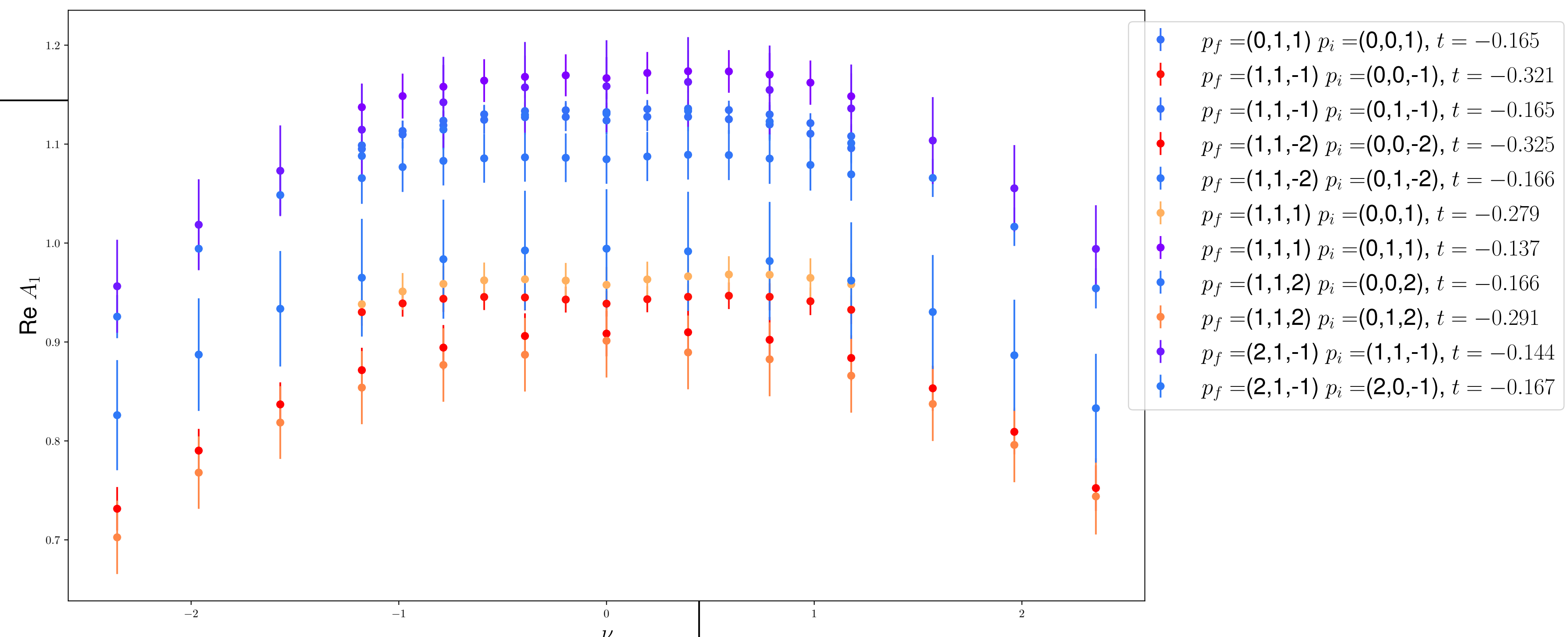
- A_1 amplitude from skewness=0



GPD fits

- Skewness=0
- Used DGLAP matching and fit each (p_f, p_i)
- Fit to $H(x, \xi = 0, t) = Nx^a(1-x)^b$





GPD fits

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