Electron-positron production in kinematic conditions of PrimEx

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We consider photoproduction of e^+e^- pairs on a nucleus with atomic weight A and atomic number Z:

$$\gamma + A \to e^+ + e^- + A'$$

at photon energies ω a few GeV's and very small momentum transfer $|\vec{Q}|$ relevant for PrimEx.

Cross section of this process consists of the contributions listed below in order of importance.

- Bethe-Heitler mechanism of pair production on the nucleus (coherent process) with screening effects due to atomic electrons and Coulomb distortion.

- Pair production on atomic electrons with excitation of all atomic states. It contains correlation effects due to presence of other electrons and nucleus.

- Quantum Electrodynamical (QED) radiative corrections (of order α/π with respect to dominant contributions): (i) virtual-photon loops and (ii) real-photon process $\gamma + A \to e^+ + e^- + A + \gamma$, where the final photon has the energy $\omega' \leq \delta\omega$ (energy resolution in experiment).

- Nuclear incoherent contribution – quasi-elastic, or quasi-free process on the proton $\gamma + p \longrightarrow e^+ + e^- + p$.

- Nuclear coherent contribution, or virtual Compton Scattering (CS) two-step mechanism $\gamma + A \longrightarrow \gamma^* + A \longrightarrow e^+ + e^- + A$.

Below these contributions are reviewed in some detail.

Pair production on nucleus

The exclusive cross section on the nucleus has the form

$$\frac{d^4\sigma_A}{d\epsilon_+ d\theta_- d\theta_+ d\phi} = Z^2 \frac{\alpha^3}{2\pi\omega^3 \vec{Q}^4} |F_A(\vec{Q}^2) - f_{at}(\vec{Q}^2)|^2 |T|^2,$$

where $\alpha = 1/137.036$ is the fine-structure constant,
θ_+, θ_- are the lepton polar angles,
ϕ is the azimuthal angle between the plane spanned by the momenta $\vec{k}$, $\vec{p}_+$ and the plane spanned by $\vec{k}$, $\vec{p}_-$,
$k = (\omega, \vec{k})$ is the photon 4-momentum,
$p_+ = (\epsilon_+, \vec{p}_+)$ and $p_- = (\epsilon_-, \vec{p}_-)$ is the positron and electron 4-momentum respectively and

m_e is the electron mass.
$\vec{Q} = \vec{k} - \vec{p}_+ - \vec{p}_-$ is 3-momentum transfered to the nucleus.
In "no-recoil" approximation $Q_0 = \omega - \epsilon_+ - \epsilon_- = 0$.

$|T|^2$ is a kinematic factor

$$
\begin{aligned}
|T|^2 \;=\; & p_+ p_- \sin\theta_+ \sin\theta_- [-\frac{p_+^2}{\xi_+^2}(4\epsilon_-^2 - \vec{Q}^2)\sin^2\theta_+ \\
& -\frac{p_-^2}{\xi_-^2}(4\epsilon_+^2 - \vec{Q}^2)\sin^2\theta_- + \frac{2\omega^2}{\xi_+\xi_-}(p_+^2 \sin^2\theta_+ \\
& + p_-^2 \sin^2\theta_-) \\
& + \frac{2p_+p_-}{\xi_+\xi_-}(2\epsilon_+^2 + 2\epsilon_-^2 - \vec{Q}^2)\sin\theta_+ \sin\theta_- \cos\phi]
\end{aligned}
$$

where $\xi_\pm = \epsilon_\pm - p_\pm \cos\theta_\pm$ and $p_\pm = (\epsilon_\pm^2 - m_e^2)^{1/2}$.

$x_+ \equiv \epsilon_+/\omega$ ($x_- \equiv \epsilon_-/\omega$) is fraction of energy carried by positron (electron), and $x_+ + x_- = 1$.

$f_{at}(\vec{Q}^2)$ is the atomic form factor describing charge distribution of electrons $\rho_{at}(r)$. Screening of the nucleus charge by atomic electrons becomes important if $|\vec{Q}| < a_{at}^{-1}$, where $a_{at} \approx 137 Z^{-1/3}/m_e \sim 3 \times 10^{-9}$ cm is the atomic size. Since the main contribution to cross section comes from the region close to the minimal possible value

$$|\vec{Q}|_{min} \sim m_e(\frac{m_e\omega}{2\epsilon_+\epsilon_-}) = \frac{m_e}{2x_+x_-}(\frac{m_e}{\omega}),$$

then at high energies, where $\omega >> m_e$ and $x_+,\ x_-$ are not too close to zero, $|\vec{Q}|_{min}$ is always less than $a_{at}^{-1} \sim 10$ keV, and the screening is very important.

In calculation the Thomas-Fermi-Moliere model [Moliere] is used

$$f_{at}(\vec{Q}^2) = 1 - \sum_{i=1}^{3} \frac{a_i\vec{Q}^2}{\vec{Q}^2 + b_i^2/b_0^2},$$

with parameters $b_0,\ b_i$ and a_i.

Further, $F_A(\vec{Q}^2)$ is the nuclear charge form factor (Fourier transform of $\rho_A(r)$) which behaves like

$$F_A(\vec{Q}^2) \approx 1 - \frac{1}{6}\vec{Q}^2\langle r^2\rangle_A$$

where $\langle r^2\rangle_A$ is the m.s.r. of the nucleus. In the conditions of PrimEx this form factor can always be replaced by unity since $|\vec{Q}| < a_{at}^{-1} << r_{nucl}^{-1}$.

Energy distribution of the positrons has the form [Bethe and Heitler]:

$$\frac{d\sigma_A}{d\epsilon_+} = \int \frac{d^4\sigma_A}{d\epsilon_+ d\theta_- d\theta_+ d\phi} \, \mathrm{d}\theta_+ \, \mathrm{d}\theta_- \, \mathrm{d}\phi$$
$$= Z^2 \frac{\alpha^3}{m_e^2 \omega^3} [(\epsilon_+^2 + \epsilon_-^2)(\phi_1 - \frac{4}{3}\log Z - 4f)$$
$$+ \frac{2}{3}\epsilon_+\epsilon_-(\phi_2 - \frac{4}{3}\log Z - 4f)].$$

$f \equiv f((\alpha Z)^2)$ is the Coulomb distortion function [Bethe and Maximon]

$$f((\alpha Z)^2) = (\alpha Z)^2 \sum_{n=1}^{\infty} \frac{1}{n[n^2 + (\alpha Z)^2]}.$$

For the ^{12}C nucleus $f \approx 2.3 \times 10^{-3}$.

Functions $\phi_{1,2}$ depend on the parameter
$\gamma = 100 Z^{-1/3} \frac{m_e}{\omega x_+ x_-}$ and account for the screening effect.
If $\gamma >> 1$ then screening is unimportant, while
for $\gamma \approx 0$ screening is essential.

In the conditions of PrimEx screening leads to reduction of cross sections by a factor $\sim 1.3-1.7$ depending on x_+. The functions $\phi_{1,2}$ are calculated in the Thomas-Fermi-Moliere model.

Note that the recoil of the nucleus can be neglected as the energy of the final nucleus is of the order ~ 1 keV in the conditions of PrimEx.

Pair production on atomic electrons

The corresponding cross section has the form

$$\frac{d^4\sigma_e}{d\epsilon_+ d\theta_- d\theta_+ d\phi} = Z\frac{\alpha^3}{2\pi\omega^3\vec{Q}^4}H(\vec{Q}^2)|T|^2,$$

where $H(\vec{Q}^2)$ is a correlation factor related to all atomic excitations. One can also say that it accounts for screening effects due to the presence of other electrons and the nucleus. It can be chosen as [Tsai]:

$$H(\vec{Q}^2) = \frac{a'^4\vec{Q}^4}{(1+a'^2\vec{Q}^2)^2}$$

with $a' = 1194 Z^{-2/3}/(e^{1/2} m_e)$ which follows from the value of radiation logarithm L'_{rad} in (see below).

The energy distribution of positrons has the form

$$\frac{d\sigma_e}{d\epsilon_+} = Z\frac{\alpha^3}{m_e^2\omega^3}[(\epsilon_+^2+\epsilon_-^2)(\psi_1-\frac{8}{3}\log Z)+\frac{2}{3}\epsilon_+\epsilon_-(\psi_2-\frac{8}{3}\log Z)].$$

The functions $\psi_{1,2}$ [Wheeler and Lamb] depend on the parameter $\varepsilon = 100 Z^{-2/3}\frac{m_e}{\omega x_+ x_-}$.

Values $\phi_1(0)$ and $\psi_1(0)$ are directly related to the unit radiation length of material

$$X_0 = \frac{716.405\ A}{Z^2(L_{rad} - f) + Z L'_{rad}},$$

where in the Thomas-Fermi-Moliere model

$$\begin{aligned} L_{rad} &= \frac{1}{4}\phi_1(0) - \frac{1}{3}\log(Z) = \log(184.15\ Z^{-1/3}), \\ L'_{rad} &= \frac{1}{4}\psi_1(0) - \frac{2}{3}\log(Z) = \log(1194\ Z^{-2/3}). \end{aligned}$$

For example, for carbon one obtains $X_0 = 42.6983\,\mathrm{g/cm^2}$.

Cross section can be further corrected for recoil of the target electron

$$\frac{d\sigma_e}{d\epsilon_+} \longrightarrow \frac{d\sigma_e}{d\epsilon_+}(1 - \delta_{rec}),$$

where

$$\delta_{rec} = (\frac{m_e}{\omega}) \frac{4L^3/3 - 3L^2 + 6.84L - 21.51}{28L/9 - 218/27}$$

with $L \equiv \log(2\omega/m_e)$. This is a small effect of order 4×10^{-3} at high photon energies.

Radiative corrections

QED radiative corrections are included in the method of Weizsacker-Williams following the work of Mork.

There are 16 Feynman diagrams which describe virtual-photon loops (vertex modification, lepton self-energy insertions, box diagrams and vacuum polarization), and real-photon emission. The latter contribution depends on the energy resolution. Specifically, if the energy difference

$\omega - \epsilon_+ - \epsilon_-$ is determined with accuracy $\delta\omega$ then the emitted photon may have the energy $\omega' \leq \omega'_{max} = \delta\omega$. The energy resolution in pair production experiments $\delta\omega/\omega$ varies from 1.70% to 1.85%.

The radiative corrections modify the lowest-order cross section

$$\frac{d\sigma_{A,e}}{d\epsilon_+} \rightarrow \frac{d\sigma_{A,e}}{d\epsilon_+}(1 + \delta_{RC}),$$

and for the correction factor δ_{RC} we obtain

$$\delta_{RC} = G_1 + G_2 \log(\frac{\delta\omega}{\omega}) + G_3(\log(R))^{-1}.$$

Function G_2 originates from real-photon contribution, G_1 – from real- and virtual-photon contributions and G_3 – from vacuum-polarization diagrams.

The quantity R is defined as

$$R = [(\frac{m_e}{2\omega x_+ x_-})^2 + (\frac{Z^{1/3}}{183})^2]^{-1/2},$$

and $R \approx 2x_+x_-\omega/m_e$ if screening is unimportant, while $R \approx 183Z^{-1/3}$ for complete screening relevant for PrimEx.

Radiative correction due to vacuum polarization is small and therefore δ_{RC} virtually depends only on x_+.

Typical radiation correction is shown on Fig. 1. For the other photon energies the corresponding δ_{RC} practically coincides with the curve in Fig. 1.

Nuclear incoherent contribution

The photon interacts with the nucleon inside the nucleus as with a free nucleon. The cross section of this quasi-free (quasi-elastic) incoherent process in the Fermi-gas model can approximately be written as follows

$$\frac{d^4\sigma_N}{d\epsilon_+ d\theta_- d\theta_+ d\phi} = Z\frac{\alpha^3}{2\pi\omega^3\vec{Q}^4}|F_p(\vec{Q}^2)|^2 I_F(\vec{Q}^2)|T|^2.$$

The factor $I_F(\vec{Q}^2)$ takes into account Pauli blocking for the final proton

$$\begin{aligned} I_F(\vec{Q}^2) &= \frac{3|\vec{Q}|}{4p_F}(1-\frac{\vec{Q}^2}{12p_F^2}), \qquad \mathrm{if} \quad |\vec{\mathrm{Q}}| \le 2\mathrm{p_F}, \\ &= 1, \quad \mathrm{if} \quad |\vec{\mathrm{Q}}| > 2\mathrm{p_F}. \end{aligned}$$

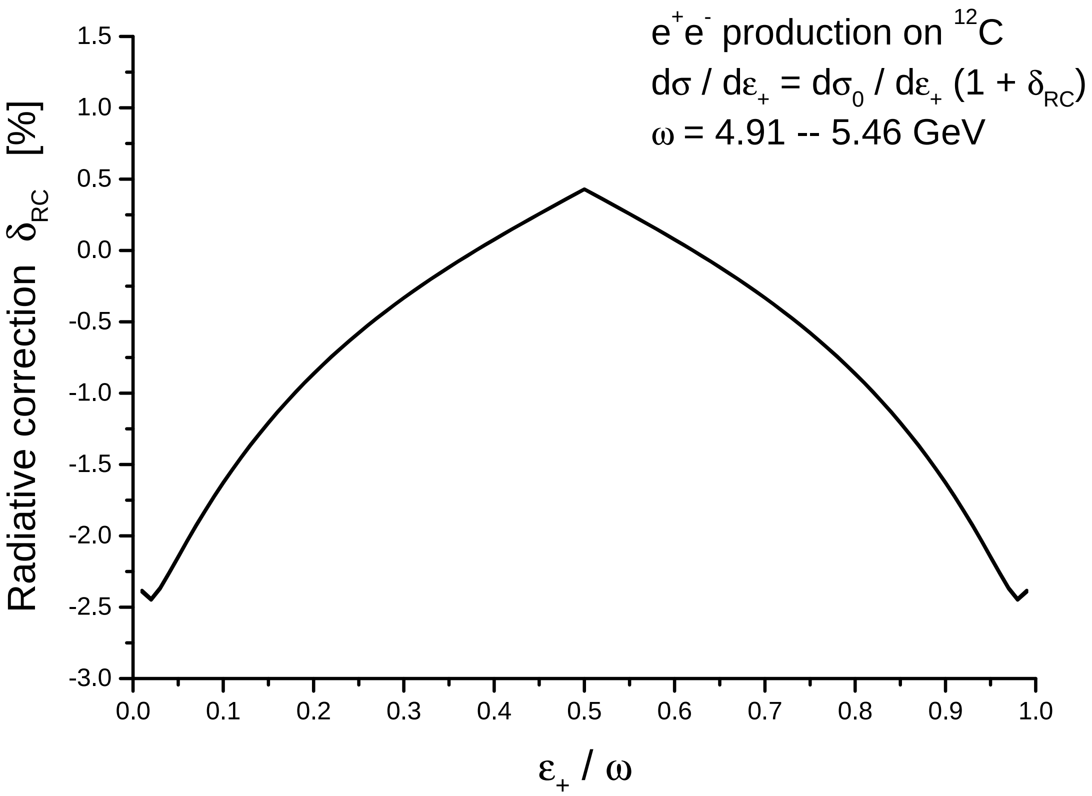

Figure 1: Radiative correction δ_{RC} as a function of the fraction of energy carried by positron.

Here p_F is the Fermi momentum for the nucleus A, in particular, for ^{12}C nucleus, one finds from electron scattering $^{12}C(e,e')$ experiments [Whitney et al.] $p_F = 221$ MeV.

The contribution from the neutrons is neglected, as well as magnetic moments of the nucleons. The latter contribution is proportional to a very small factor $\vec{Q}^2/(4M_N^2)$.

$F_p(\vec{Q}^2)$ is the proton charge form factor (for example, in the dipole parameterization)

$$F_p(\vec{Q}^2) = (1 + \frac{\vec{Q}^2}{0.71 \text{ GeV}^2})^{-2}.$$

This form factor at very small values of $|\vec{Q}|$ is practically equal to unity.

The energy distribution is obtained by integration of this cross section over the angles of the leptons

$$\frac{d\sigma_N}{d\epsilon_+} = Z\frac{4\alpha^3}{m_e^2\omega^3}\frac{\beta_F}{2}\{-(\epsilon_+^2+\epsilon_-^2)B$$

$$+4\epsilon_+\epsilon_-[\frac{1}{2}+\beta_F+\beta_F(1+\beta_F)B]\},$$

where

$$\beta_F = \frac{3m_e}{2p_F}(1-\frac{m_e\omega}{2\epsilon_+\epsilon_-})^2, \qquad B=\log(\frac{\beta_F}{1+\beta_F}).$$

At very small $|\vec{Q}|$, of the order m_e or less, the incoherent cross section is suppressed due to the Pauli blocking effect. Rough estimate gives

$$\beta_F \sim 3.4\times 10^{-3}, \qquad \frac{\beta_F}{2}\log(\beta_F)\sim 10^{-2},$$

therefore it is at most $Z^{-1}\times 10^{-2}$ of the dominant Bethe-Heitler cross section on the nucleus.

Nuclear coherent contribution

The nuclear coherent contribution is the most complicated and least known mechanism of pair production. The corresponding amplitude of the virtual Compton scattering (CS) on the nucleus, $\mathcal{M}_{VCS}$, describes the process

$\gamma A \to \gamma^* A \to e^+e^- A$. We need this amplitude for almost forward scattering in the energy region a few GeV's, i.e. above the region of excitation of baryon resonances. At these enrgies this is diffractive process which can be viewed as the $t-$channel exchanges by Regge trajectories associated with the so-called Pomeron and a few scalar and tensor mesons.

At very high energies and small momentum transfer we can express the virtual CS amplitude on the nucleus in terms of the spin-averaged Compton scattering amplitude on the free proton (neglecting the neutron contributions):

$$\mathcal{M}_{VCS} \approx Z\mathcal{M}^{(p)}_{VCS}.$$

The magnitude of $|\mathcal{M}_{VCS}|^2$ is extremely small, therefore it is sufficient to keep only the interference between the dominant Bethe-Heitler amplitude on the nucleus and virtual CS amplitude $2\mathrm{Re}(\mathcal{M}_{\mathrm{A}}\mathcal{M}^*_{\mathrm{VCS}})$. The virtual CS amplitude on the proton, $\mathcal{M}^{(p)}_{VCS}$, can be related to forward scattering Compton amplitude with *real* photons $f_1(\omega)$. The energy dependence of this amplitude is known from high-energy CS experiments [Armstrong et al.]. In particular, for the real part we obtain parameterization

$$\mathrm{Re}\ \mathrm{f}_1(\omega) = -\frac{\alpha}{\mathrm{M_p}}[0.76 + 1.88(\frac{\omega}{\omega_0})^{1/2}],$$

where ω_0 =1 GeV and M_p = 938.3 MeV is the proton mass.

Here we consider only kinematics for pair production at very small (but not equal) angles

$$\delta_+ \sim \delta_- \sim 1, \qquad \delta_\pm = \frac{\epsilon_\pm \theta_\pm}{m_e},$$

and the region

$$|\delta_+ - \delta_-| < \frac{m_e}{\epsilon_\pm}, \qquad |\phi - \pi| < \frac{m_e}{\epsilon_\pm},$$

which gives the dominant "logarithmic" contribution to the energy distribution in the conditions of PrimEx.

In these conditions we find the ratio of the interference cross section and the nucleus Bethe-Heitler cross section

$$R \approx (x_- - x_+)\,\frac{m_e^2}{\omega M_p}\,[0.76 + 1.88(\frac{\omega}{\omega_0})^{1/2}].$$

At photon energy 5 GeV, for example, one obtains

$$R \approx 2.6 \times 10^{-7}(x_- - x_+).$$

For the energy distribution we get the estimate

$$\frac{d\sigma_{inter}}{d\epsilon_+} \sim 10^{-7}\ (x_- - x_+) \times \frac{d\sigma_A}{d\epsilon_+}.$$

This minor contribution can be completely neglected.

Note that the interference results in asymmetry between the positron and electron yields. The cross section is not anymore symmetrical in variables $x_+,\ x_-$ and depends on which lepton (e^+ or e^-) has greater energy, i.e.

$$\sigma(x_+, x_-)|_{x_- > x_+} > \sigma(x_+, x_-)|_{x_+ > x_-}.$$

This is an example of the charge asymmetry caused by the BH – virtual CS interference in general case [Bjorken, Drell and Frautschi]

$$\frac{2\mathrm{Re}(\mathcal{M}_{\mathrm{A}}\mathcal{M}^*_{\mathrm{VCS}})}{|\mathcal{M}_A|^2} = \frac{\sigma(e^+, e^-) - \sigma(e^-, e^+)}{\sigma(e^+, e^-) + \sigma(e^-, e^+)}.$$

mechanism	contribution in %
nuclear Bethe-Heitler	82.789
atomic electrons	17.185
nuclear incoherent (quasielastic)	0.026
nuclear coherent (virtual CS)	$\sim 10^{-5}$
total	100.00

Table 1: Various contributions to cross section at photon energy 4.91 GeV. $x_+ = 0.4$ and $x_- = 0.6$

In order to minimize the virtual CS contribution the "symmetric" conditions for the leptons

$$\epsilon_+ = \epsilon_-, \qquad \theta_+ = \theta_- \quad \text{and} \quad \phi = \pi$$

are the most appropriate. In this case the interference term $2\mathrm{Re}(\mathcal{M}_{\mathrm{BH}}\mathcal{M}^*_{\mathrm{VCS}})$ vanishes *identically* and one is left with virtual CS amplitude squared $|\mathcal{M}_{VCS}|^2$, which can be safely omitted.

The cross section (energy distribution) including all above contributions is thus

$$\frac{d\sigma}{d\epsilon_+} = \frac{d\sigma_A}{d\epsilon_+} + \frac{d\sigma_e}{d\epsilon_+} + \frac{d\sigma_N}{d\epsilon_+}.$$

Finally, the total cross section

$$\sigma_{tot} = \int_{m_e}^{\omega - m_e} \frac{d\sigma}{d\epsilon_+}\, \mathrm{d}\epsilon_+$$

is presented in Table 2.

photon energy in GeV	σ_{tot} in mb
4.91	348.8
4.97	348.9
5.03	349.1
5.08	349.0
5.13	349.1
5.18	349.3
5.23	349.2
5.28	349.5
5.34	349.3
5.41	349.5
5.46	349.5

Table 2: Total cross section of pair production on carbon at various photon energies

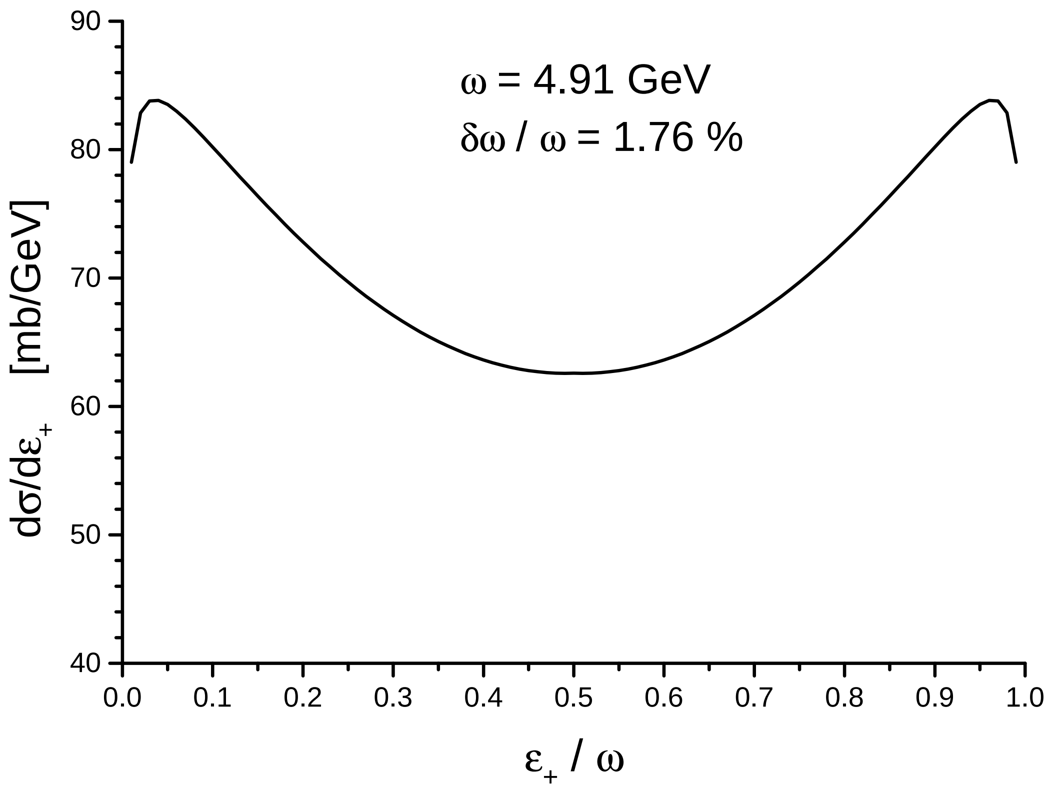

Figure 2: Energy distribution of positrons in e^+e^- production on carbon. The photon energy is 4.91 GeV.

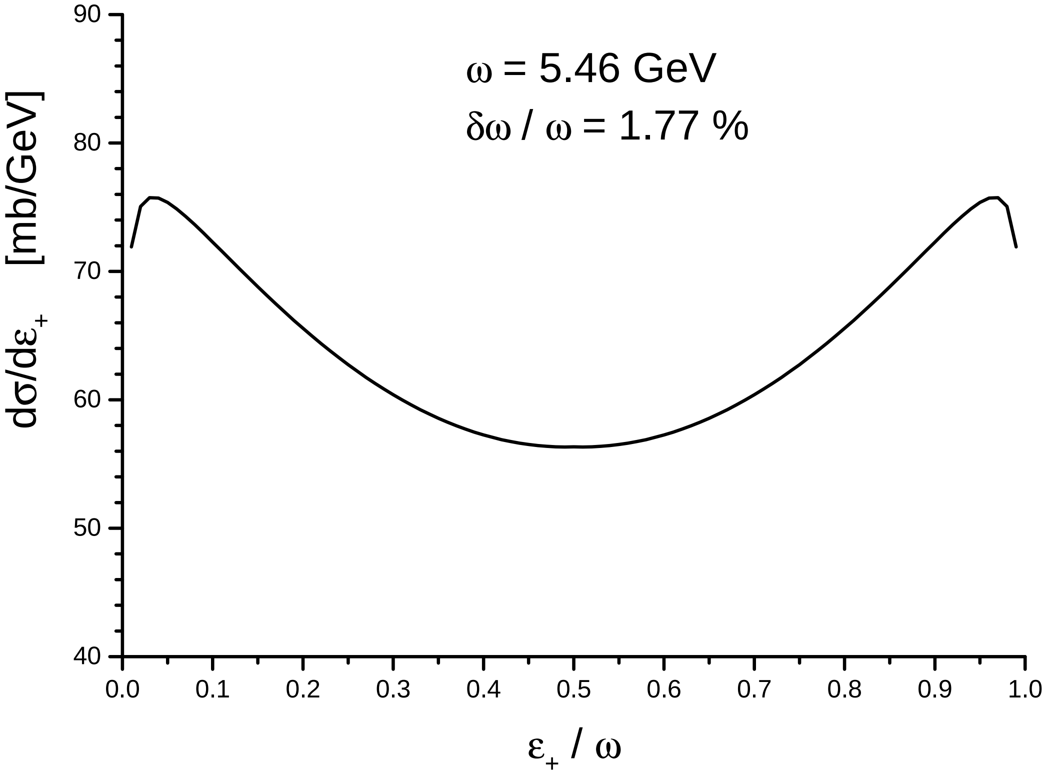

Figure 3: The same as in Fig. 2 but for the photon energy 5.46 GeV.