Considerations on using single electron scattering on tungsten, for the calibration of the HPS ECal

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Outline

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- Single electon scattering cross section
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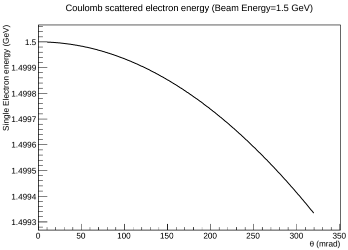
  - Integrated cross section
- Event rate
- Summary and conclusions

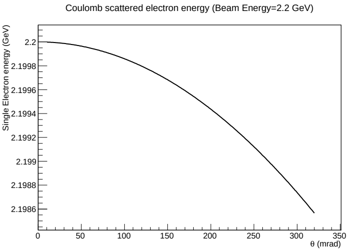
Kinematis

For ultrarealativistic electrons $E \gg mc^2$ and $E \approx |\mathbf{p}|$ and the following approximation for the elastically scattered electron energy E' from a nucleus in the laboratory system:

$$E' = \frac{E}{1 + \frac{E}{M}(1 - cos\theta)} \qquad (1)$$

where E is the beam energy, M is the target mass and θ is the scattering angle.





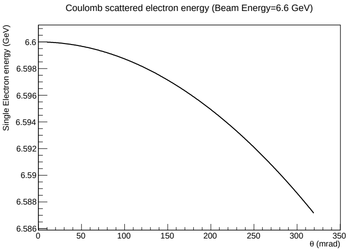


Figure: Scattered electron energy (GeV) VS scattering angle (mrad)

Beam Energy (GeV)	Ecal resolution δE (MeV)	$\Delta E'$ (MeV)
1.5	30	0.7
2.2	20	1.4
6.6	6	14

Table: Expected energy resolution of ECal ($\frac{4\%}{\sqrt{E}}$) compared with scattered electrons energy spread within ECal acceptance at different beam energies

- At low energy we can look for electrons with the same energy of the beam, at high energy we have to take into account the "angular position"

Cross section

The elastic scattering cross section may be computed starting from the Mott cross section, taking into account the electric form factor for the tungsten target:

$$\frac{d\sigma}{d\Omega}(E,\theta) = \frac{(Ze^2)^2}{(4\pi\epsilon_0)^2 4E^2 \sin^4\frac{\theta}{2}}(1-\beta^2\sin^2\frac{\theta}{2})|F(Q)|^2 \quad (2)$$

where:

$$F(Q) = \frac{3}{(QR)^3}(\sin(QR) - QR\cos(QR)) \quad (3)$$

with $Q^2 = -(P-P')^2$, $Q = \sqrt{Q^2} \approx |\vec{p}|$ being the transferred four-momentum and R the nucleus radius, given by $R = 1.21 fm A^{\frac{1}{3}}$. For a tungsten target we have $A \approx 183$ and $Z = 74$.

Writing the magnetic permeability of vacuum ϵ_0 as:

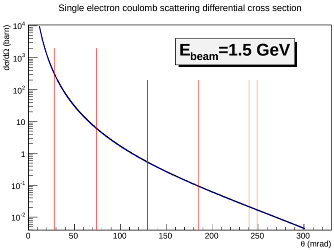
$$\epsilon_0 = \frac{e^2}{2\alpha hc} = \frac{e^2}{2\alpha 2\pi \hbar c} \tag{4}$$

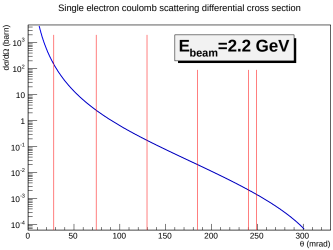
and using natural units:

$$\epsilon_0 = \frac{e^2}{4\pi\alpha} \text{ or } \alpha = \frac{e^2}{4\pi\epsilon_0} \tag{5}$$

the differential cross section then reads:

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2(1-\beta^2\sin^2\frac{\theta}{2})}{4E^2\sin^4\frac{\theta}{2}}\left(\frac{3}{(QR)^3}(\sin(QR) - QR\cos(QR))\right)^2 \tag{6}$$





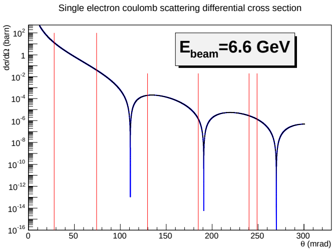


Figure: Differential cross sections of electron scattering off tungsten nucleus. Vertical red lines define the angular spread corresponding to each of the sectors defined in Figure3

Integrated cross sections

We want to estimate the number of elastic electron scattering events that would be detected by each ECal crystal. The rates are calculated according to the following expression:

$$\frac{dN_{E'}}{dt} = \frac{I_e}{q_e}\frac{\rho \cdot N_{av} \cdot l}{A}\Delta\sigma \tag{7}$$

where

- $\frac{dN_{E'}}{dt}$ number of scattered electrons per unit of time
- I_e is the beam current (200 nA at 2.2 GeV and 450 nA at 6.6 GeV)
- $q_e = 1.6\ 10^{-19} C$ is the electron charge
- $\rho = 19,3\ \frac{gm}{cm^3}$ is the tungsten density
- $N_{av} = 6.022\ 10^{23}(gm^{-1})$ is the Avogadro number, equal to the inverse of atomic mass unit expressed in grams
- $l = 5\mu m$ is the target thickness

- $A = 183.35$ is the average tungsten atomic number for natural isotope composition
- $\Delta\sigma$ is the differential cross section from equation (6) integrated over the acceptance of a single ECal crystal.

Substituting all numbers one obtains:

- $\mathbb{L} = 3.95\ 10^{31}\ cm^{-2}s^{-1}$ is the luminosity at 1.5 and 2.2 GeV ($I_{beam} = 200nA$)
- $\mathbb{L} = 8.93\ 10^{31}\ cm^{-2}s^{-1}$ is the luminosity at 6.6 GeV ($I_{beam} = 450nA$)

We need $\Delta\sigma$:

- Integration of the cross section (6) over each single ECal crystal geometrical acceptance is not straight forward.
- Geometrical and mathematical simplifications have been performed to calculate $\Delta\sigma$
  - The cross section depends on the polar electron scattering angle θ while it is independent from the azimuthal angle ϕ;
  - therefore the ECal geometry has been divided into five sectors corresponding to fixed intervals of θ, according to Figure 3.

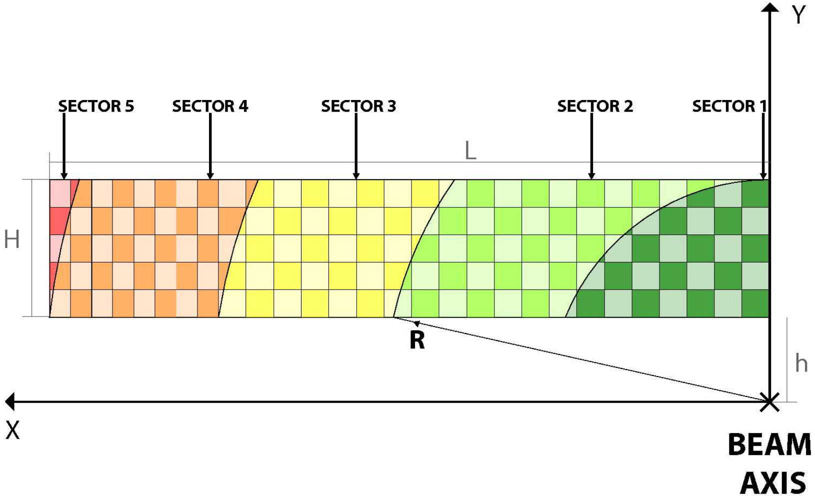


Figure: The upper-left face of the calorimeter showing the crystal granularity and divided into five sectors of fixed polar angle intervals.

The following geometrical dimensions have been considered:

- L=34 cm (half-calorimeter widh)
- h= 4 cm (distance of the bottom of the ECal quadrant from the x-z plane equal to half of the ECal vertical opening)
- H=6.5 cm (half of calorimenter height = 1.3 cm x 5 crystals)
- d=139.7 cm (distance of the calorimeter face from the target)
- r $= d\tan(\theta)$ (radial distance of the crystals from the beam axis)

For each sector a maximum and a minimum value of both the polar angle θ and the azimuthal angle ϕ have been identified, over which the differential cross section (6) may be averaged and integrated. Table 14 shows the intervals of the the polar angles θ and the variation of the corresponding azimuthal angles ϕ as a function of θ (and r), for each of the five sectors.

sector	$\theta_{min}(mrad)$	θ_{max} (mrad)	$\Delta\theta$ (mrad)	ϕ_{min}	ϕ_{max}
sector 1	28	74.31	46.31	$arcsin(\frac{h}{r})$	$\pi/2;$
sector 2	74.31	129.64	55	$arcsin(\frac{h}{r})$	$arcsin(\frac{h+H}{r})$
sector 3	129.64	184.98	55	$arcsin(\frac{h}{r})$	$arcsin(\frac{h+H}{r})$
sector 4	184.93	240.32	55	$arcsin(\frac{h}{r})$	$arcsin(\frac{h+H}{r})$
sector 5	240.32	249	8.68	$arccos(\frac{L}{r})$	$arcsin(\frac{h+H}{r})$

Where $r = d\tan(\theta)$.

The following formula for the numerical integration over all crystal of each sector, has been used:

$$\sigma_n = \sum_{i=1}^{1000} \frac{d\sigma(\theta_i, E)}{d\Omega} sin\theta_i \left(\phi_{max}(\theta_i, n) - \phi_{min}(\theta_i, n)\right) \Delta\theta_i = \\ = \sum_{i=1}^{1000} \delta\sigma_i(E, \theta_i, n) \tag{8}$$

where

- n is the geometrical sector number $n = 1, .., 5$
- σ_n is the integrated cross section over each sector
- $\frac{d\sigma(\theta_i)}{d\Omega}$ is the differential cross section from equation (6)
- θ_i assumes values from $\theta_{min}(mrad)$ to θ_{max}, in steps of $\Delta\theta_i = \frac{\Delta\theta}{1000}$ (mrad) for each sector, according to Table 14;
- $\phi_{max}(\theta_i, n)$ and $\phi_{min}(\theta_i, n)$, are the maximum and minimum values of the azimuthal angles

Considering the cross section constant (valid at 10% level) we can divide σ_n by the number N of crystal in that sector and obtain the integrated cross section over one crystal acceptance $\Delta\sigma$

E_{beam} (GeV)	$\Delta\sigma_1$ (mb)	$\Delta\sigma_2(\mu b)$	$\Delta\sigma_3$ (μb)	$\Delta\sigma_4$(nb)	$\Delta\sigma_5$ (nb)
1.5	2.91	200	x 20.09	3210	221
2.2	1.29	85.7	5.787	538	63.666
6.6	0.083	0.571	$8.27 \cdot 10^{-3}$	0.205	0.136

Multiplying $\Delta\sigma$ for the luminosity we obtain the rate of events on each crystal

E (GeV)	$\frac{dN}{dt}(s^{-1})$ sector 1	$\frac{dN}{dt}(s^{-1})$ sector 2	$\frac{dN}{dt}(s^{-1})$ sector 3	$\frac{dN}{dt}(s^{-1})$ sector 4	$\frac{dN}{dt}(s^{-1})$ sector 5
1.5	$116 \cdot 10^3$	$7.9 \cdot 10^3$	799	127	9
2.2	$5.2 \cdot 10^4$	$3.1 \cdot 10^3$	$2.3 \cdot 10^2$	21.6	2.5
6.6	$6.9 \cdot 10^3$	50	0.73	$1.9 \cdot 10^{-2}$	1.2×10^{-2}

- quite a high rate is available at all energies in the first sector, while in the second sector it drops of about one order of magnitude
- Another reduction of at least one order of magnitude is observed, increasing the beam energy for a fixed sector
- An overall variation of 5 orders of magnitude is observed at 6.6 GeV

The use of elastic scattering events seems not possible for the calibration of the crystals in the three more external sectors for a beam energy of $E = 6.6$ GeV.

Summary and Conclusions

- At low beam energies, the electrons are elastically scattered practically at the same energy, regardless of the scattering angle, within the ECal acceptance.
- At 6.6 GeV, the energy differences become of the same order of the requested energy resolution, therefore the angular position of the absorbed scattered electron energy must be taken into account when defining a the calibration plan.
- At 6.6 GeV the effect of the electric form factor becomes relevant, lowering by several orders of magnitude the cross section at larger angles.
- The use of the elastic scattered electrons events seems possible at lower beam energies $E = 1.5$ GeV and $E = 2.2$ GeV, while events rate critically drops at $E = 6.6$ GeV for the outer sectors of ECal.