

$$\frac{\sigma}{E} = a \oplus \frac{b}{\sqrt{E}} \oplus \frac{c}{E}$$

or $\sigma^2 = a^2 E^2 + b^2 E + c^2$

Consider $E_1 = \alpha E$ and $E_2 = (1-\alpha)E$ so that $E_1 + E_2 = E$

$$\sigma_1^2 = a^2 \alpha^2 E^2 + b^2 \alpha E + c^2$$

$$\sigma_2^2 = a^2 (1-\alpha)^2 E^2 + b^2 (1-\alpha) E + c^2$$

$$\sigma_1^2 + \sigma_2^2 = a^2 E^2 [\alpha^2 + (1-\alpha)^2] + b^2 E + 2c^2$$

$$\Rightarrow \boxed{\sigma_1^2 + \sigma_2^2 = \sigma^2 - 2\alpha(1-\alpha)a^2 E^2 + c^2}$$

• If a is large, one can have situations where

$$\sigma_1^2 + \sigma_2^2 < \sigma^2$$

This would be the case in particular if there are significant intercalibration errors between crystals.
(relative)

• Note that if one considers only the statistical term ($a=c=0$)

$$\sigma_1^2 + \sigma_2^2 = \sigma^2$$