

hadron spectroscopy and QCD

Jozef Dudek



OLD DOMINION
UNIVERSITY

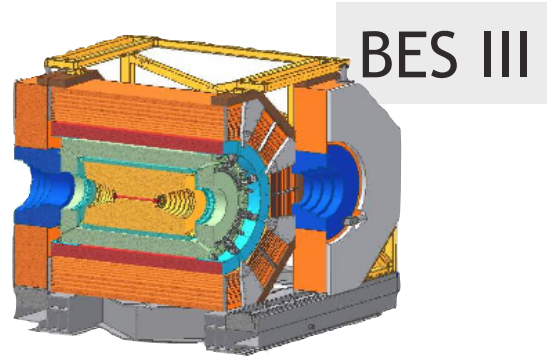


hadron spectrum
collaboration

current & future experiments

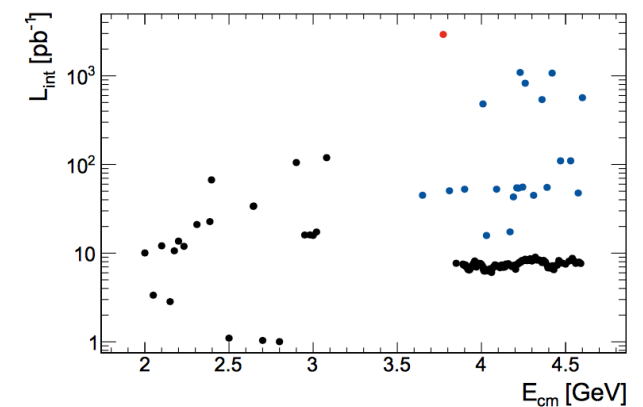
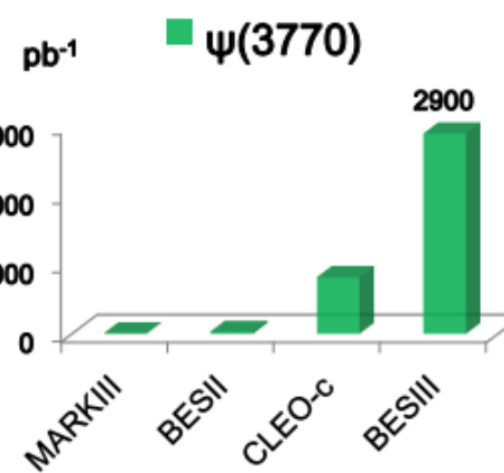
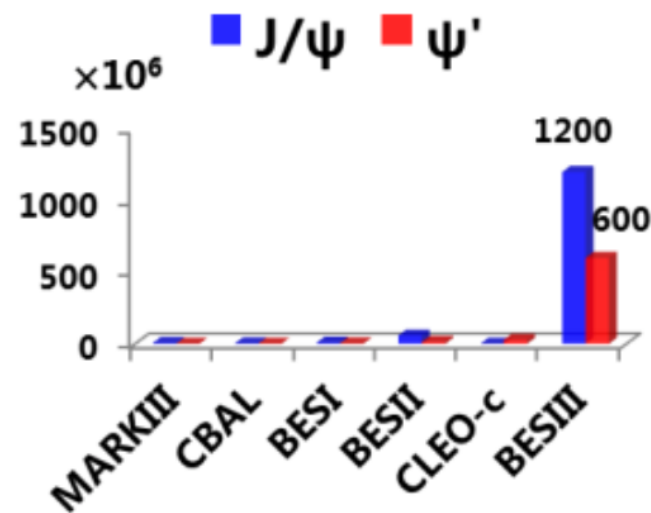
high statistics
novel production

2

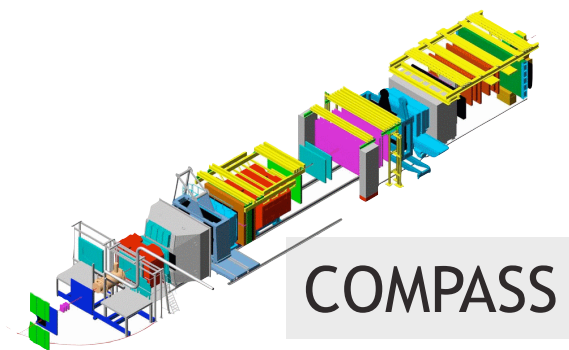


BES III

e^+e^-

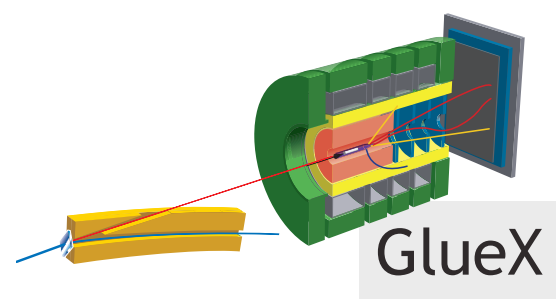
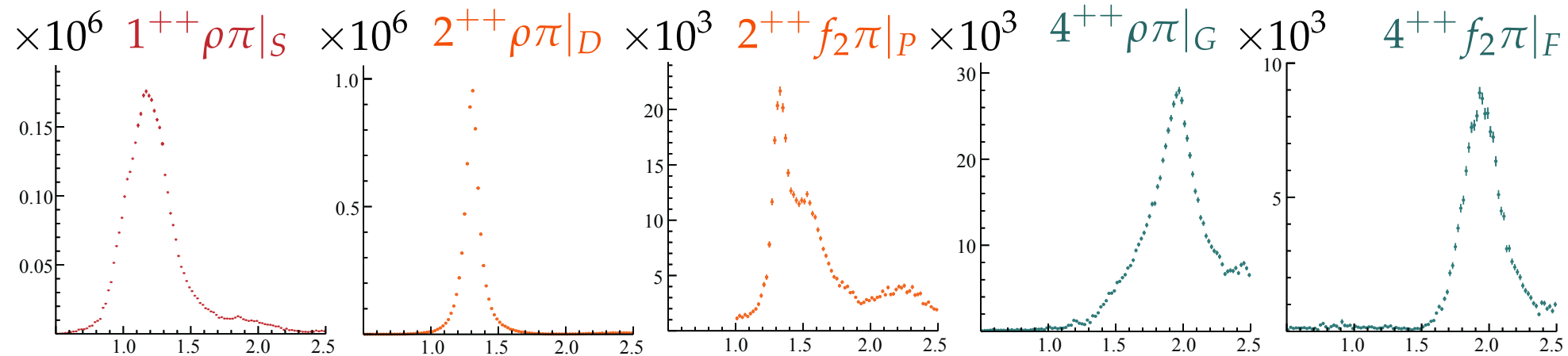


W.Gradl



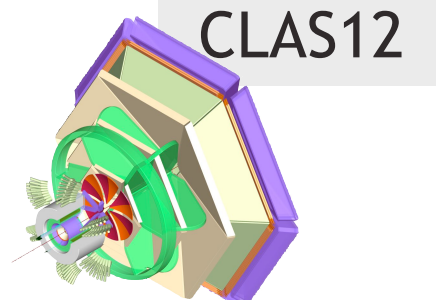
COMPASS

πN



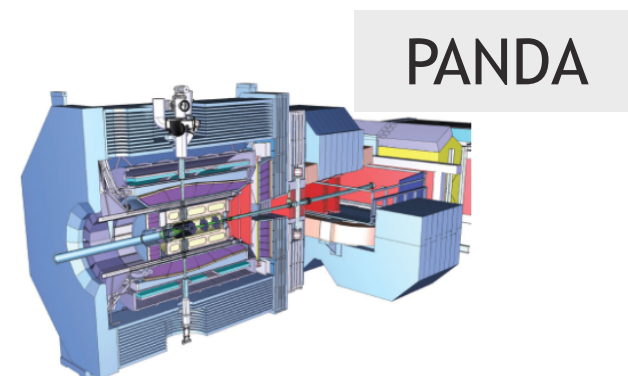
GlueX

γN



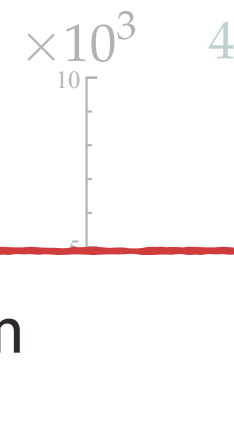
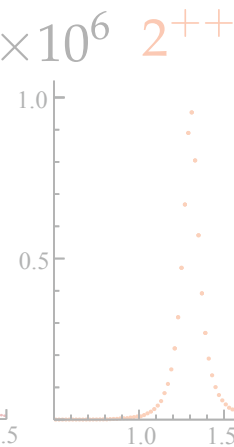
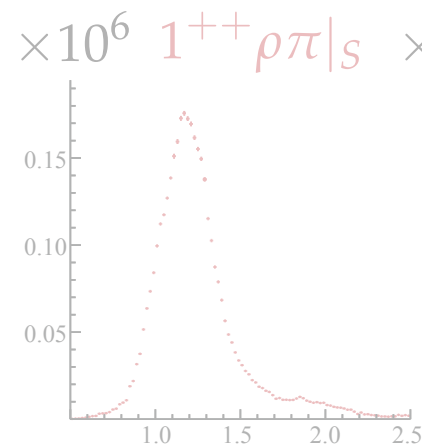
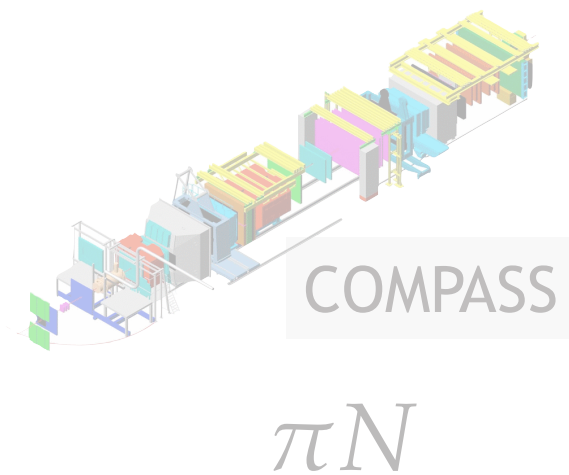
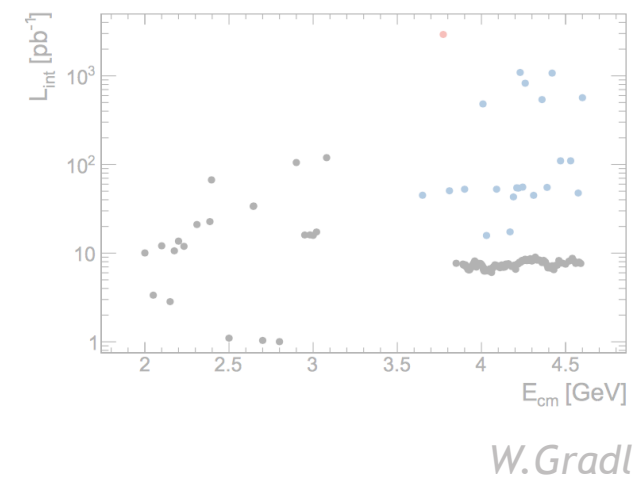
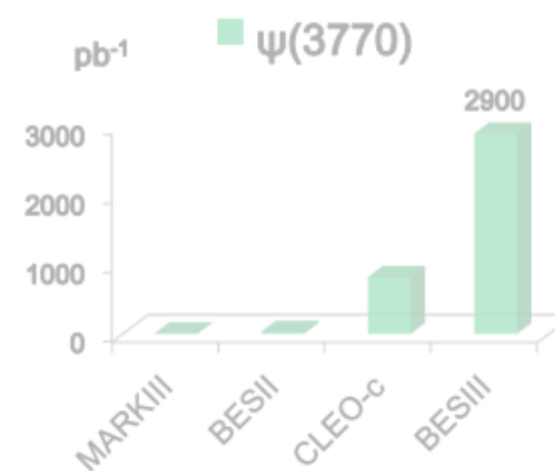
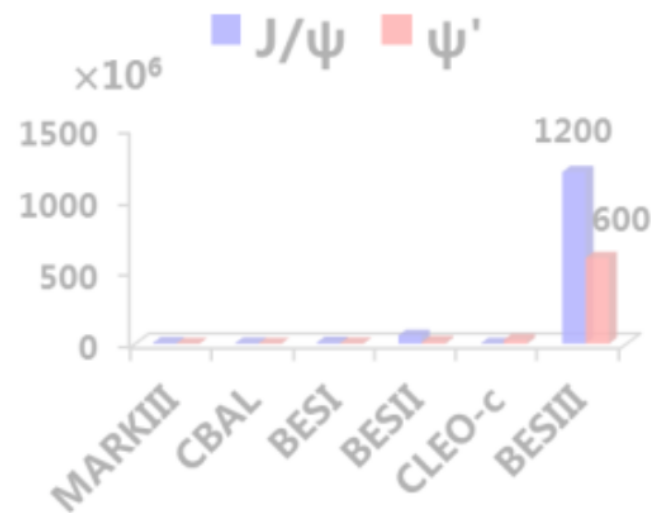
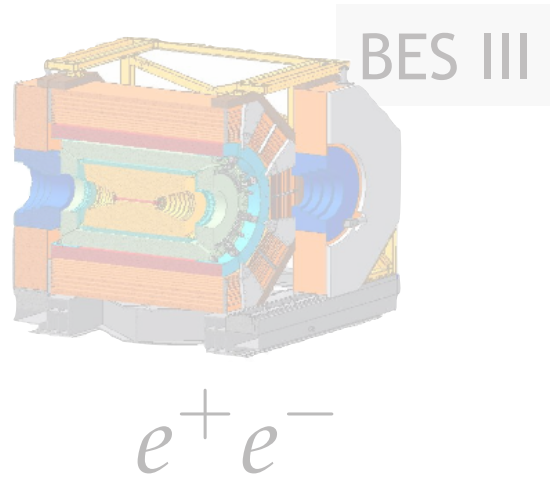
CLAS12

$e^- N$



PANDA

$\bar{p}p$



vibrant experimental program

recognition of the need for
advanced analysis tools

→ JPAC & other efforts

... but what about the connection to QCD ?

lattice QCD

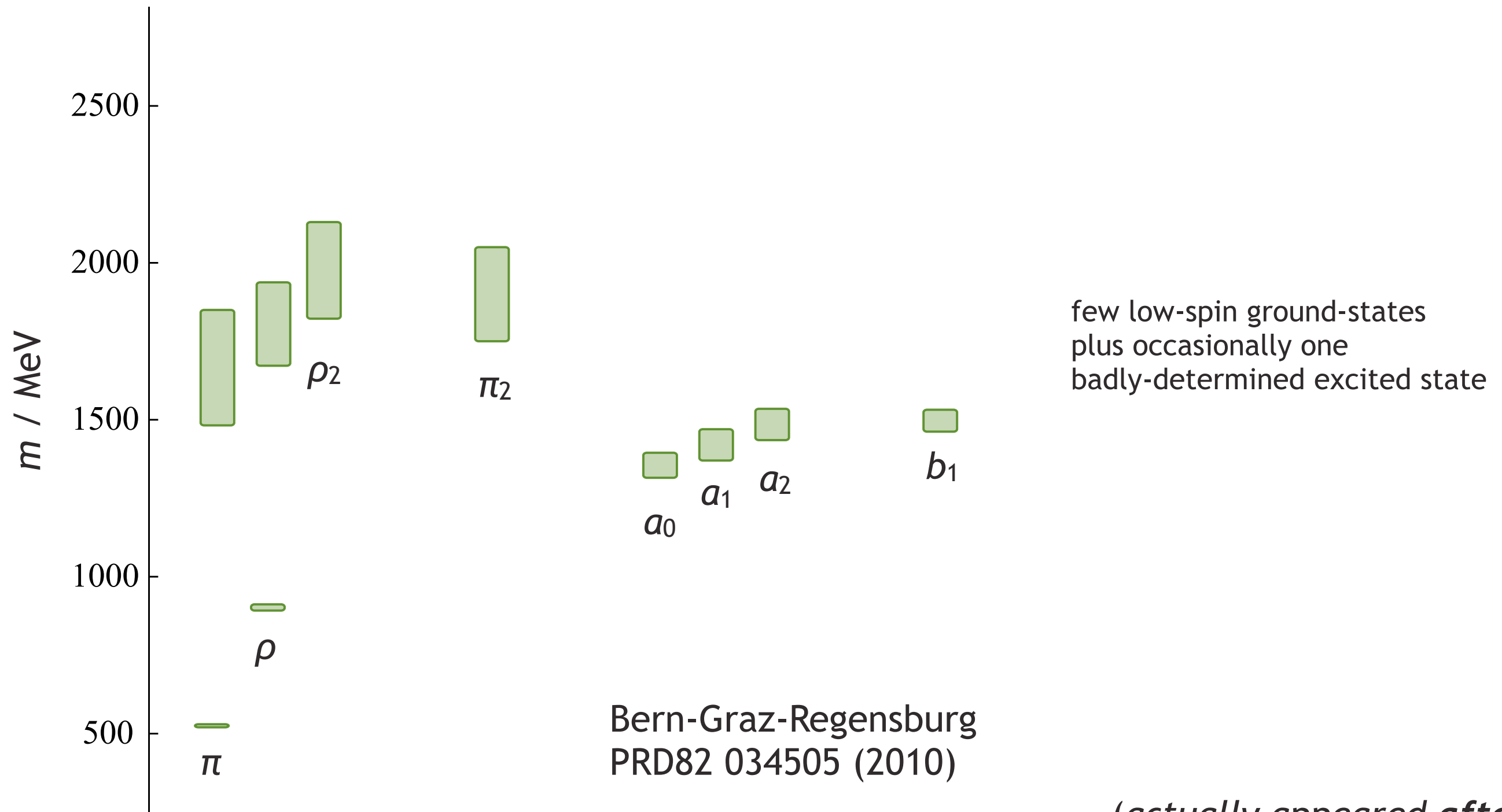
& the Hadron Spectrum Collaboration

in its current form since 2009

where the field was ... meson spectrum

4

state-of-the-art determinations of the excited meson spectrum in lattice QCD

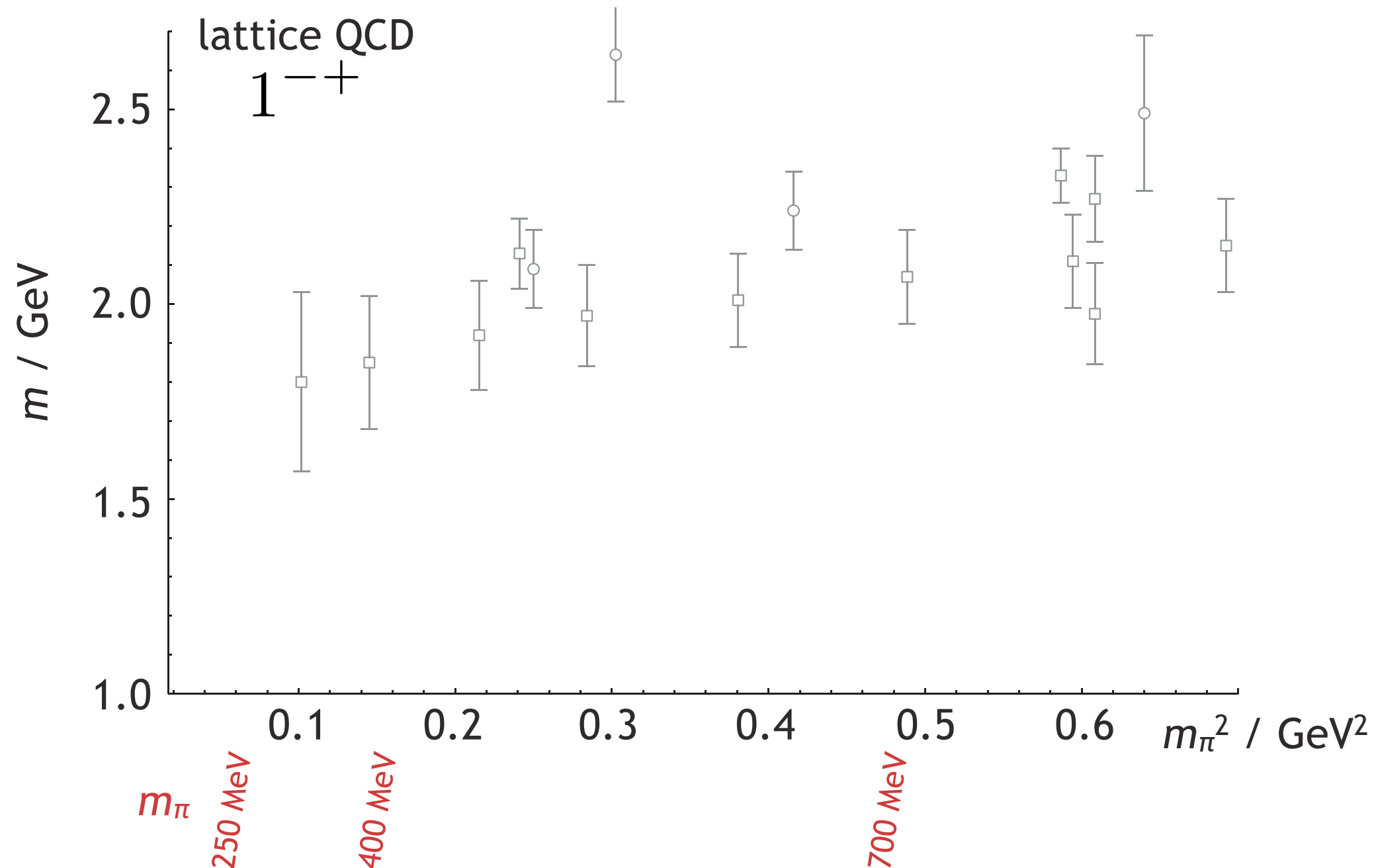


(actually appeared *after*
first HSC results)

where the field was ... hybrid mesons

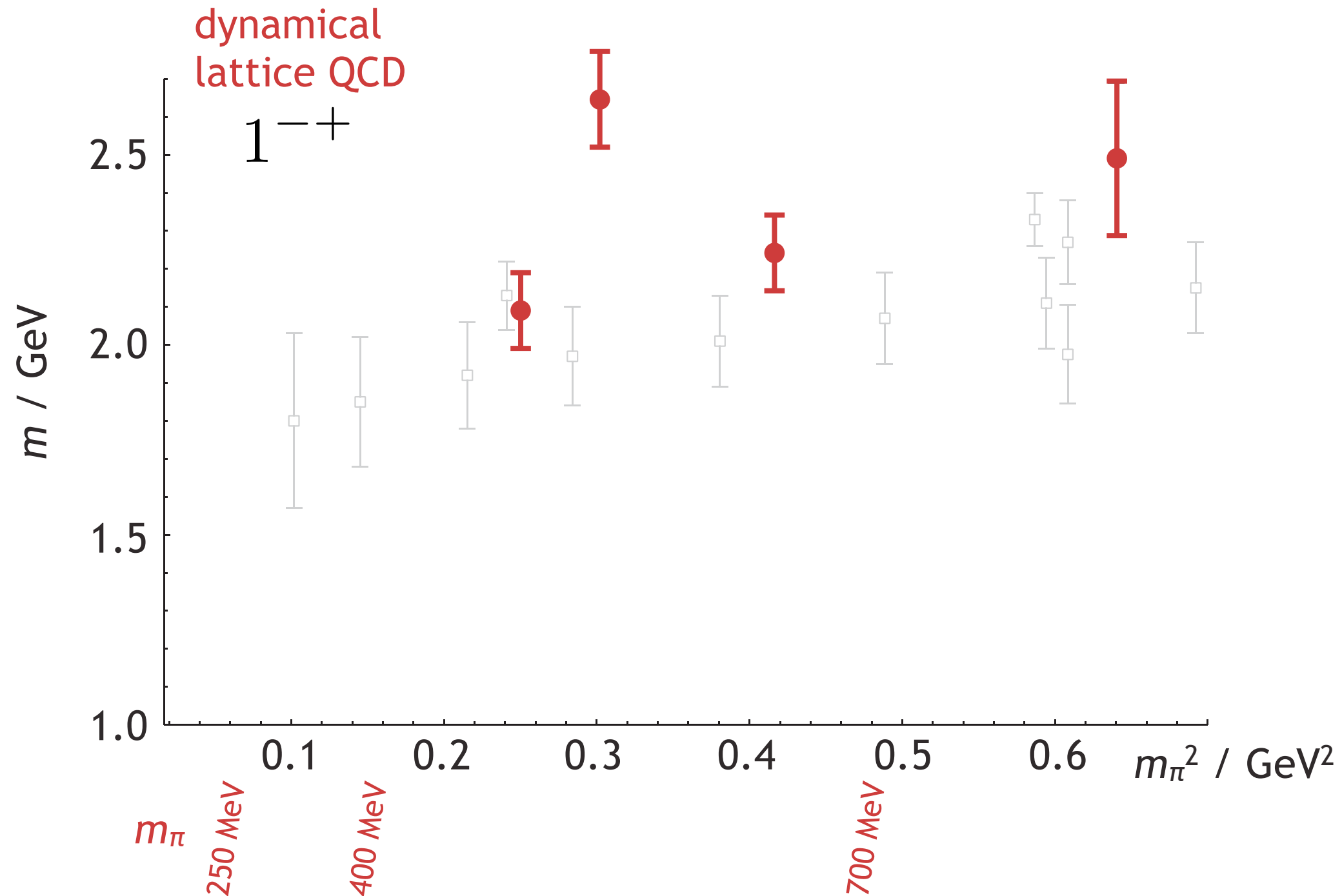
5

in 2009 flux-tube and bag models (1980s) were still state-of-the-art



where the field was ... hybrid mesons

6



distillation

efficiently evaluate a large number of correlation functions
compute quark annihilation where needed

large basis of hadron operators

began with meson operator basis $\bar{\psi}\Gamma\overleftrightarrow{D}\dots\overleftrightarrow{D}\psi$ (up to three derivatives)

‘subduced’ into the irreps of the cubic symmetry *found a workaround for the breakdown of rotational symmetry*

variational solution

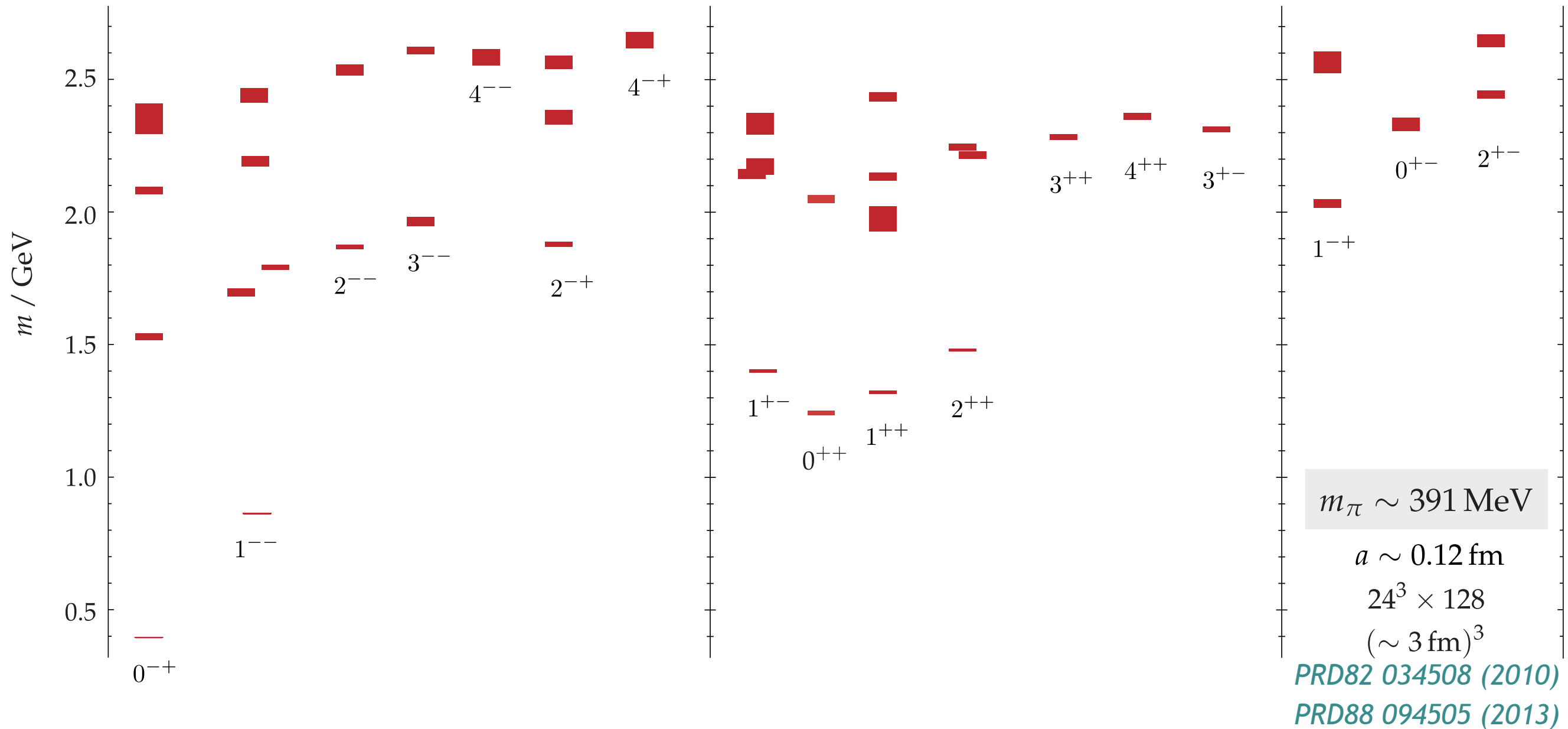
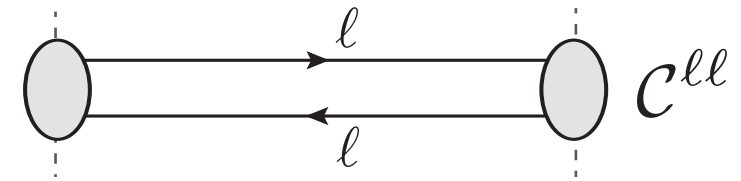
‘diagonalize’ a matrix of correlation functions $C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$

extract many excited states

$$C(t)v^n = \lambda^n(t) C(t_0)v^n$$

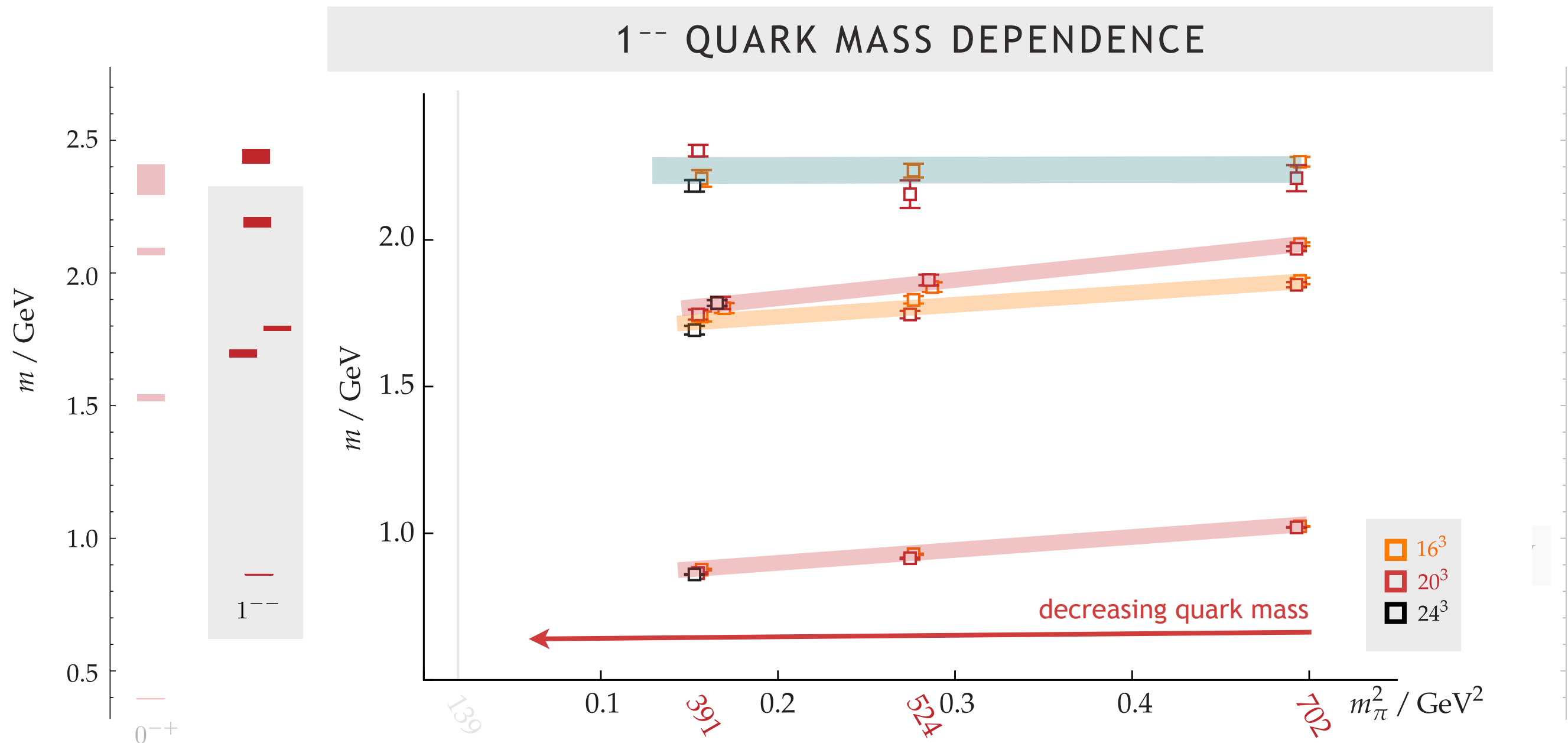
excited isovector meson spectrum

8



excited isovector meson spectrum

9



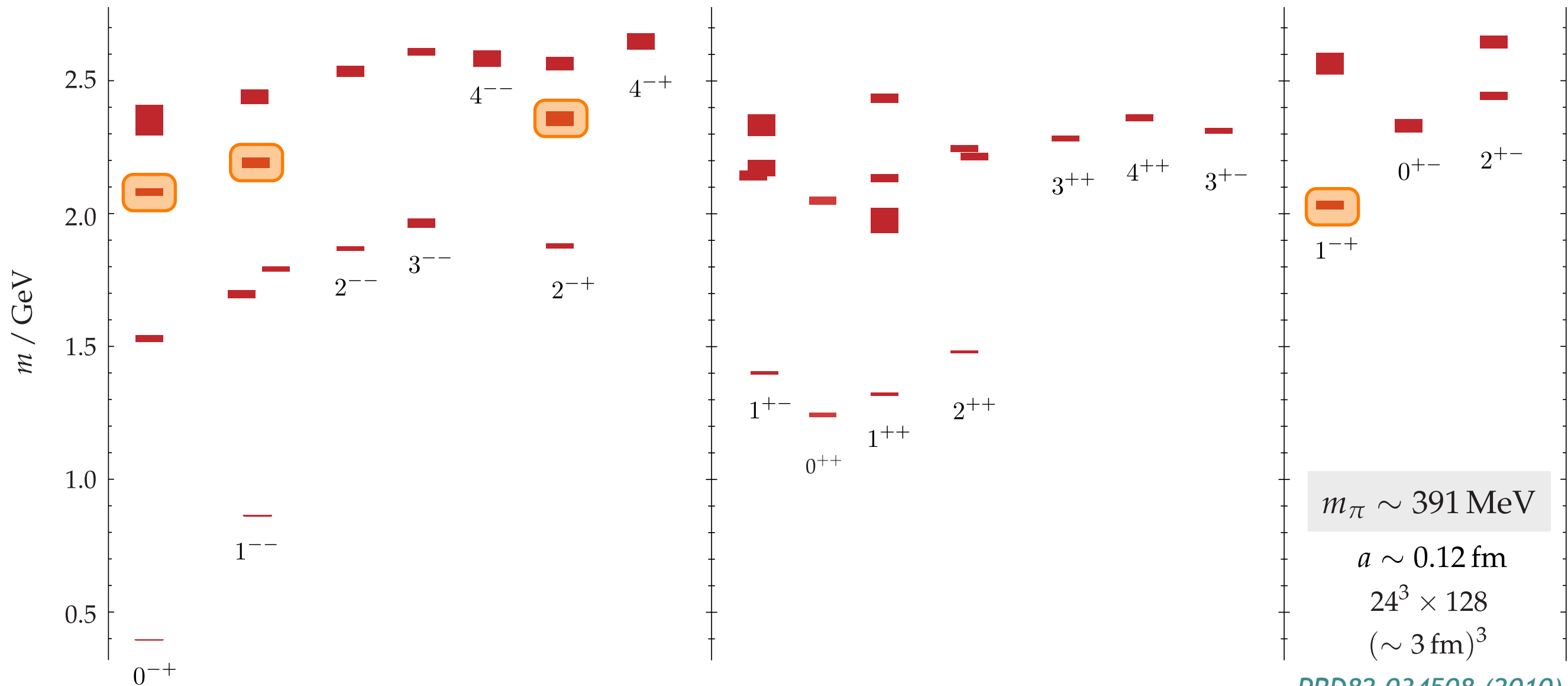
- spectrum does not change qualitatively

isovector hybrid mesons

10

- ‘super’-multiplet of **hybrid mesons** roughly 1.3 GeV above the ρ

$$(0, 1, 2)^{-+}, 1^{--}$$

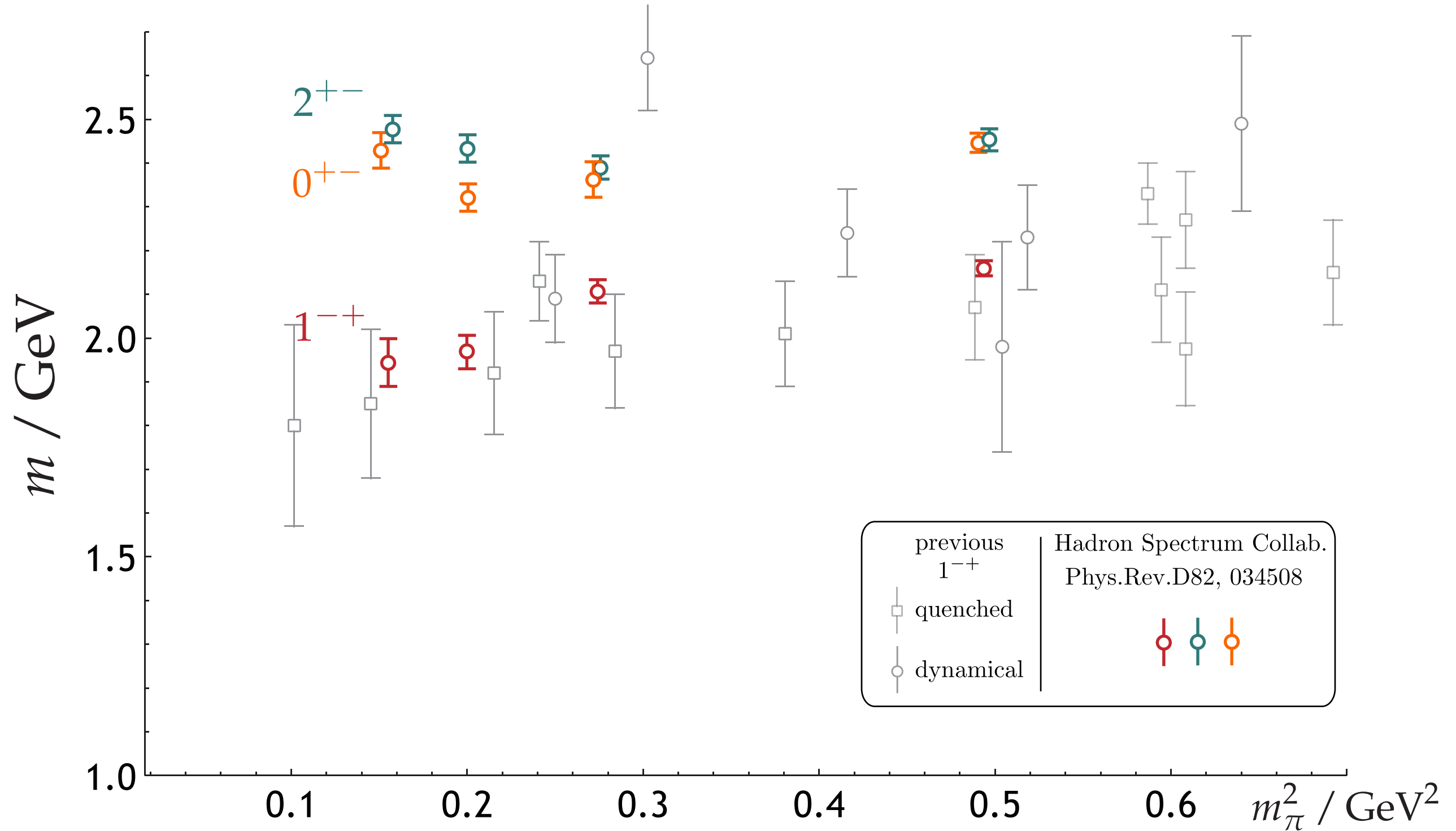


PRD82 034508 (2010)

PRD88 094595 (2013)

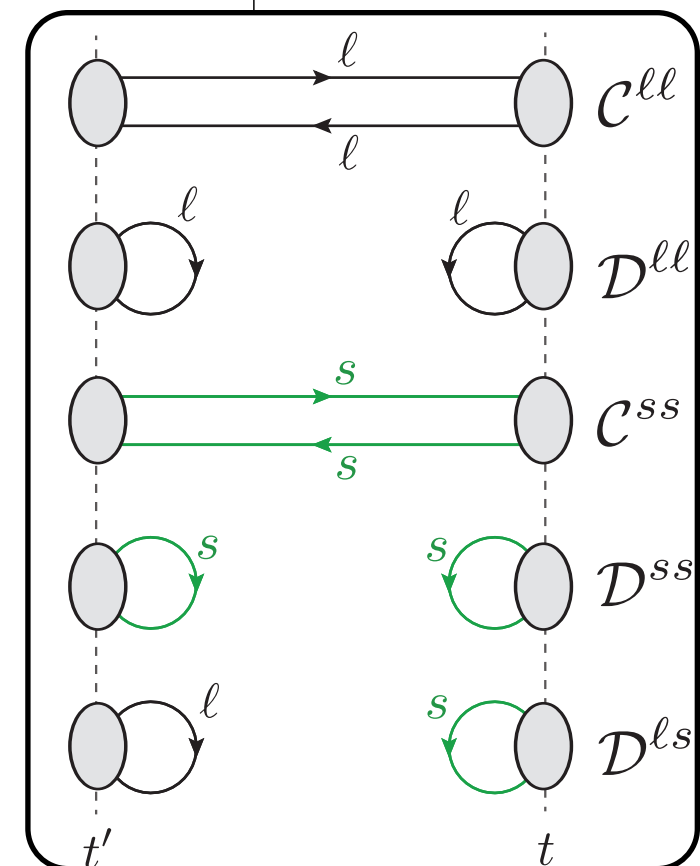
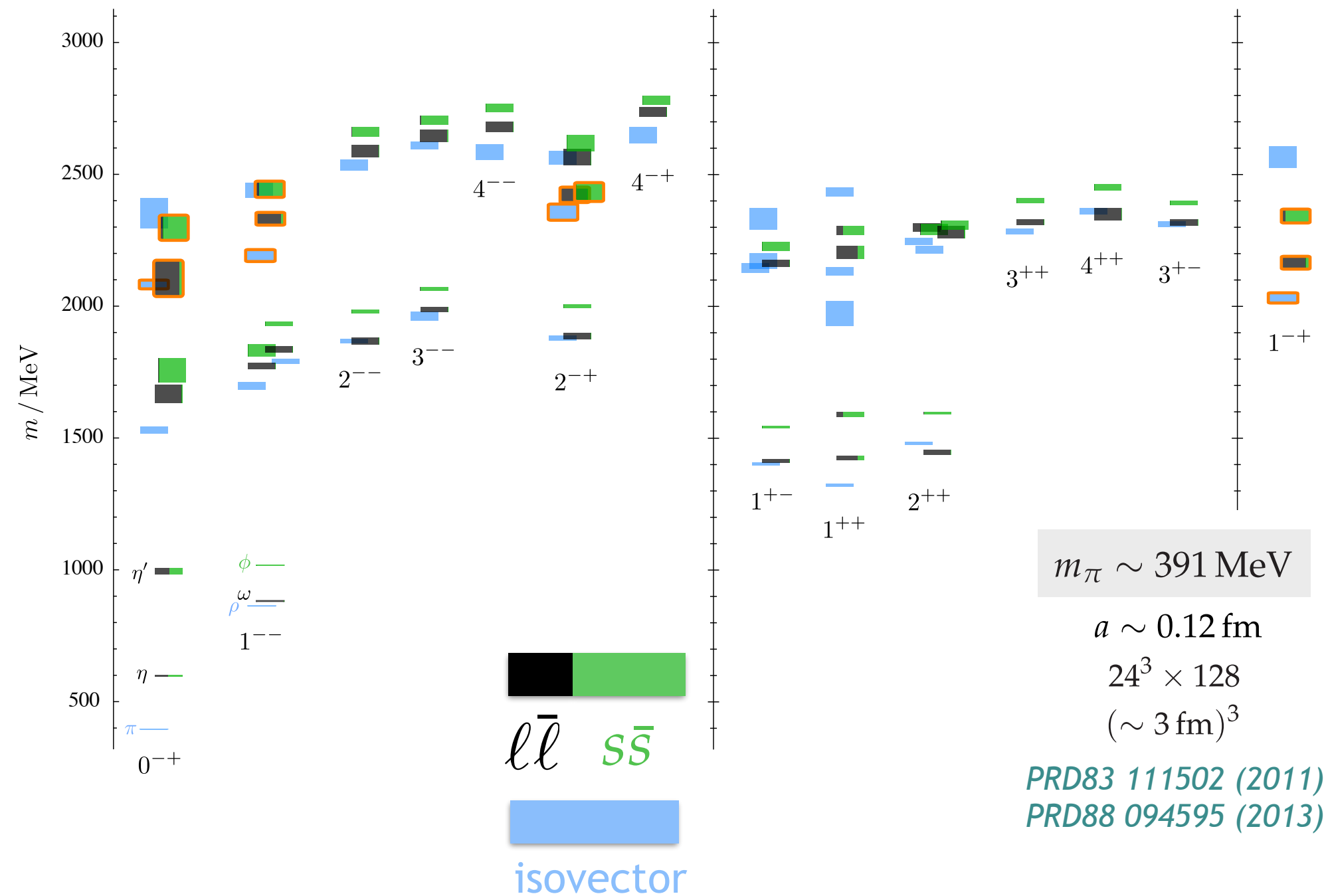
utilized overlaps with characteristic operators to identify state make-up

- these states have a dominant overlap onto $\bar{\psi}\Gamma[D, D]\psi \sim [q\bar{q}]_{8_c} \otimes B_{8_c}$



isoscalar meson spectrum

12

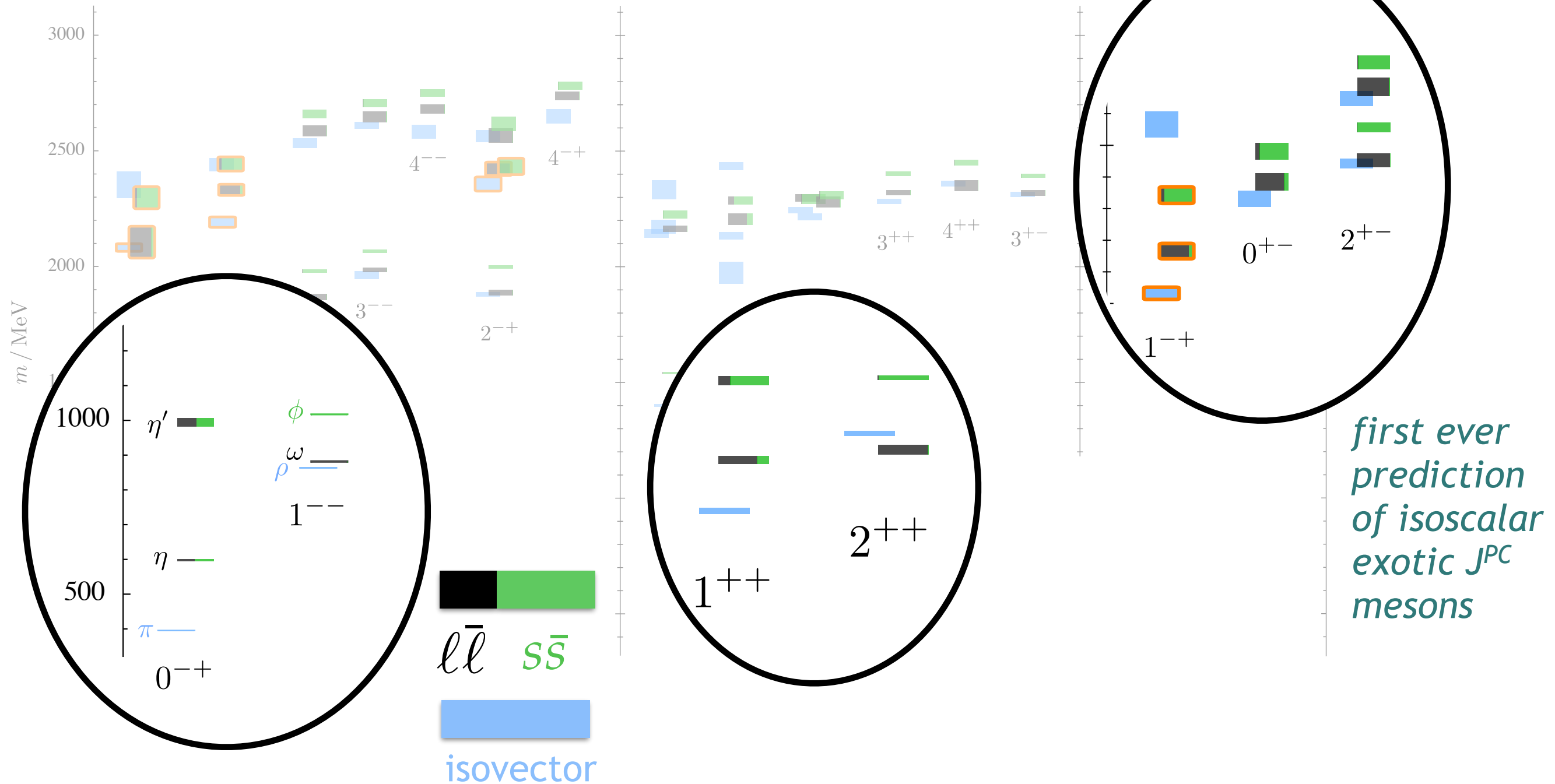


evaluated all the required annihilation diagrams
 determined light-strange mixing through operator overlaps

isoscalar meson spectrum

13

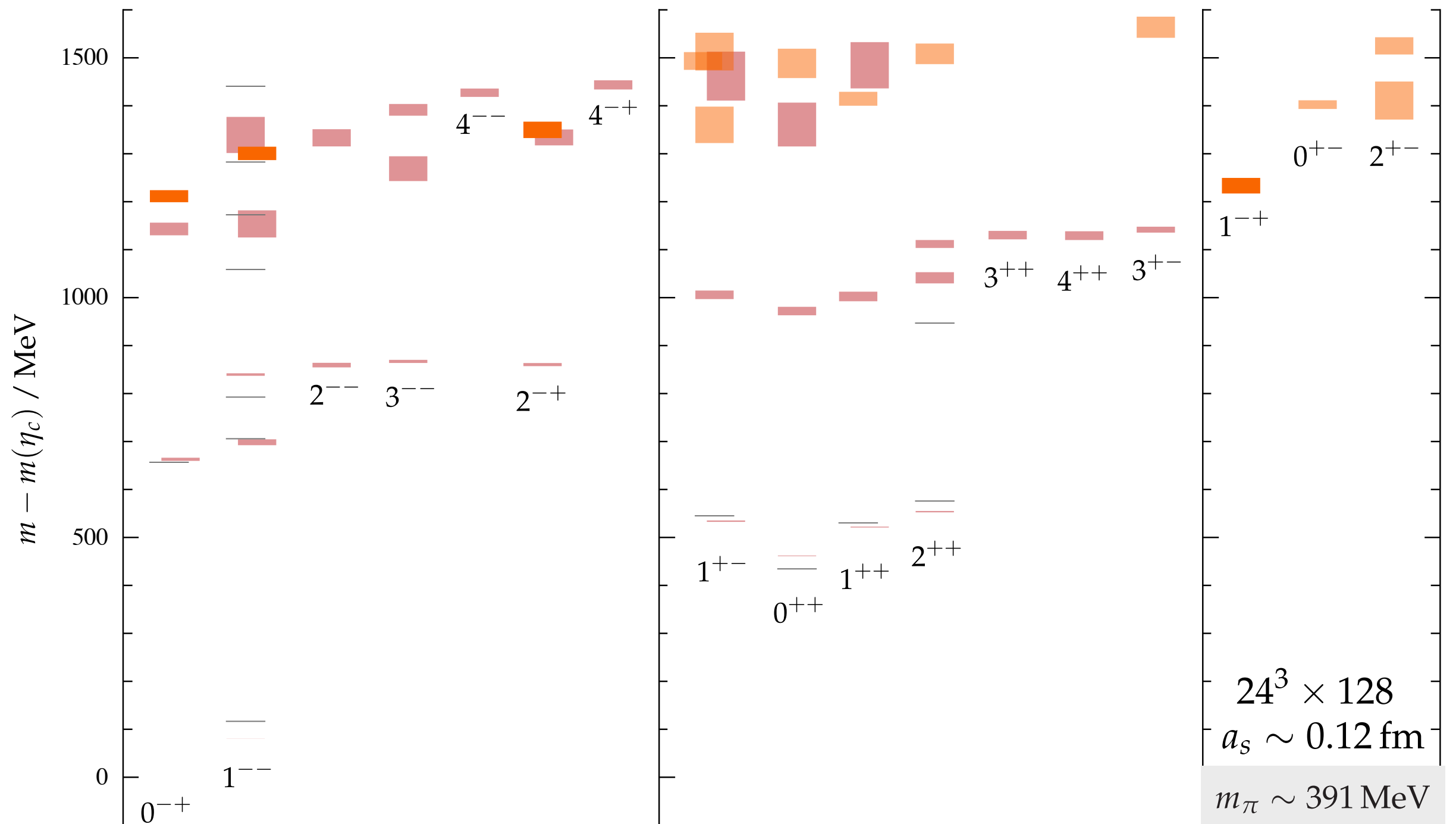
- phenomenology in qualitative agreement with experiment



rest of the lattice community still struggling with η, η' alone

- two ‘super’-multiplets of **hybrid mesons**

this work lead by our
Dublin collaborators

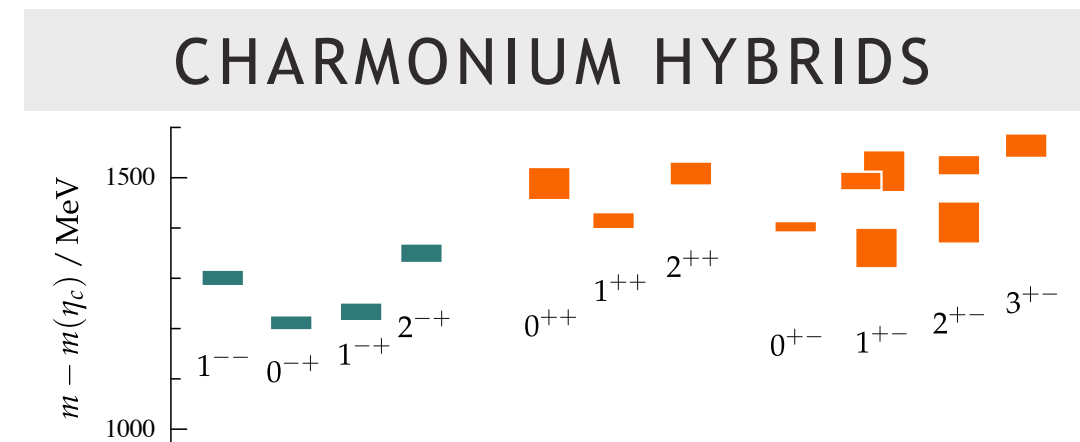


JHEP 1207 126 (2012)

- lightest set of hybrid mesons appear to contain a 1^{+-} gluonic excitation

quarks in
an S -wave

$$\left[q\bar{q}_{8_c} \left[{}^1S_0 \right] G_{8_c}^* [B] \right]_{1_c} \rightarrow 1_{\text{hyb.}}^{--}$$
$$\left[q\bar{q}_{8_c} \left[{}^3S_1 \right] G_{8_c}^* [B] \right]_{1_c} \rightarrow (0, 1, 2)_{\text{hyb.}}^{--+}$$



- some models have similar systematics
 - bag model also has 1^{+-} lowest in energy
 - 1^{+-} in a Coulomb-gauge approach

- lightest set of hybrid mesons appear to contain a 1^{+-} gluonic excitation

quarks in
an S -wave

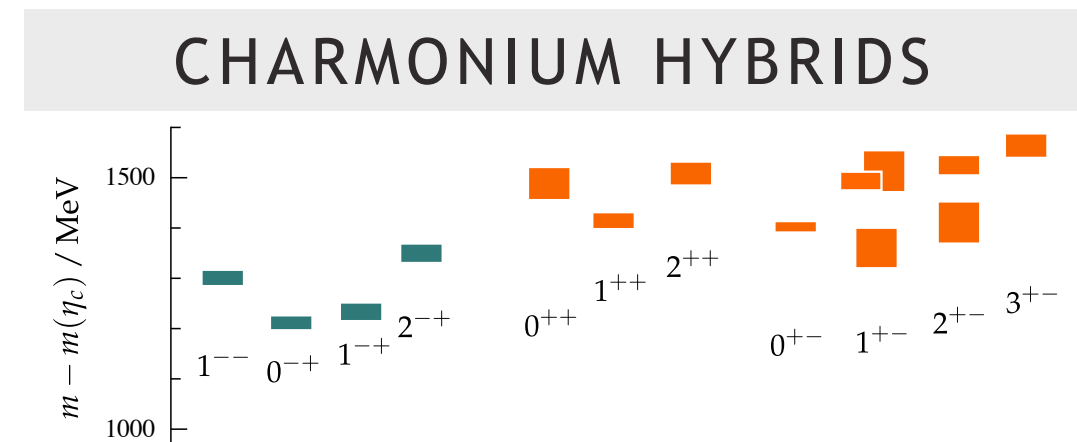
$$\left[q\bar{q}_{8_c} \left[{}^1S_0 \right] G_{8_c}^* [B] \right]_{1_c} \rightarrow 1_{\text{hyb.}}^{--}$$

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quarks in
a P -wave

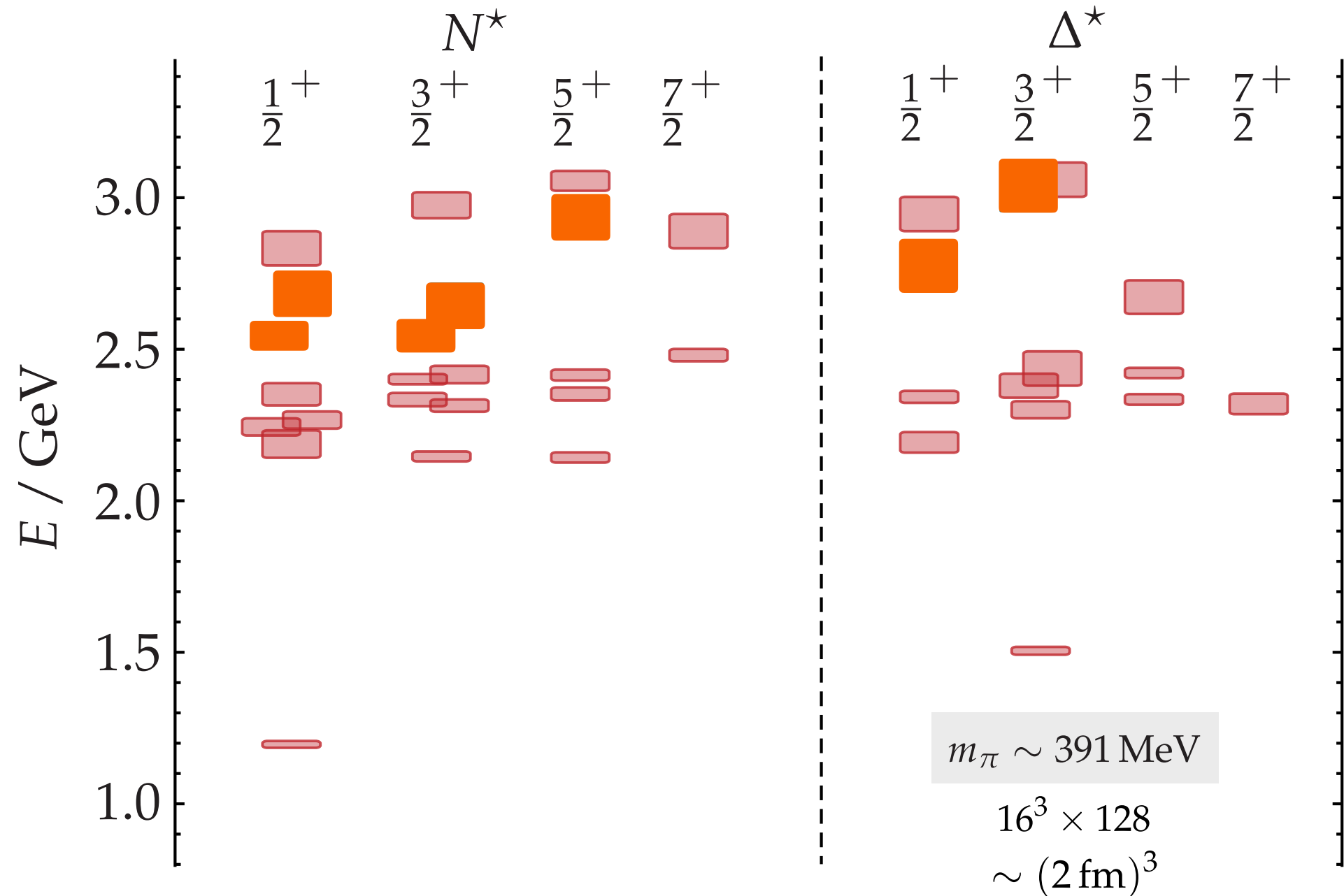
$$\left[q\bar{q}_{8_c} \left[{}^1P_1 \right] G_{8_c}^* [B] \right]_{1_c} \rightarrow (0, 1, 2)_{\text{hyb.}}^{++}$$

$$\left[q\bar{q}_{8_c} \left[{}^3P_{0,1,2} \right] G_{8_c}^* [B] \right]_{1_c} \rightarrow (0, 1^3, 2^2, 3)_{\text{hyb.}}^{+-}$$



- some models have similar systematics
 - bag model also has 1^{+-} lowest in energy
 - 1^{+-} in a Coulomb-gauge approach

- a ‘super’-multiplet of **hybrid baryons**



spectrum from large basis of baryon operators

$$\epsilon_{abc} \left(D^{n_1} \frac{1}{2} (1 \pm \gamma_0) \psi \right)^a \left(D^{n_2} \frac{1}{2} (1 \pm \gamma_0) \psi \right)^b \left(D^{n_3} \frac{1}{2} (1 \pm \gamma_0) \psi \right)^c$$

PRD84 074508 (2011)

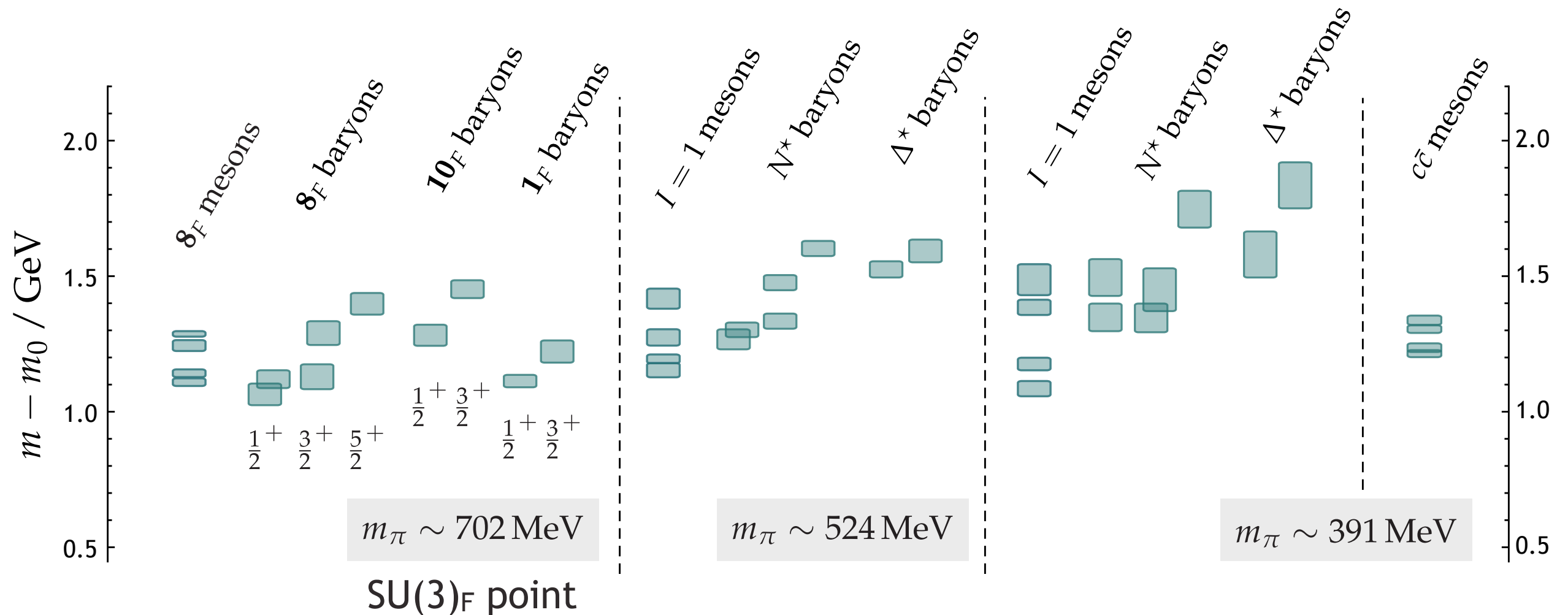
PRD85 054016 (2012)

- subtract the ‘quark mass’ contribution

$$m_0^{\text{mes}} = m_\rho$$

$$m_0^{\text{bar}} = m_N$$

$$m_0^{c\bar{c}} = m_{\eta_c}$$

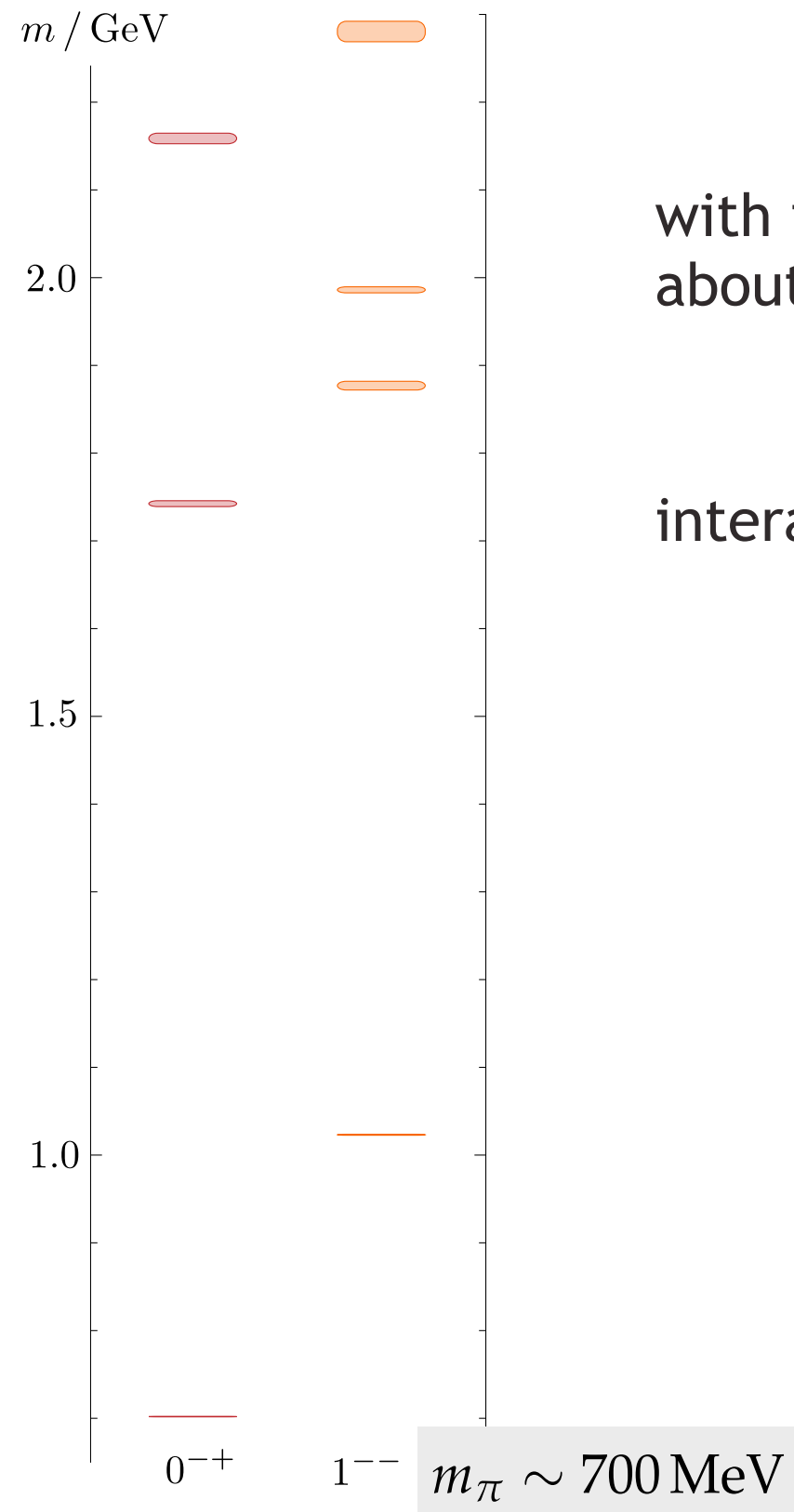


common energy scale of chromomagnetic gluonic excitation $\sim 1.3 \text{ GeV}$

lowest gluonic excitation in QCD now determined ?

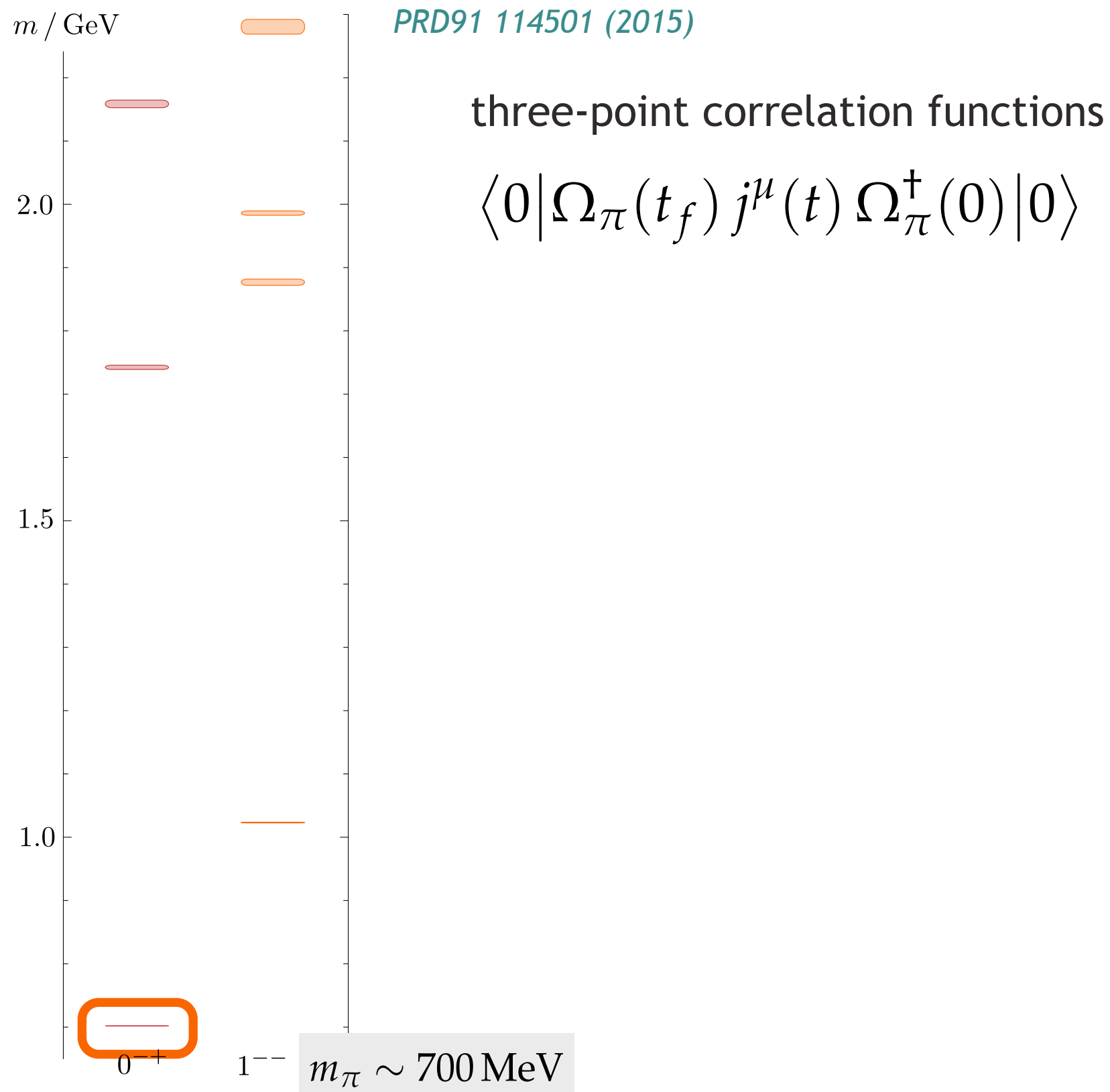
what other properties ?

18

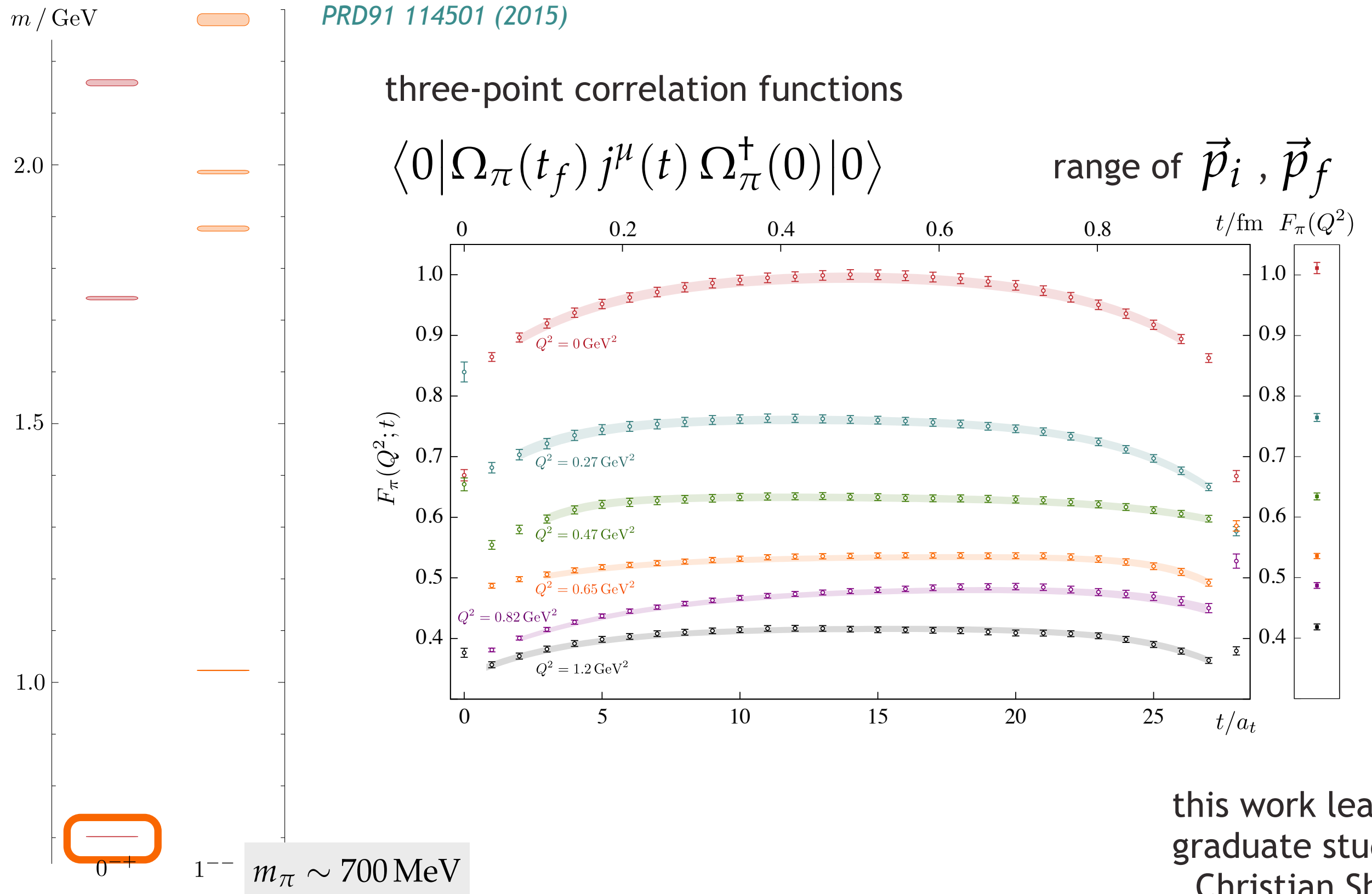


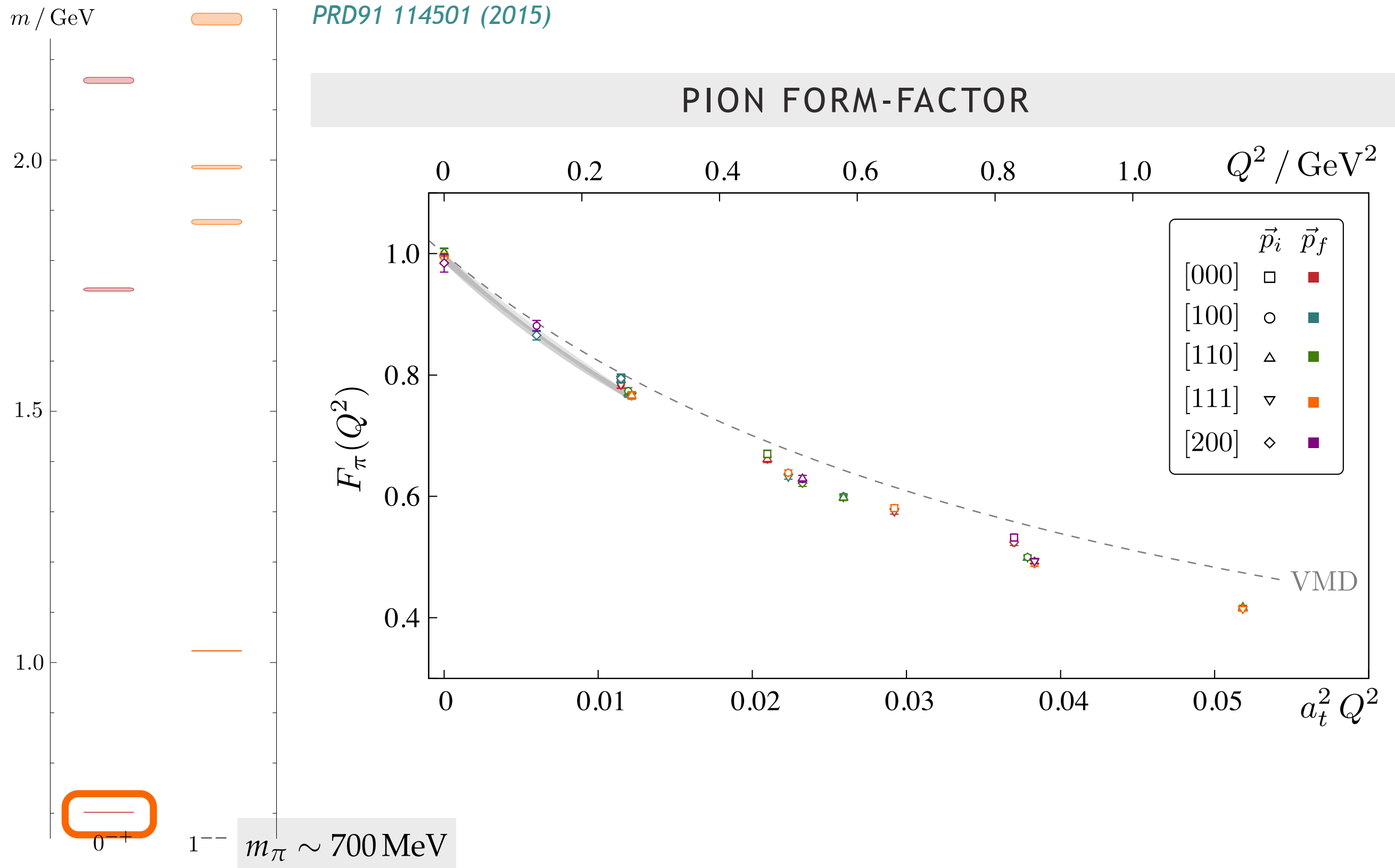
with this technology, can you obtain any other info about excited state mesons ?

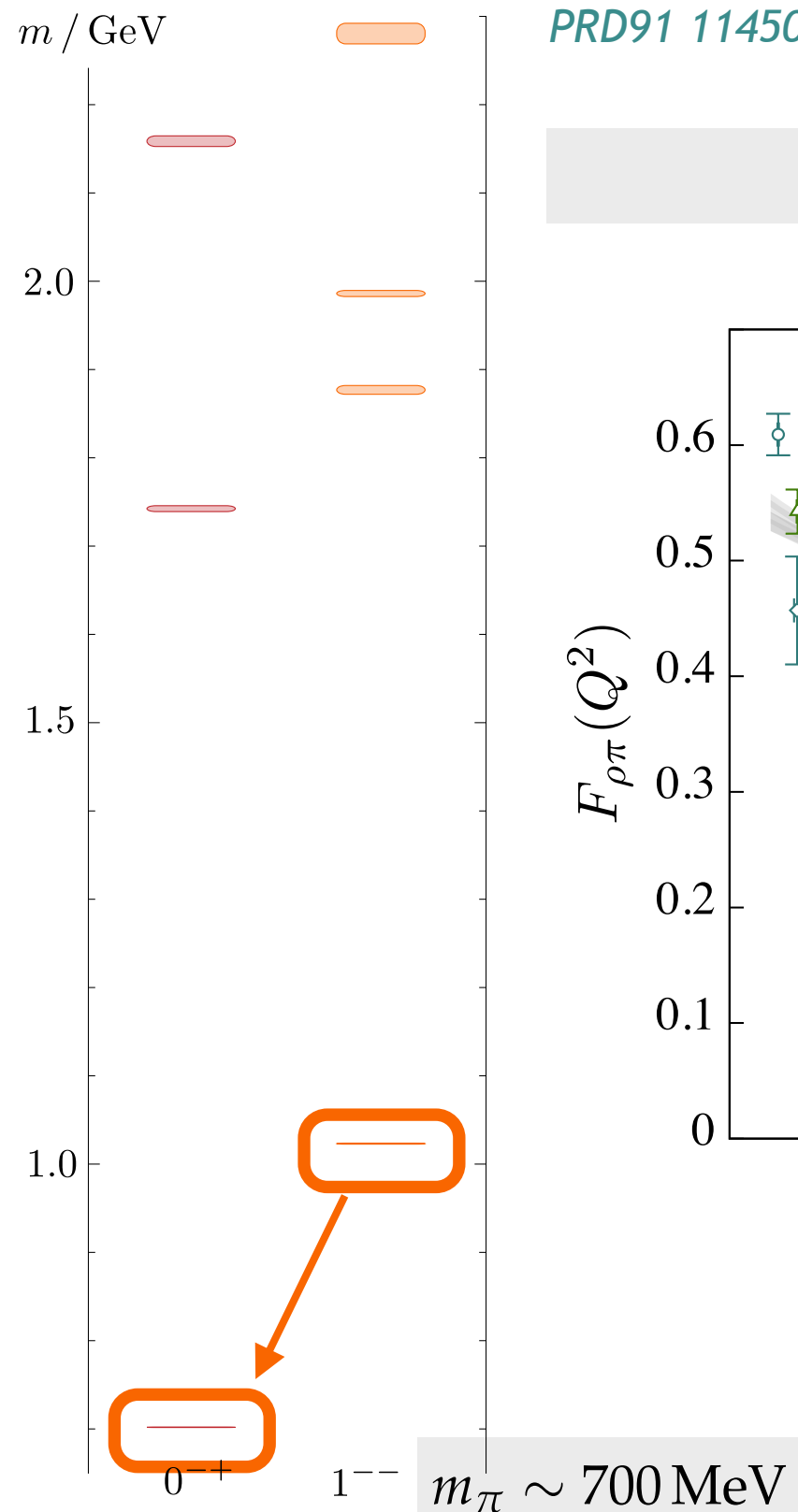
interactions with external currents ?



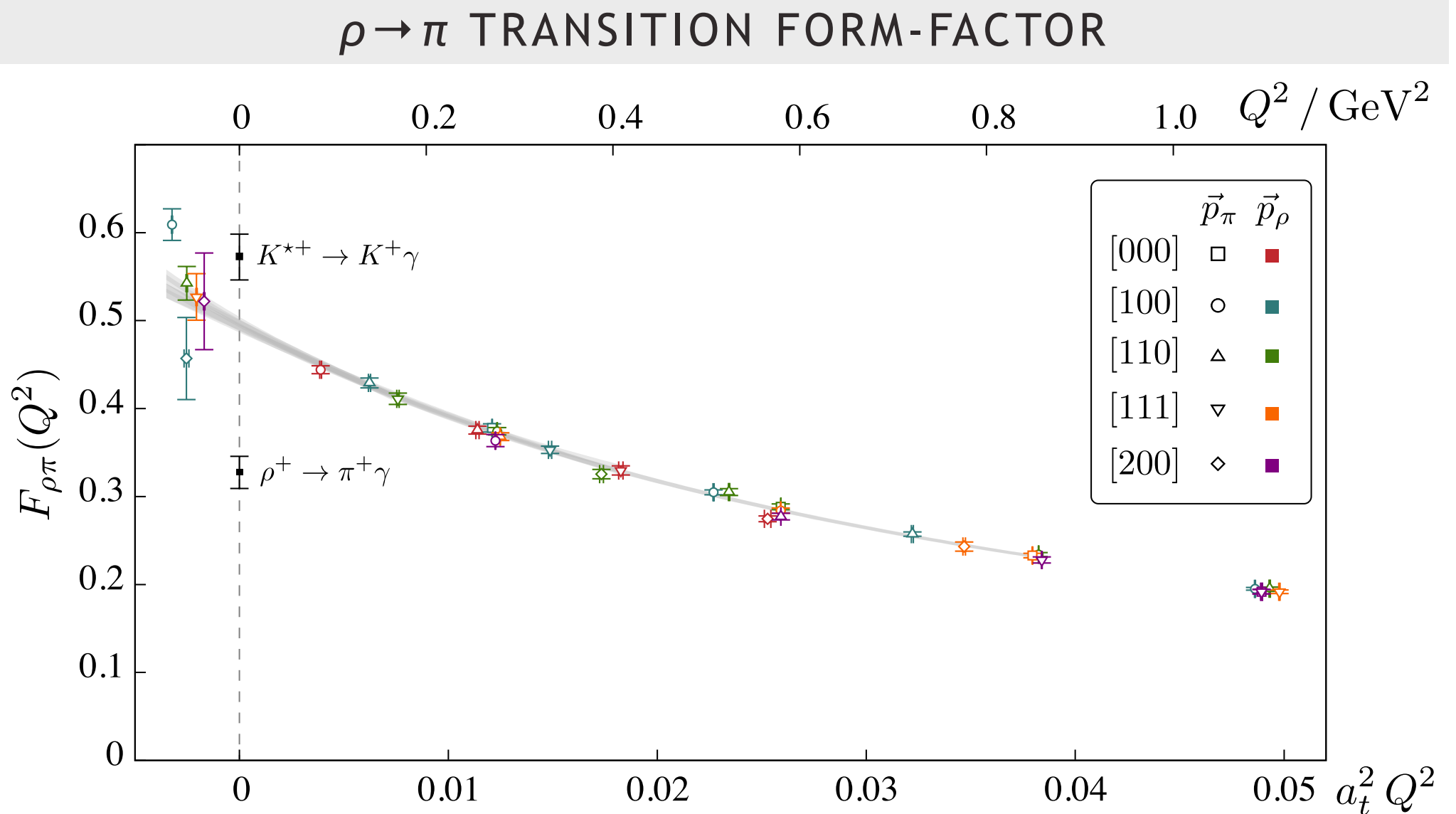
this work lead by
graduate student
Christian Shultz

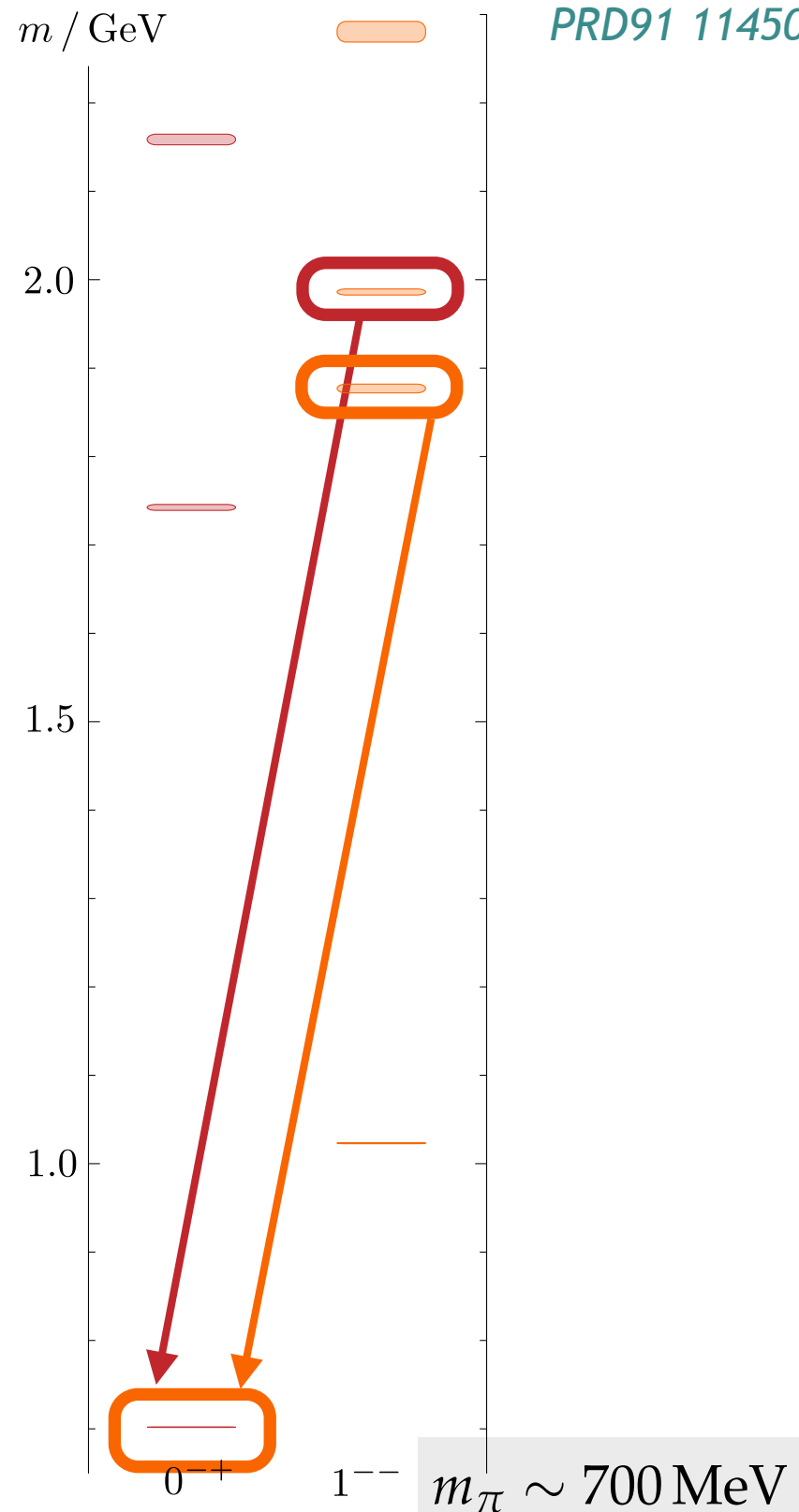




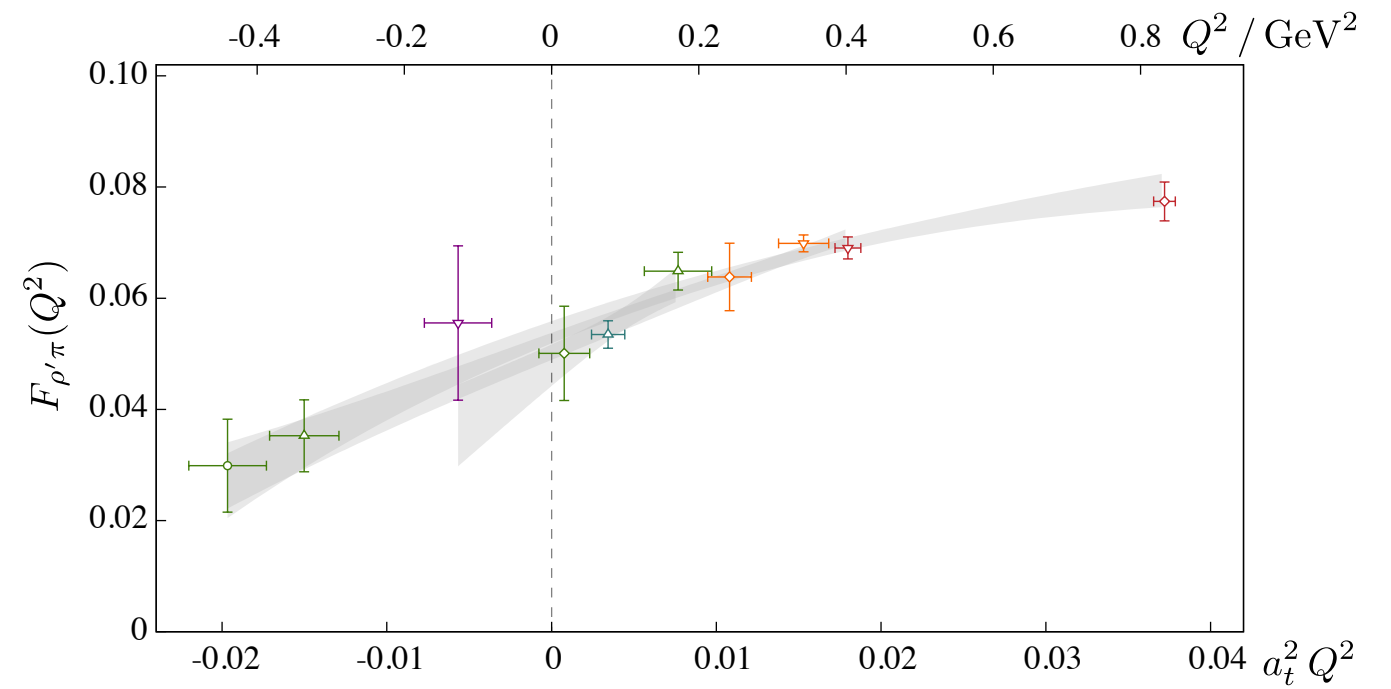


PRD91 114501 (2015)

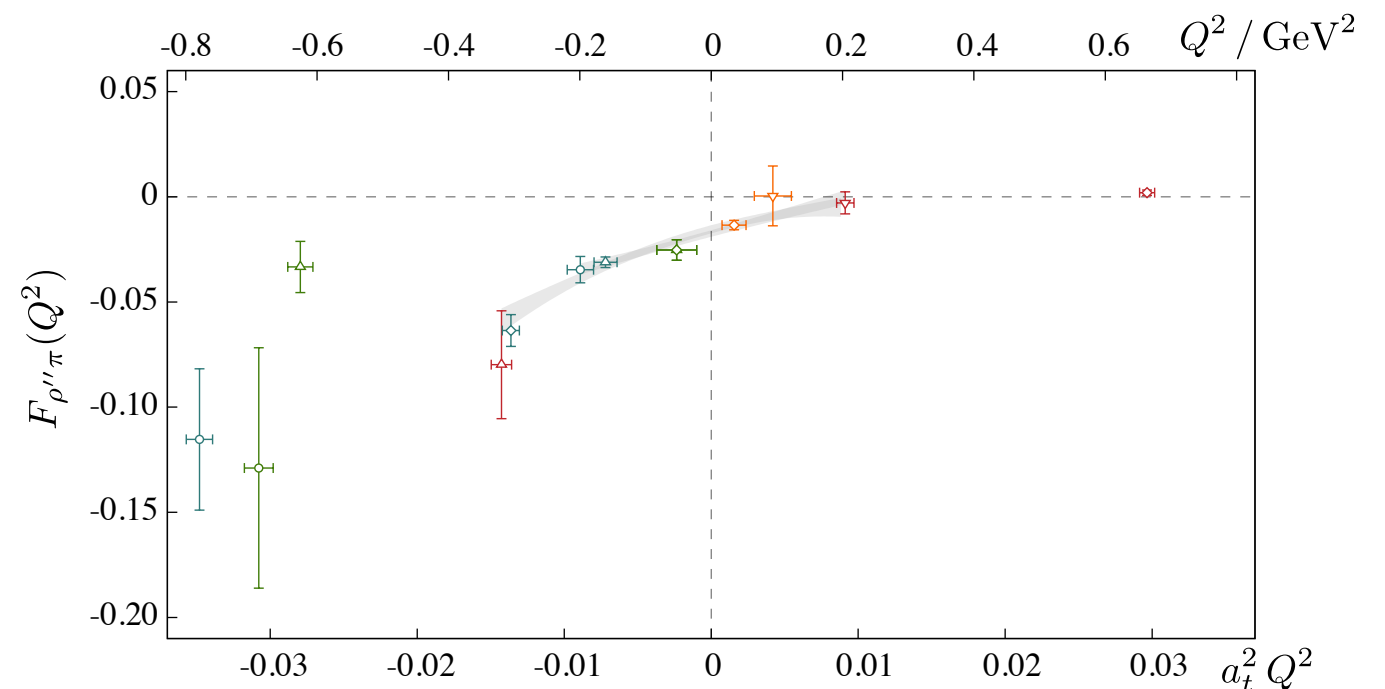




$\rho' \rightarrow \pi$ TRANSITION FORM-FACTOR



$\rho'' \rightarrow \pi$ TRANSITION FORM-FACTOR



using operators $\bar{\psi}\Gamma\overleftrightarrow{D}\dots\overleftrightarrow{D}\psi$ and $\epsilon_{abc}\left(D^{n_1}\frac{1}{2}(1\pm\gamma_0)\psi\right)^a\left(D^{n_2}\frac{1}{2}(1\pm\gamma_0)\psi\right)^b\left(D^{n_3}\frac{1}{2}(1\pm\gamma_0)\psi\right)^c$

- obtained unprecedented, and mainly still unmatched, detailed excited state spectra
- isolated the spectrum of hybrid mesons and baryons, both exotic and non-exotic
- inferred the probable nature of the lightest gluonic excitation (2009 - 2013)
- demonstrated the possibility of extracting current transitions between excited states

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(2009 - 2013)

but this comes close to exhausting what can usefully be learned from using just these operators

we learn a lot from these results, but need to move to methods that can reproduce the true physics of excited states

resonances !

even the simplest excited hadrons are not really states of a definite single mass

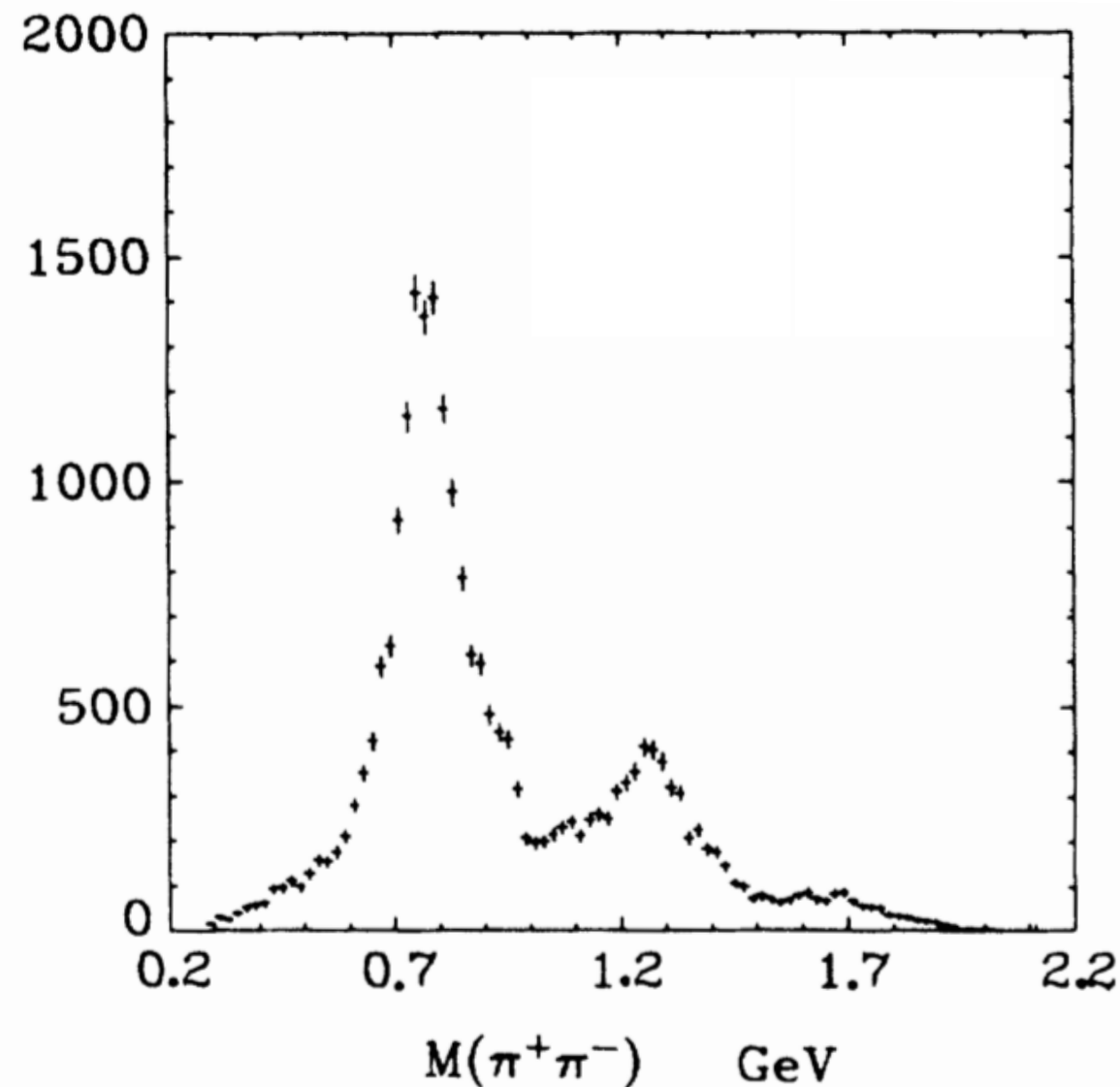
PHYSICAL REVIEW D

VOLUME 7, NUMBER 5

1 MARCH 1973

$\pi\pi$ Partial-Wave Analysis from Reactions $\pi^+p \rightarrow \pi^+\pi^-\Delta^{++}$ and $\pi^+p \rightarrow K^+K^-\Delta^{++}$ at 7.1 GeV/c†

S. D. Protopopescu,* M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté,‡
J. H. Friedman,§ T. A. Lasinski, G. R. Lynch, M. S. Rabin,|| and F. T. Solmitz
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 25 September 1972)



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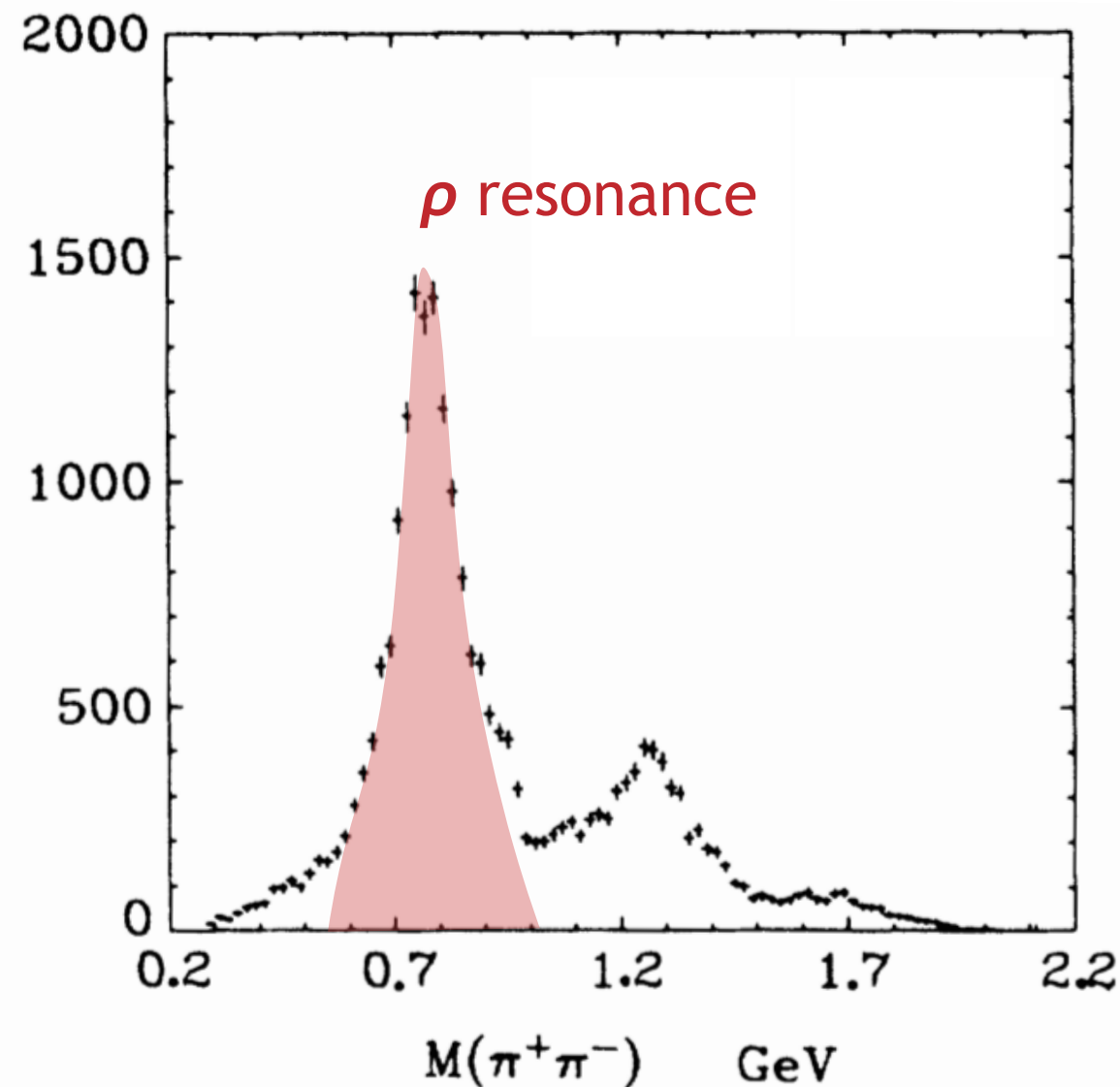
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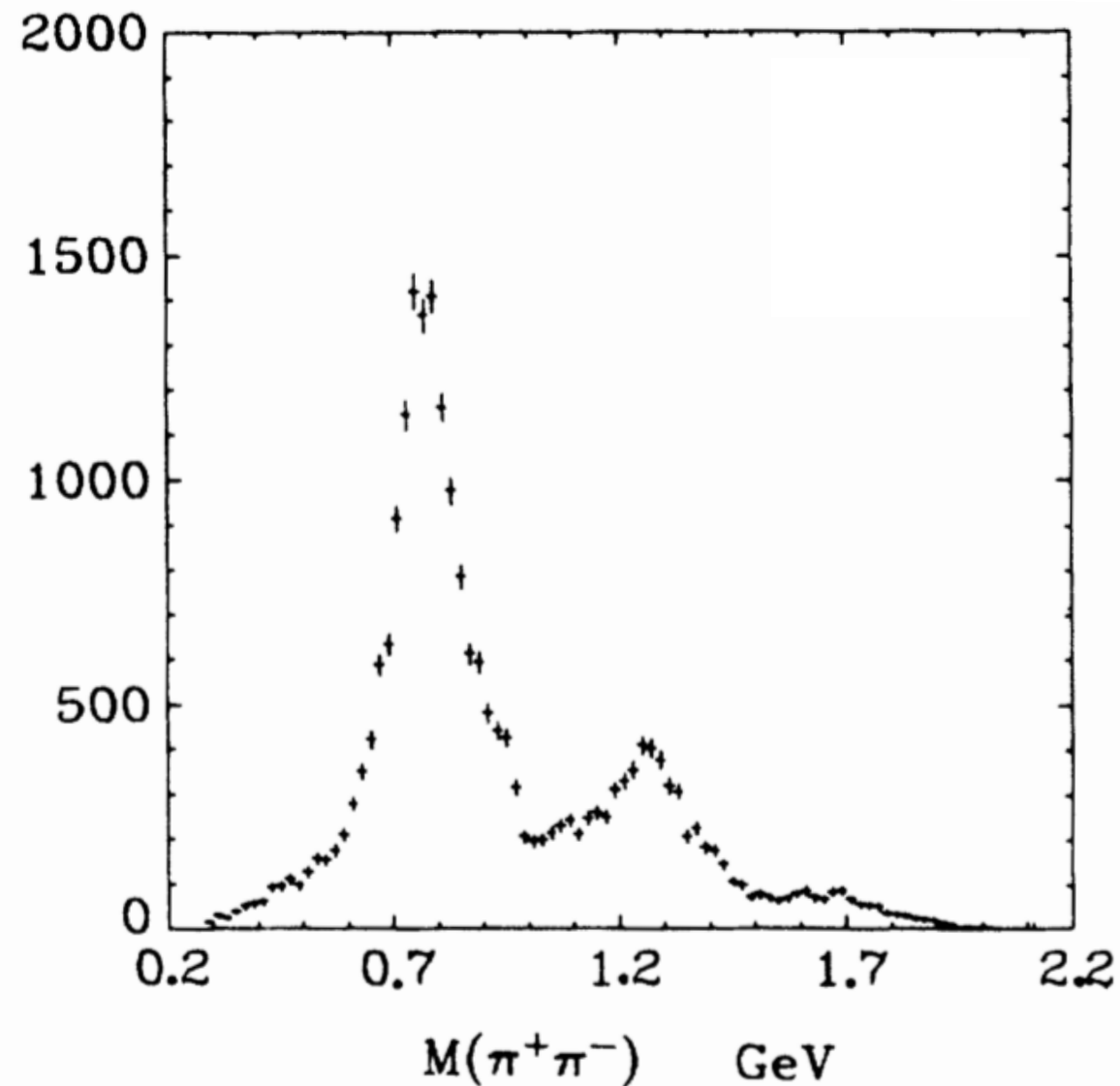
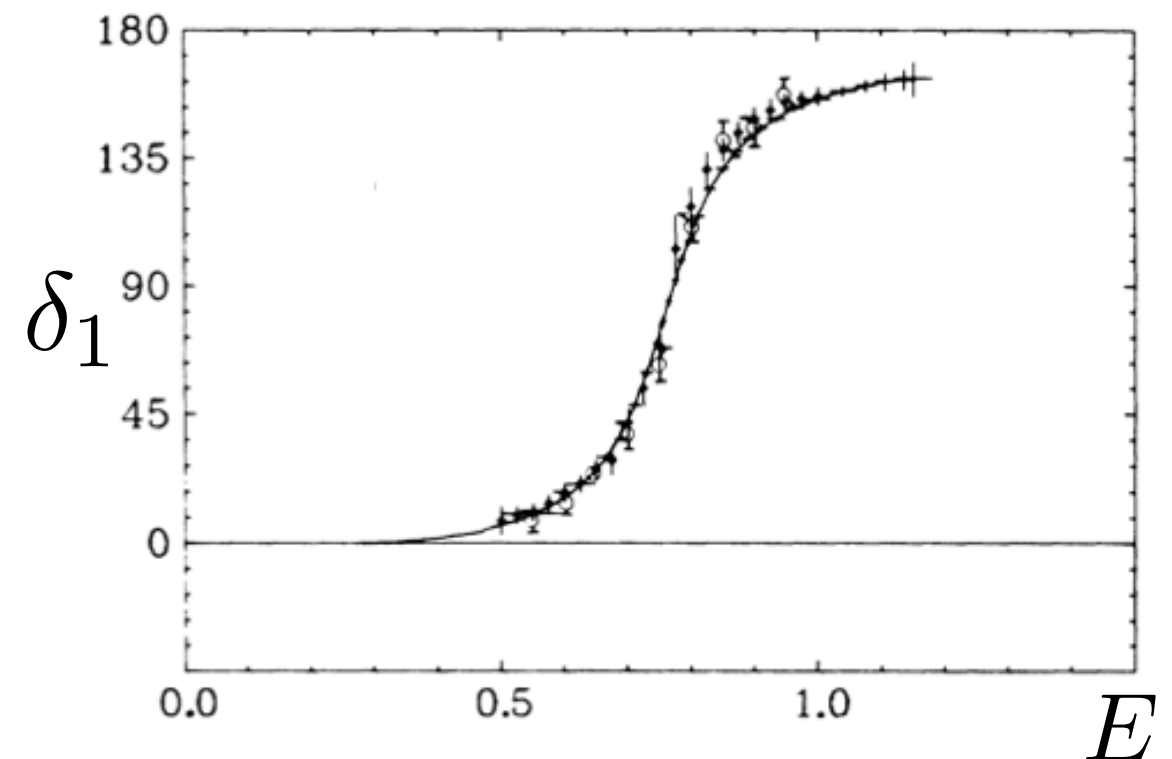
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PARTIAL WAVE AMPLITUDE

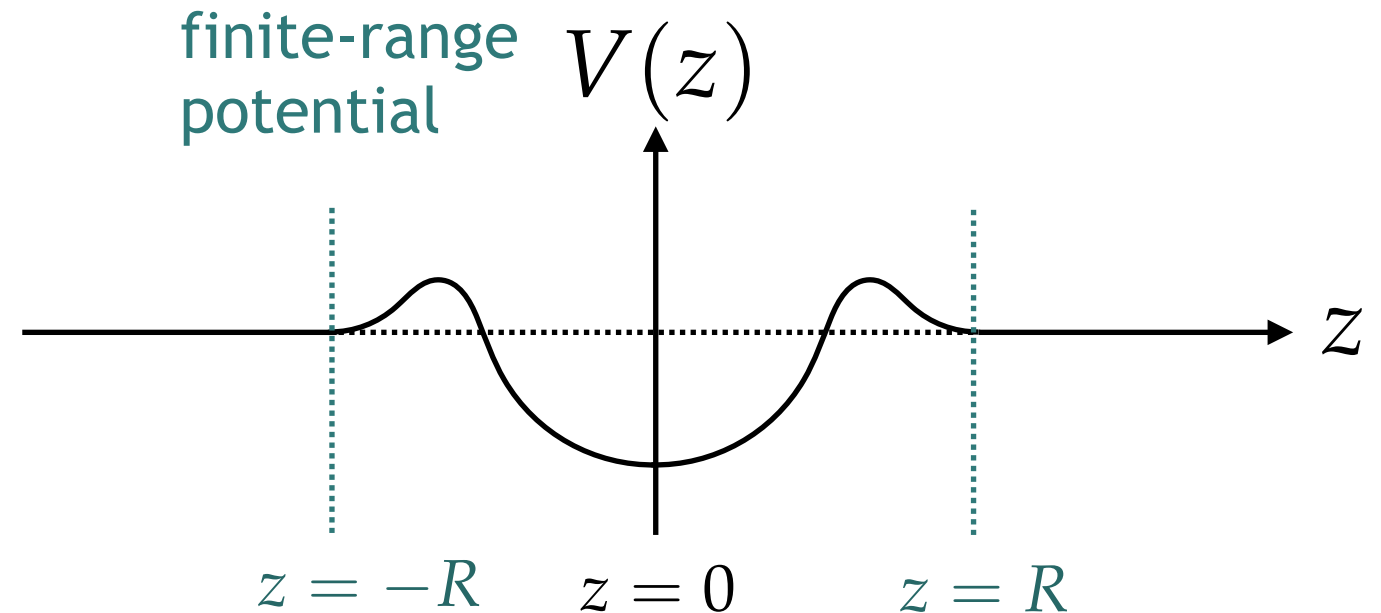
$$f_\ell(E) = \frac{1}{2i} \left(e^{2i\delta_\ell(E)} - 1 \right)$$

RESONANT PHASE SHIFT



but how can scattering amplitudes be accessed in lattice QCD ?

- consider scattering of two identical bosons



outside the well

$$\psi(|z| > R) \sim \cos(p|z| + \delta(p))$$

scattering
phase-shift

continuous
momentum

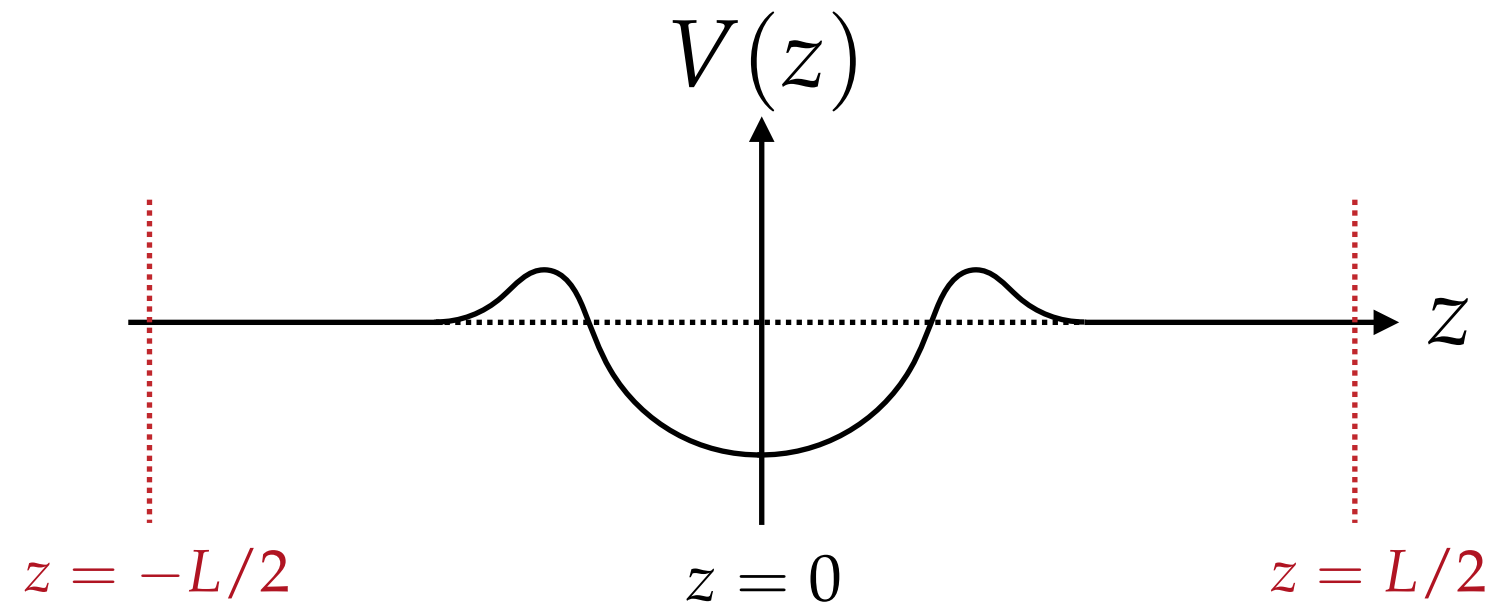


continuous spectrum
of energy eigenstates

- consider scattering of two identical bosons

outside the well

$$\psi(|z| > R) \sim \cos(p|z| + \delta(p))$$



- apply periodic boundary conditions

$$\left. \begin{aligned} \psi(-L/2) &= \psi(L/2) \\ \frac{d\psi}{dz}(-L/2) &= \frac{d\psi}{dz}(L/2) \end{aligned} \right\} \frac{pL}{2} + \delta(p) = n\pi$$

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

discrete
energy
spectrum

Lüscher:

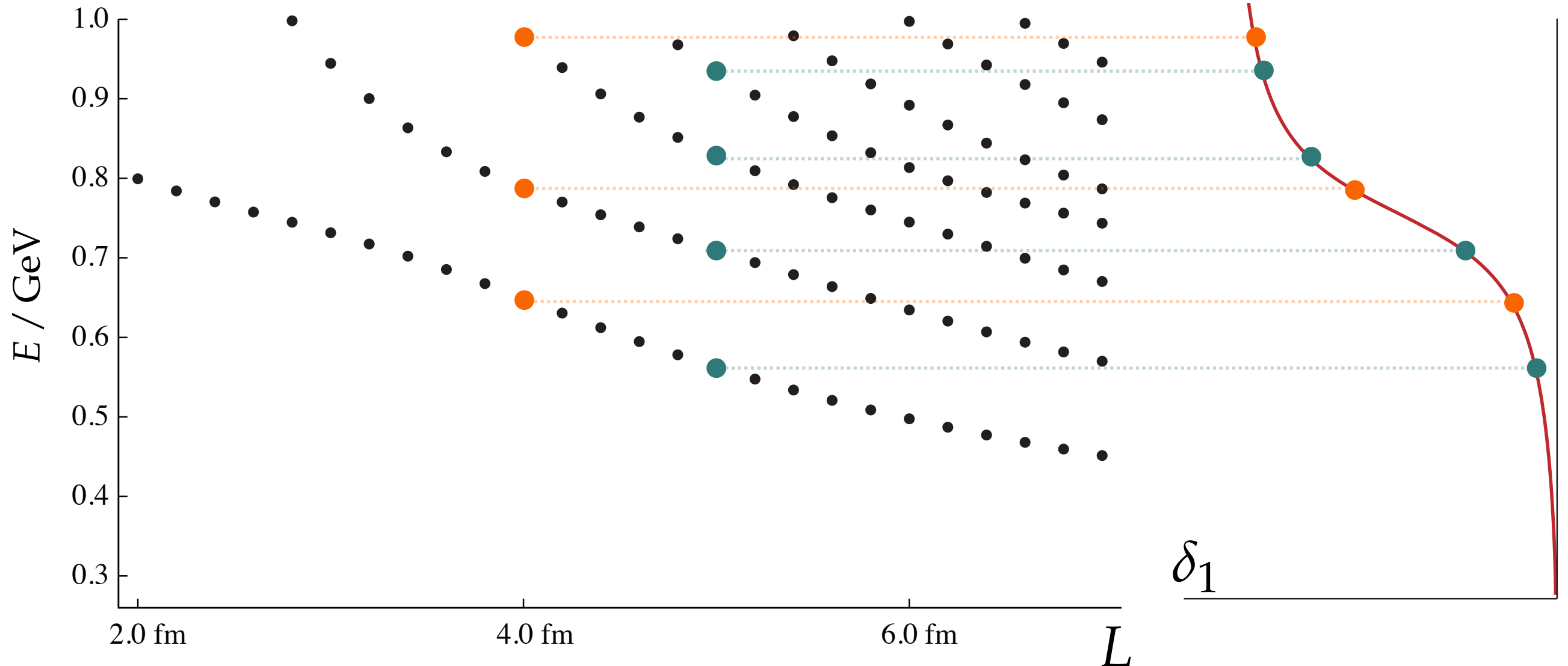
$$\cot \delta_\ell(E) = \mathcal{M}_\ell(E, L)$$

*known
functions*

*[modulo some subtleties
regarding ℓ -mixing]*

L×L×L BOX SPECTRUM

PHASE SHIFT



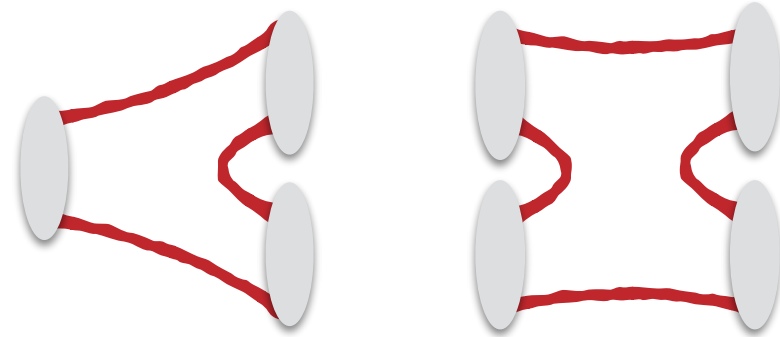
what operator basis is required ?

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supplement large $\bar{\psi}\Gamma\overleftrightarrow{D}\dots\overleftrightarrow{D}\psi$ basis with meson-meson-like operators

$$\text{e.g. } \mathcal{O}_{\pi\pi}^{|\vec{p}|} = \sum_{\hat{p}} C(\hat{p}) \mathcal{O}_{\pi}(\vec{p}) \mathcal{O}_{\pi}(-\vec{p}) \quad \text{where} \quad \mathcal{O}_{\pi}(\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{\psi}\Gamma\psi(\vec{x})$$

now need to evaluate diagrams like

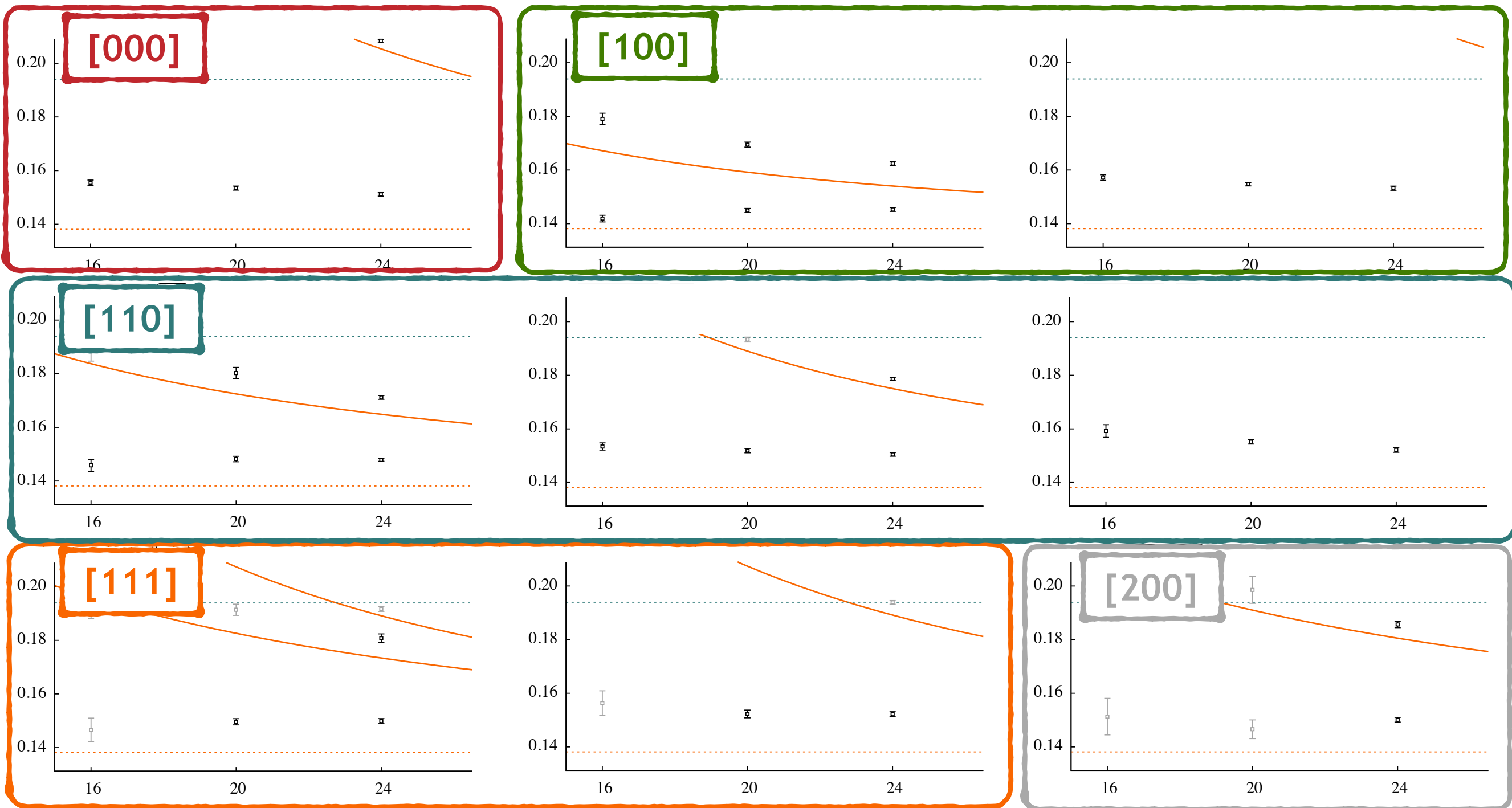


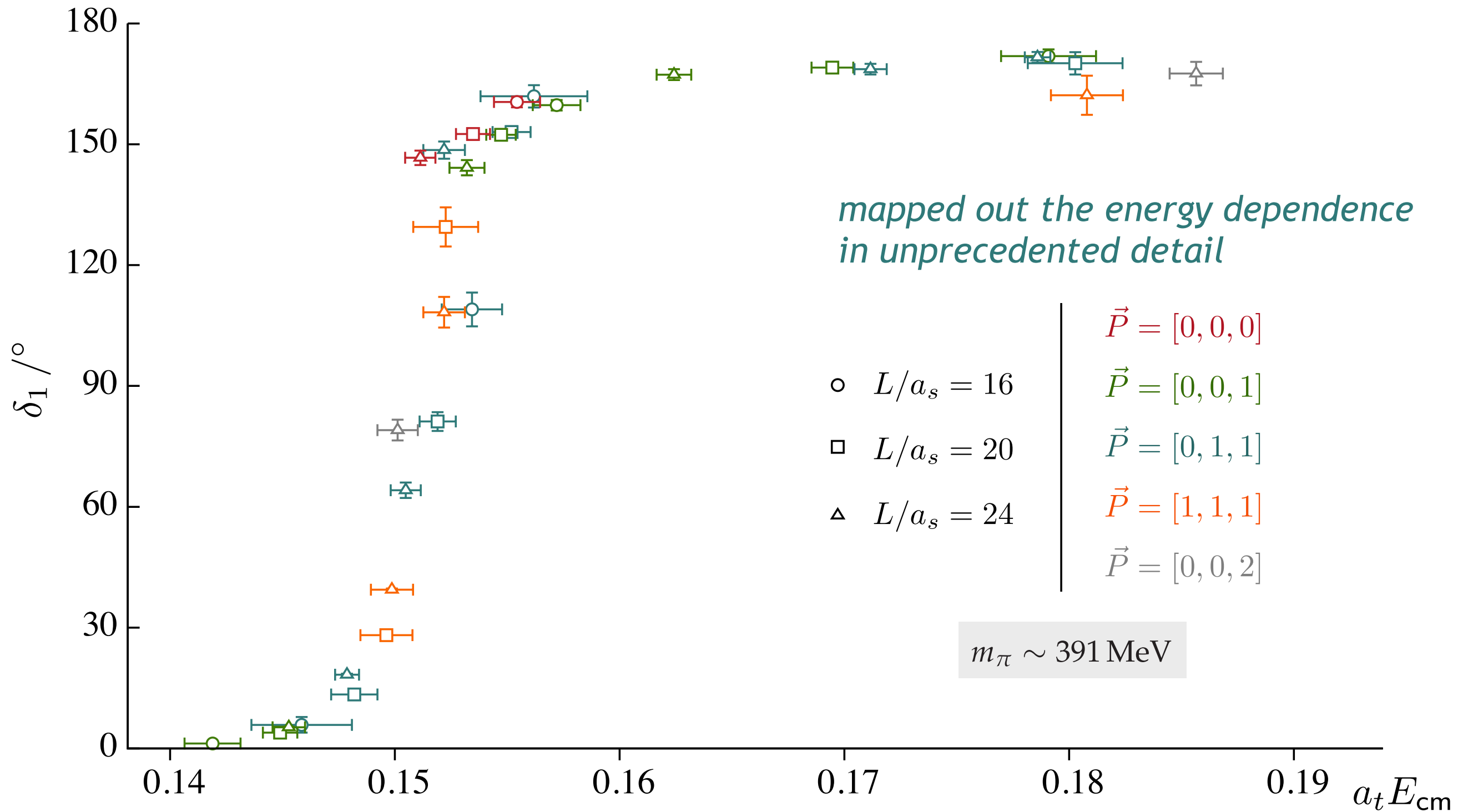
distillation can handle the annihilation lines

spectra in moving frames

$$m_{\pi} \sim 391 \text{ MeV}$$

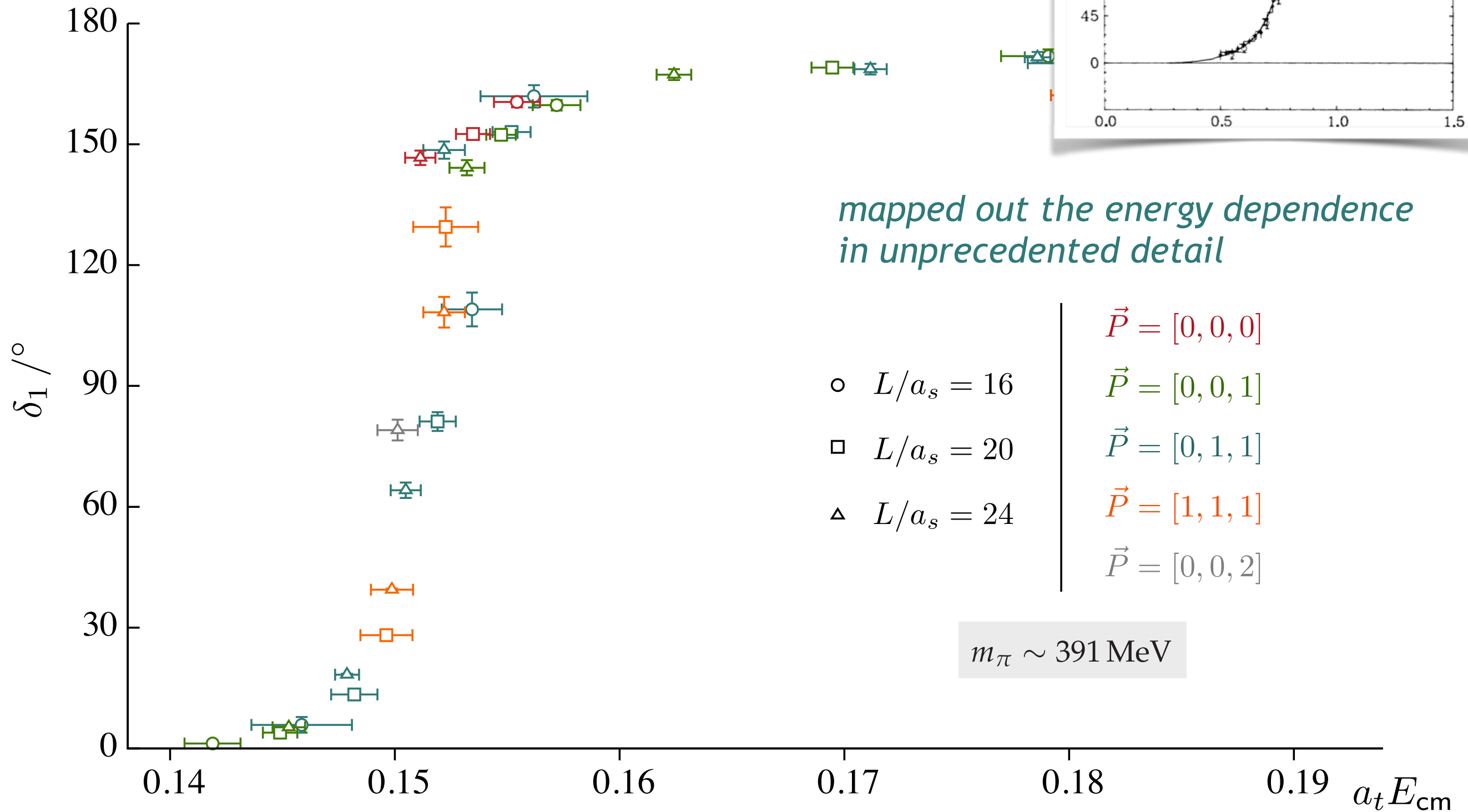
30



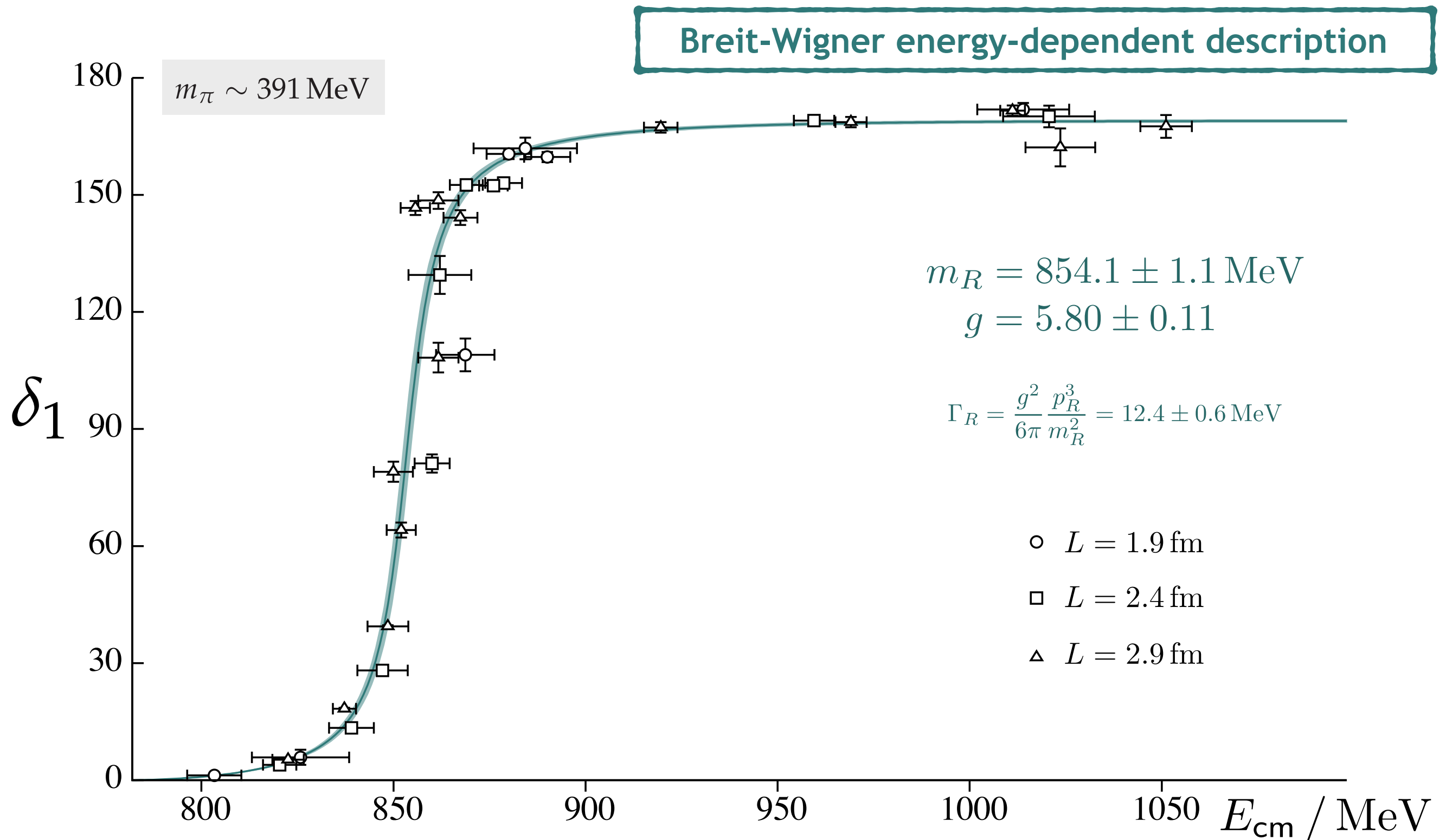


PRD87 034505 (2013)

$\pi\pi$ P -wave phase-shift

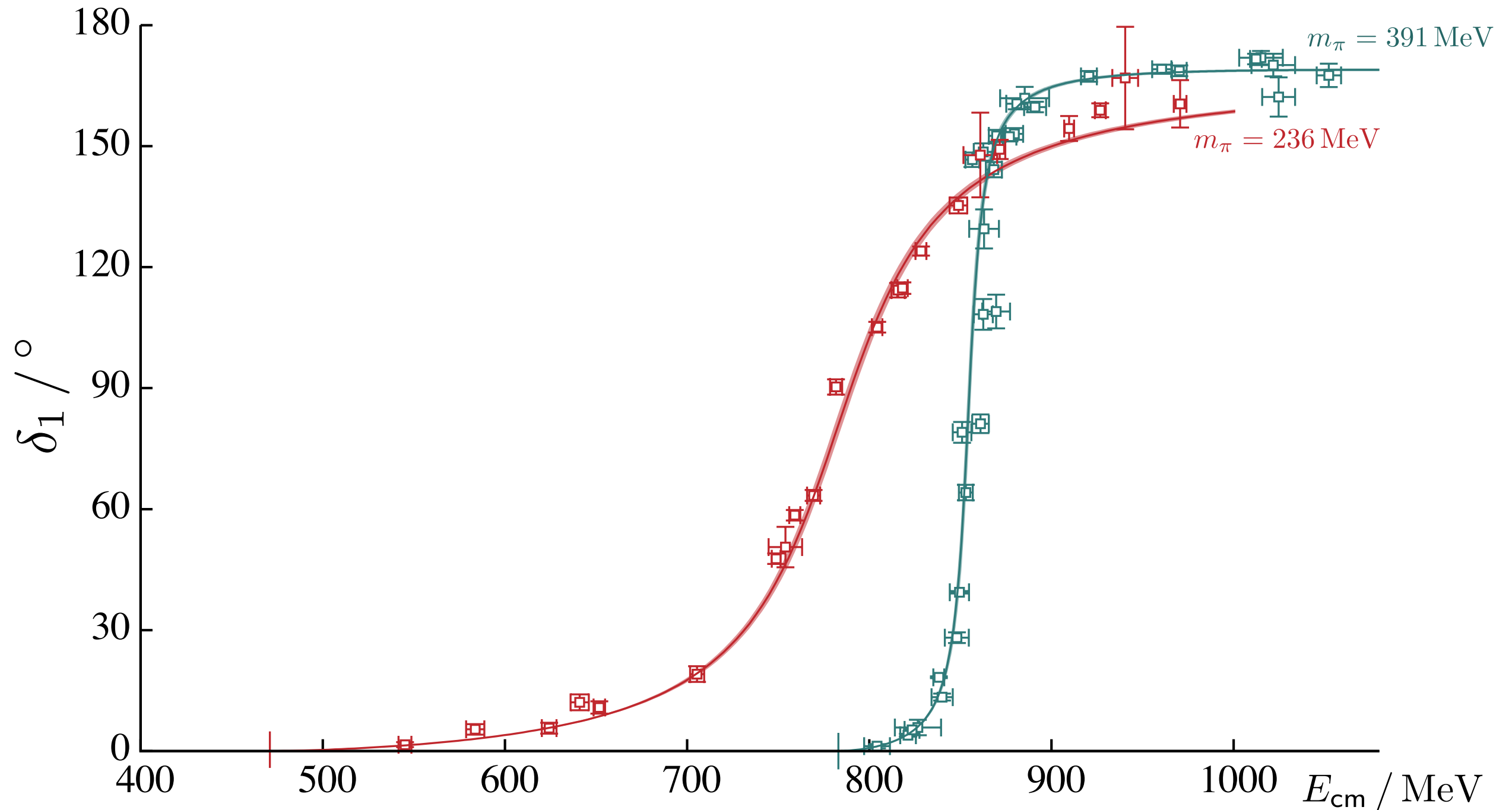


PRD87 034505 (2013)



PRD87 034505 (2013)

- reducing the pion mass moves mass, width in the right direction ...

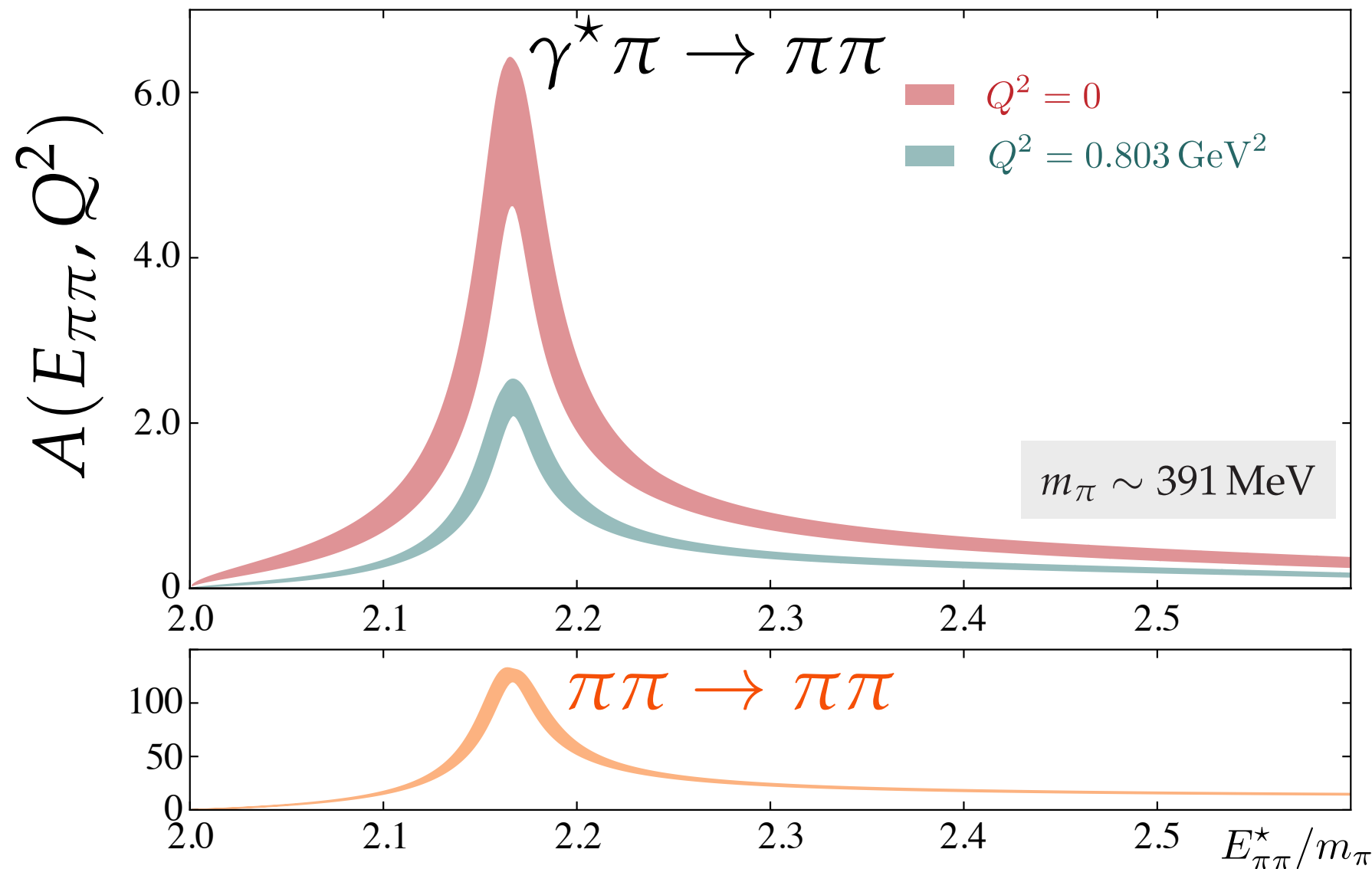


PRD87 034505 (2013)

PRD92 094502 (2015)

- e.g. GlueX/CLAS12 use photons to excite resonances - can this be studied ?
- finite-volume formalism applied to three-point functions

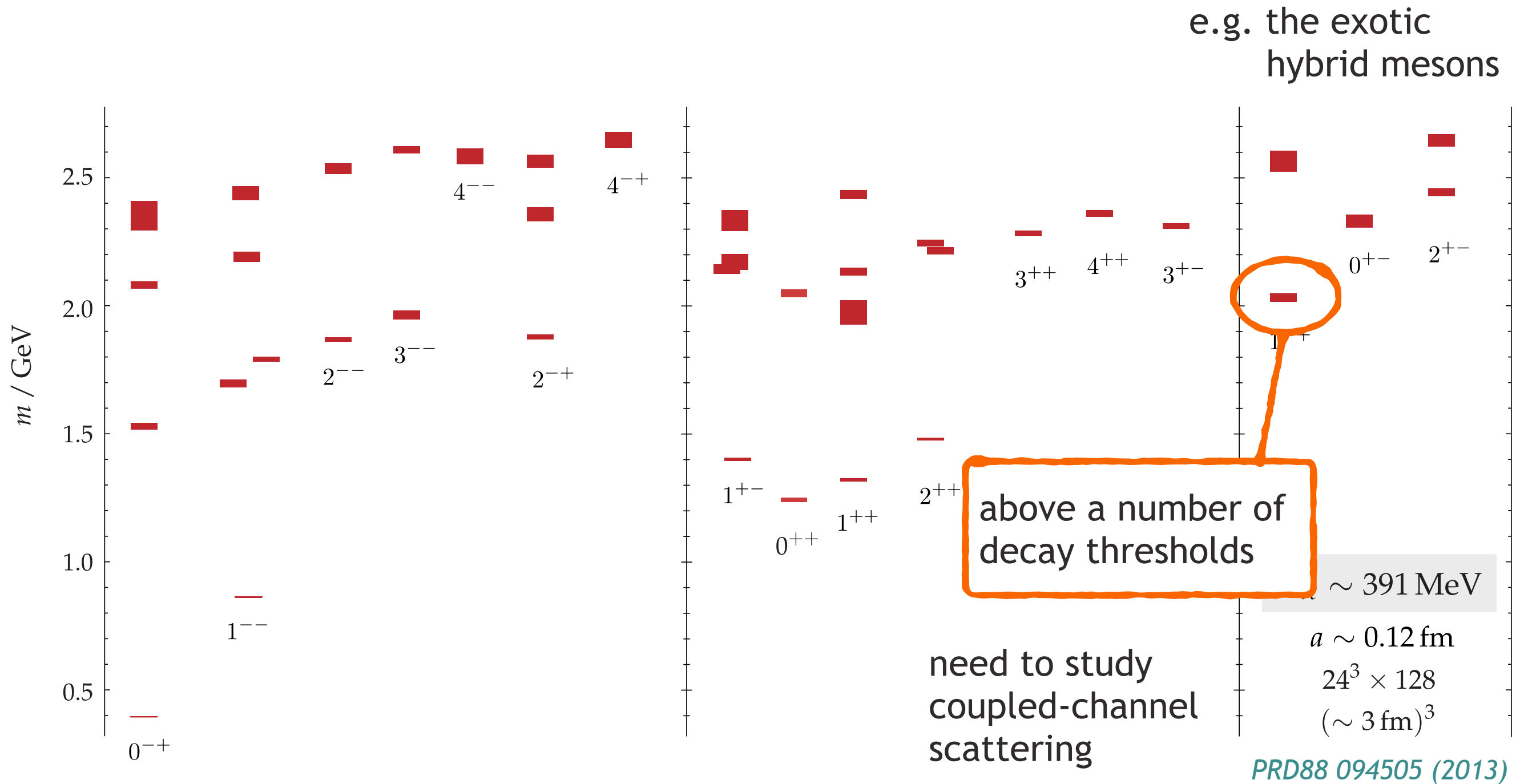
- e.g. GlueX/CLAS12 use photons to excite resonances - can this be studied ?
- finite-volume formalism applied to three-point functions
- first explicit application has appeared $\gamma^* \pi \rightarrow \rho \rightarrow \pi\pi$ *PRL 115 242001 (2015)*



Raul
Briceño



- what about decays of higher excited states ?



- more challenging analysis problem

e.g. in a **two-channel** process, **three** unknowns specify the S -matrix at each energy

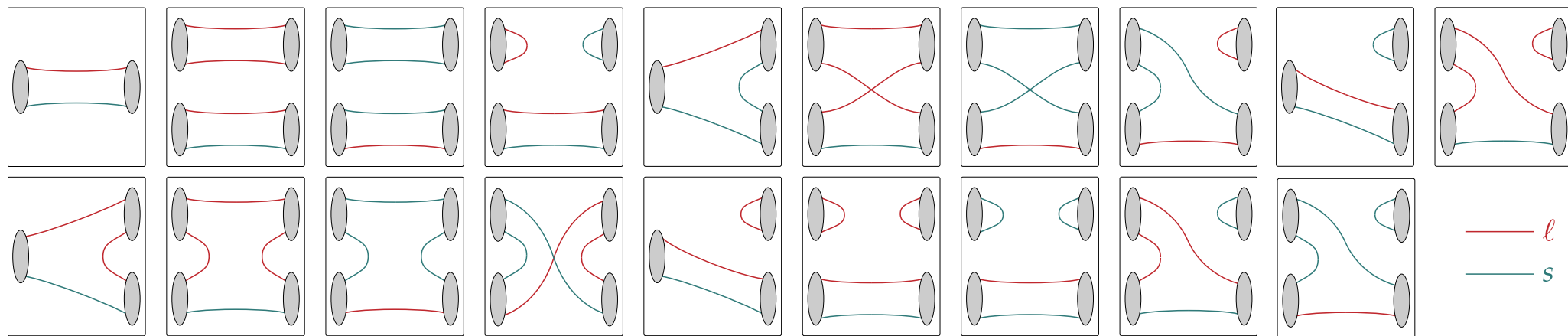
our solution: parameterize the energy dependence of the S -matrix and describe the finite-volume spectra by varying parameters

- more challenging analysis problem

e.g. in a **two-channel** process, **three** unknowns specify the S -matrix at each energy

our solution: parameterize the energy dependence of the S -matrix and describe the finite-volume spectra by varying parameters

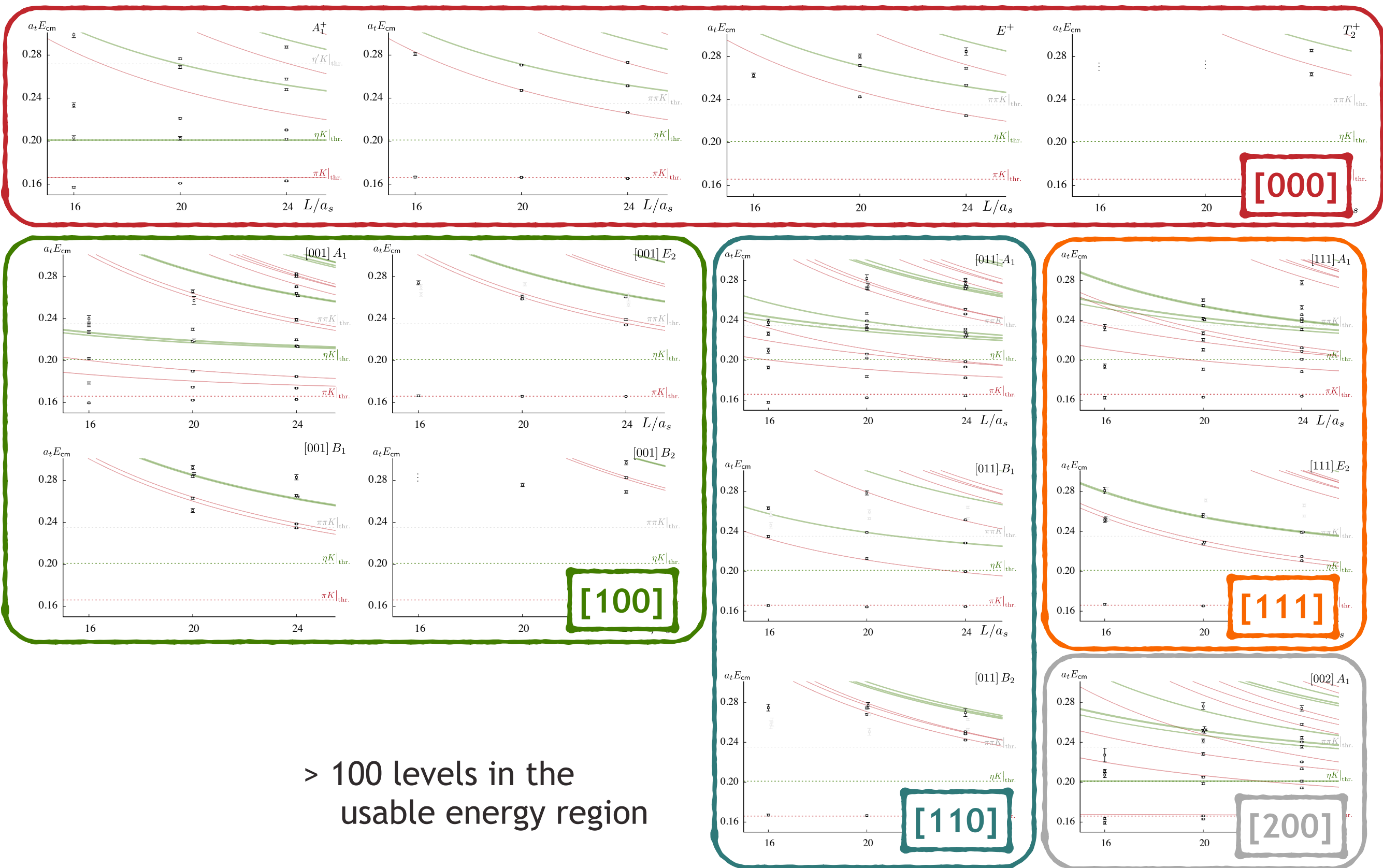
- first attempt, coupled-channel $\pi K / \eta K$ scattering
- need to compute the finite-volume spectra ... lots of Wick contractions ...



$\pi K/\eta K$ lattice QCD spectra

$m_\pi \sim 391 \text{ MeV}$

37

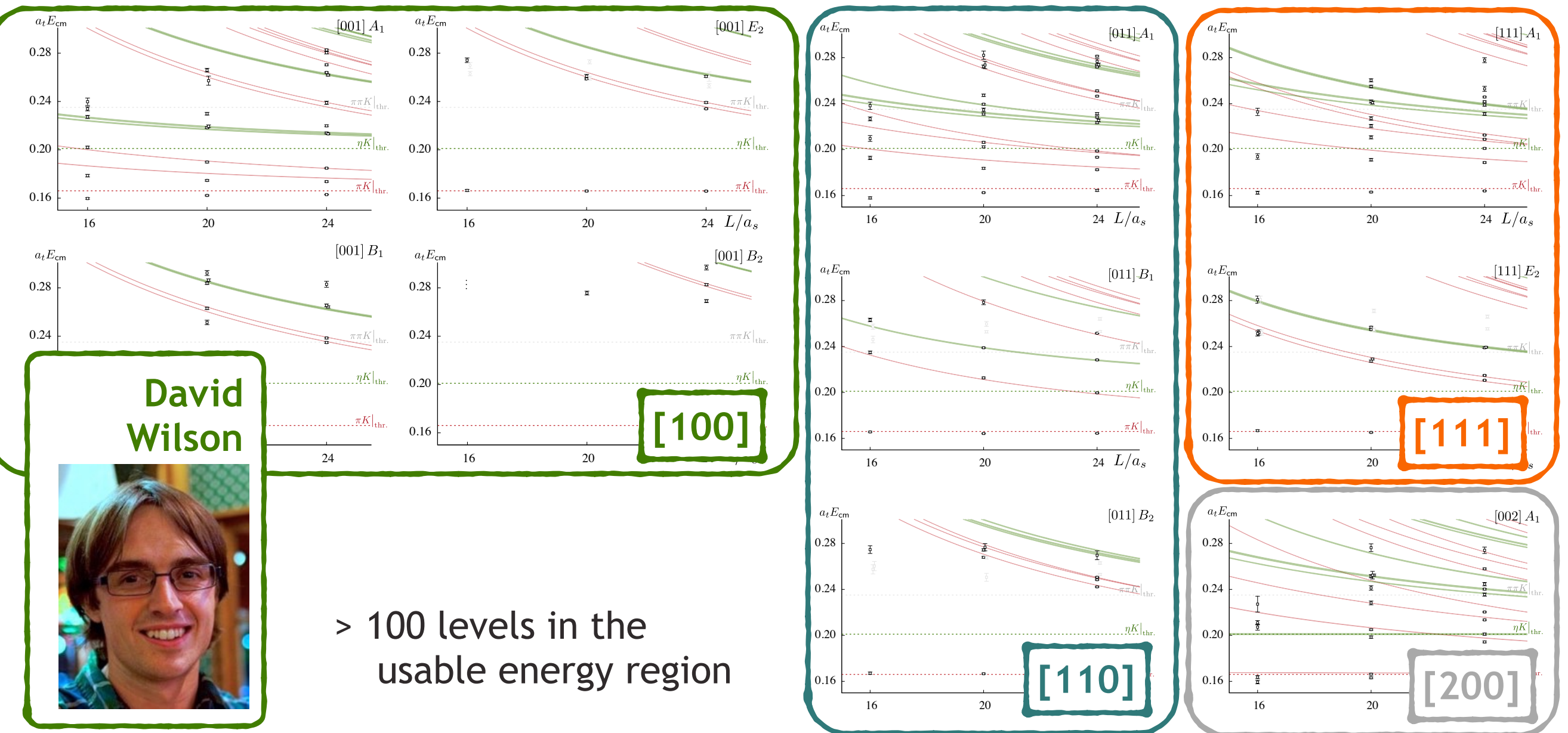
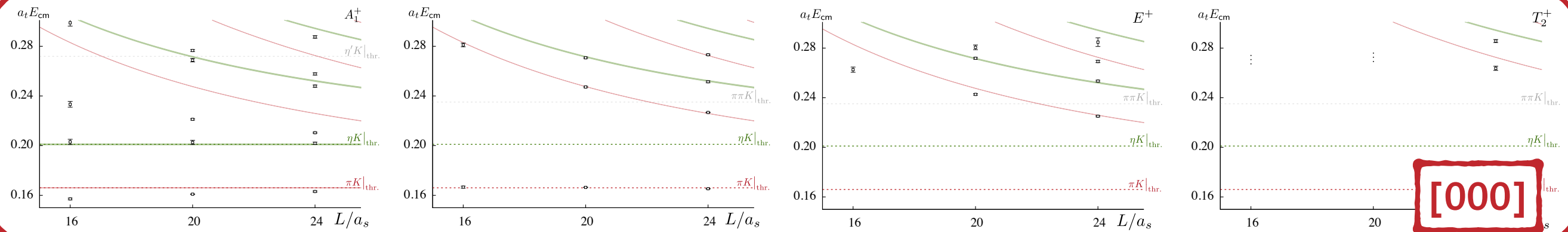


> 100 levels in the
usable energy region

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37



David
Wilson

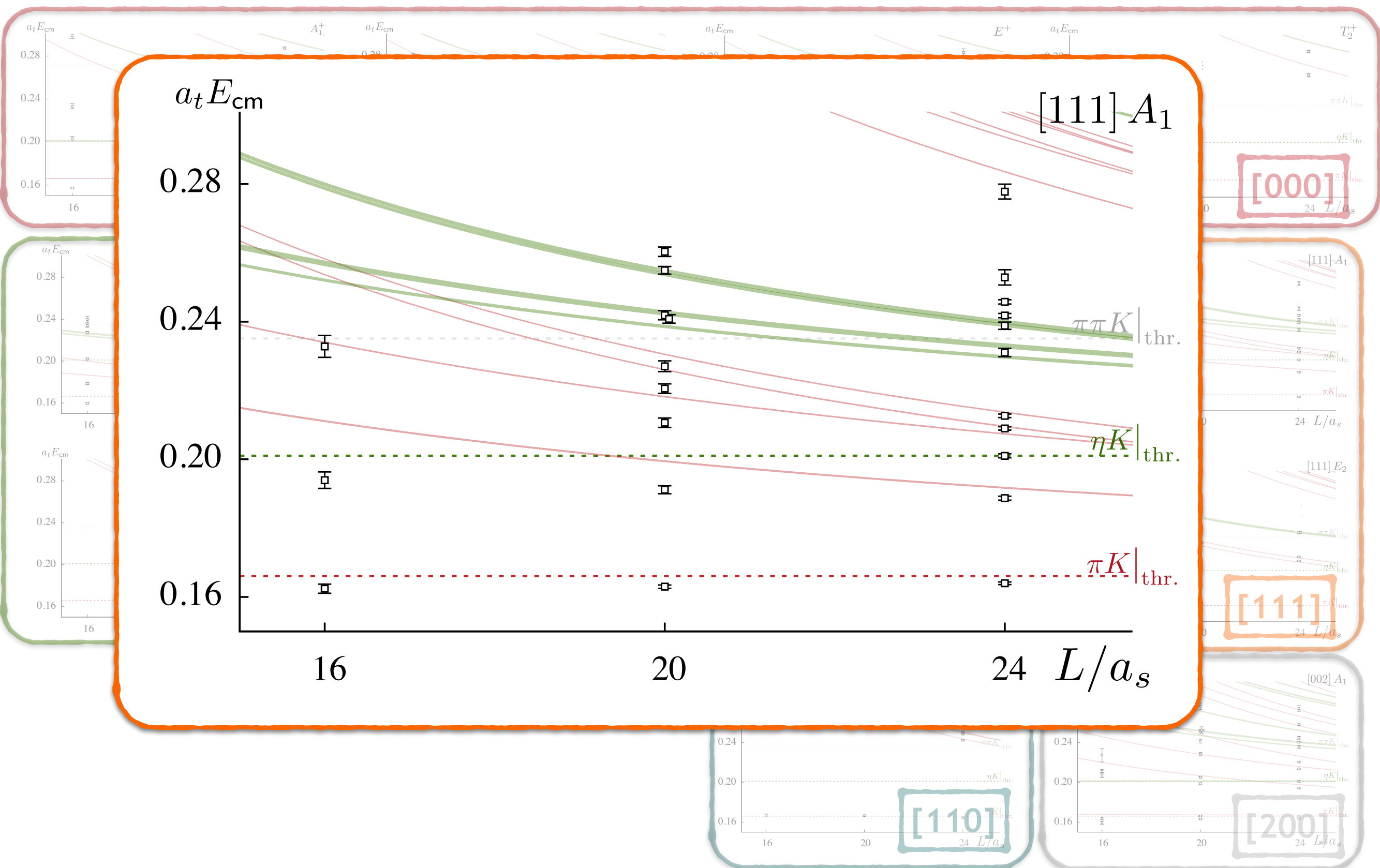


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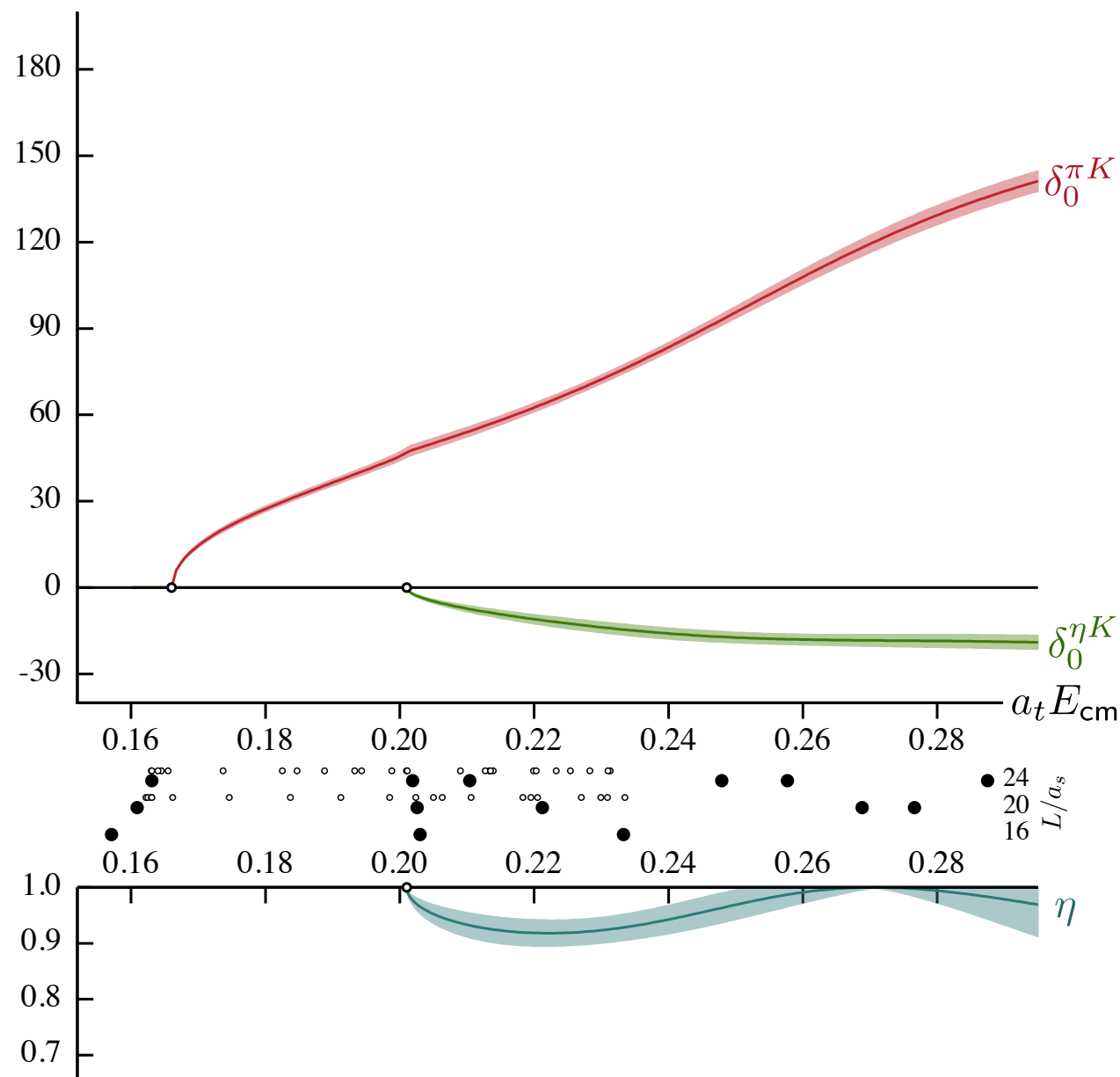
38



- describe all the finite-volume spectra

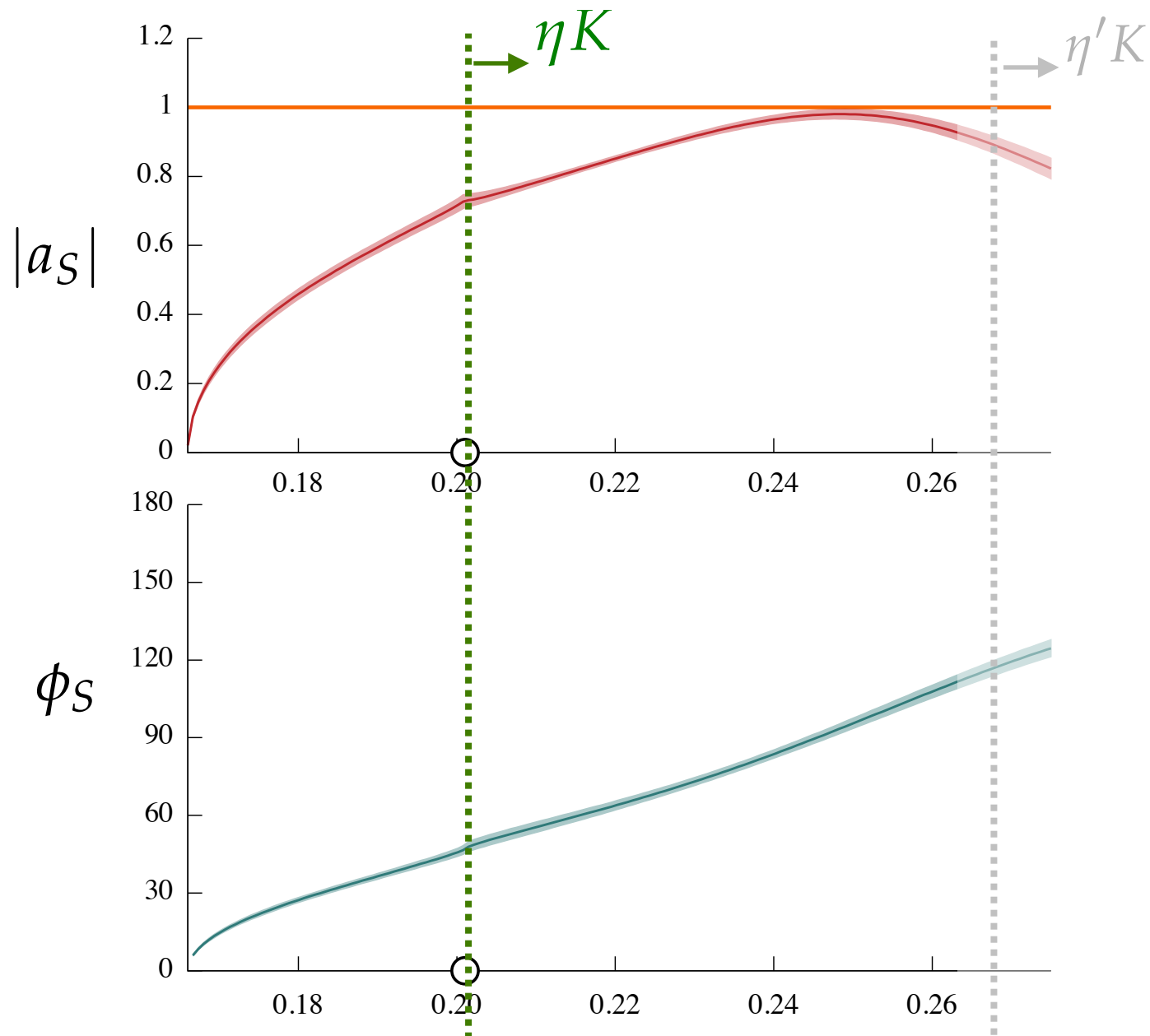
$$\chi^2/N_{\text{dof}} = \frac{49.1}{61 - 6} = 0.89$$

S-WAVE $\pi K/\eta K$ SCATTERING



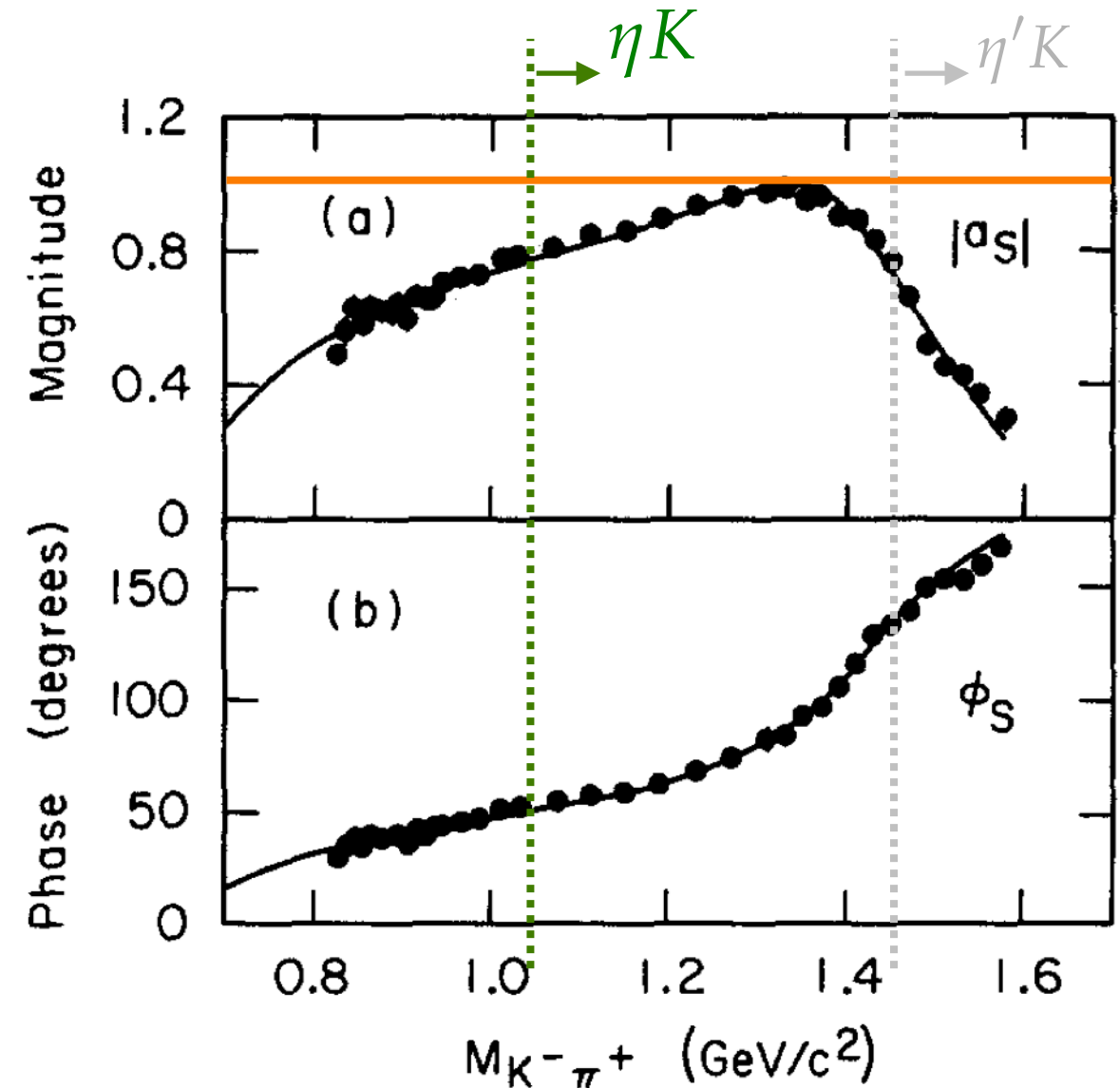
PRL 113 182001
PRD 91 054008

S-WAVE $\pi K \rightarrow \pi K$ AMPLITUDE



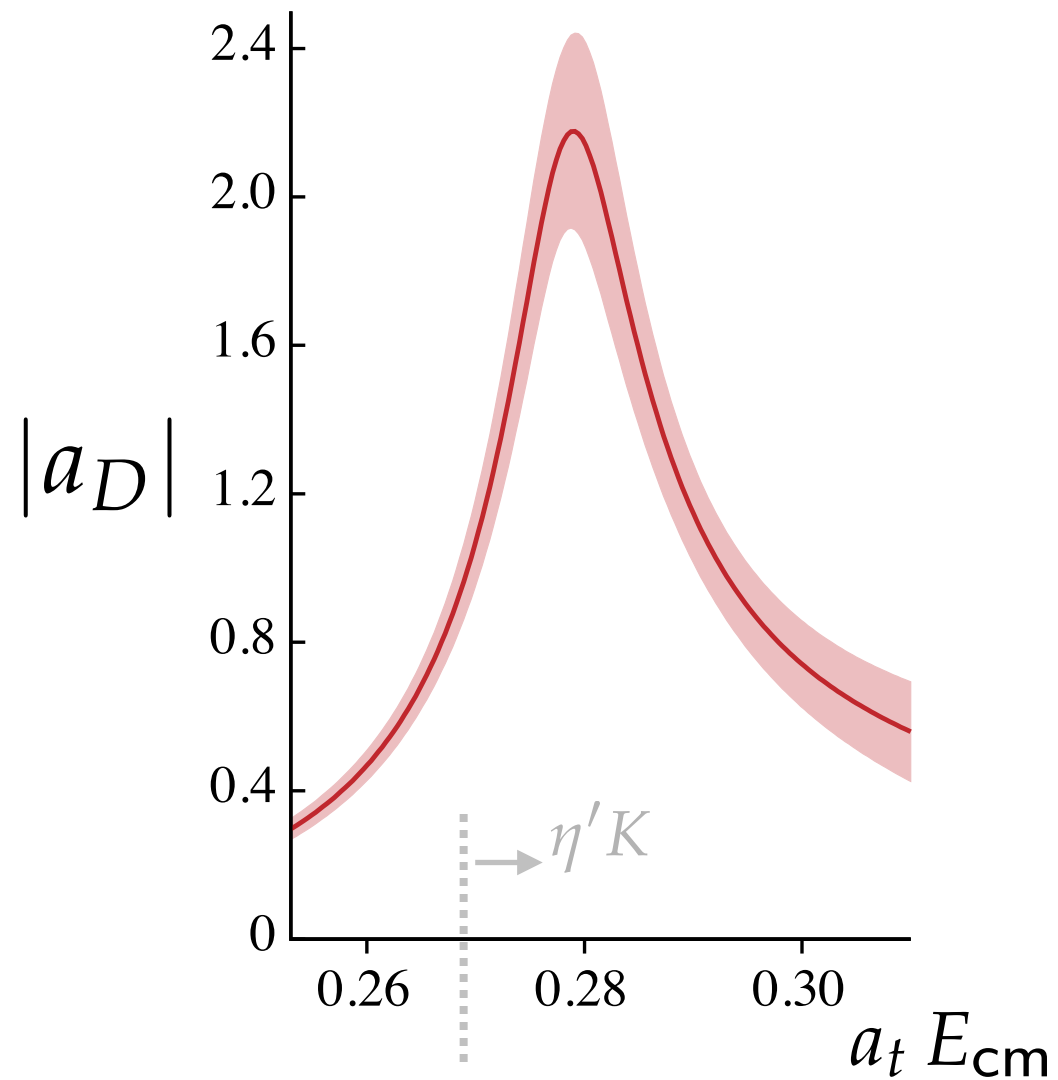
$$m_\pi \sim 391 \text{ MeV}$$

LASS S-WAVE



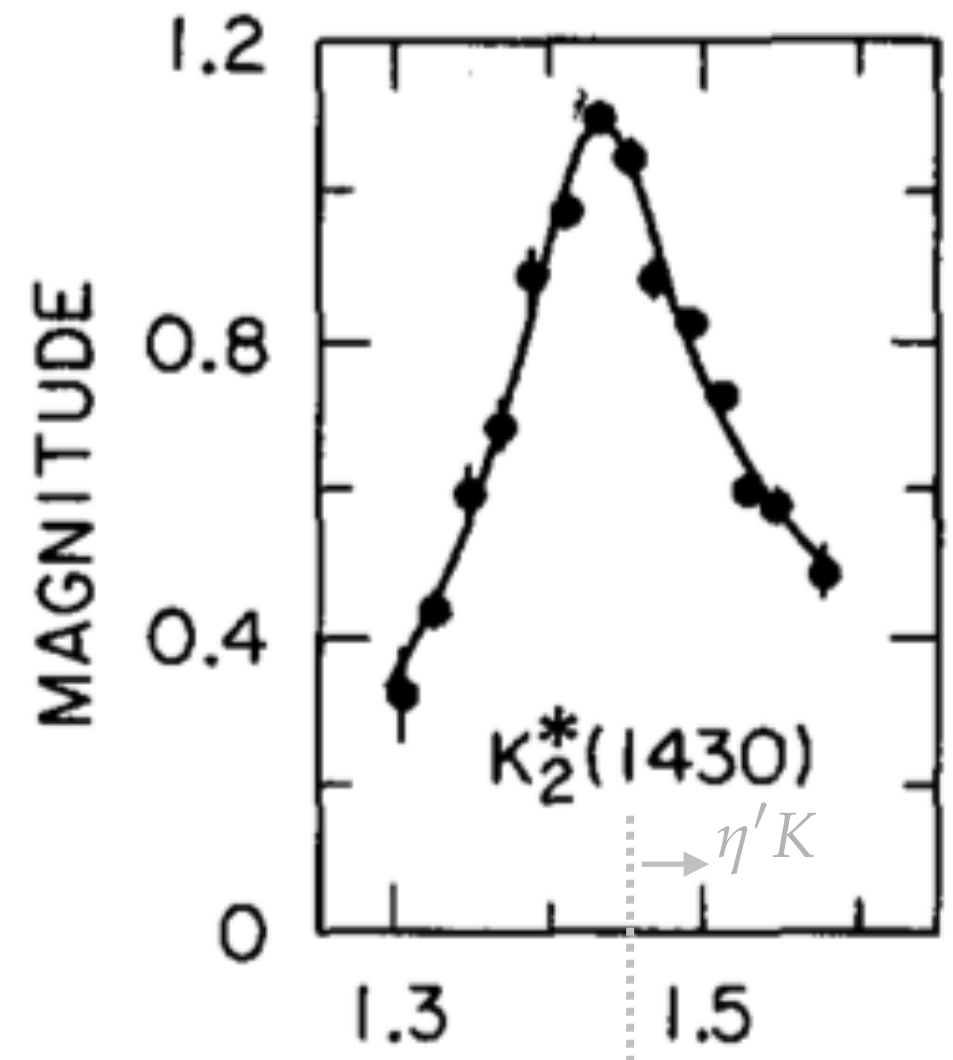
LASS, NPB296 493

D-WAVE $\pi K \rightarrow \pi K$ AMPLITUDE



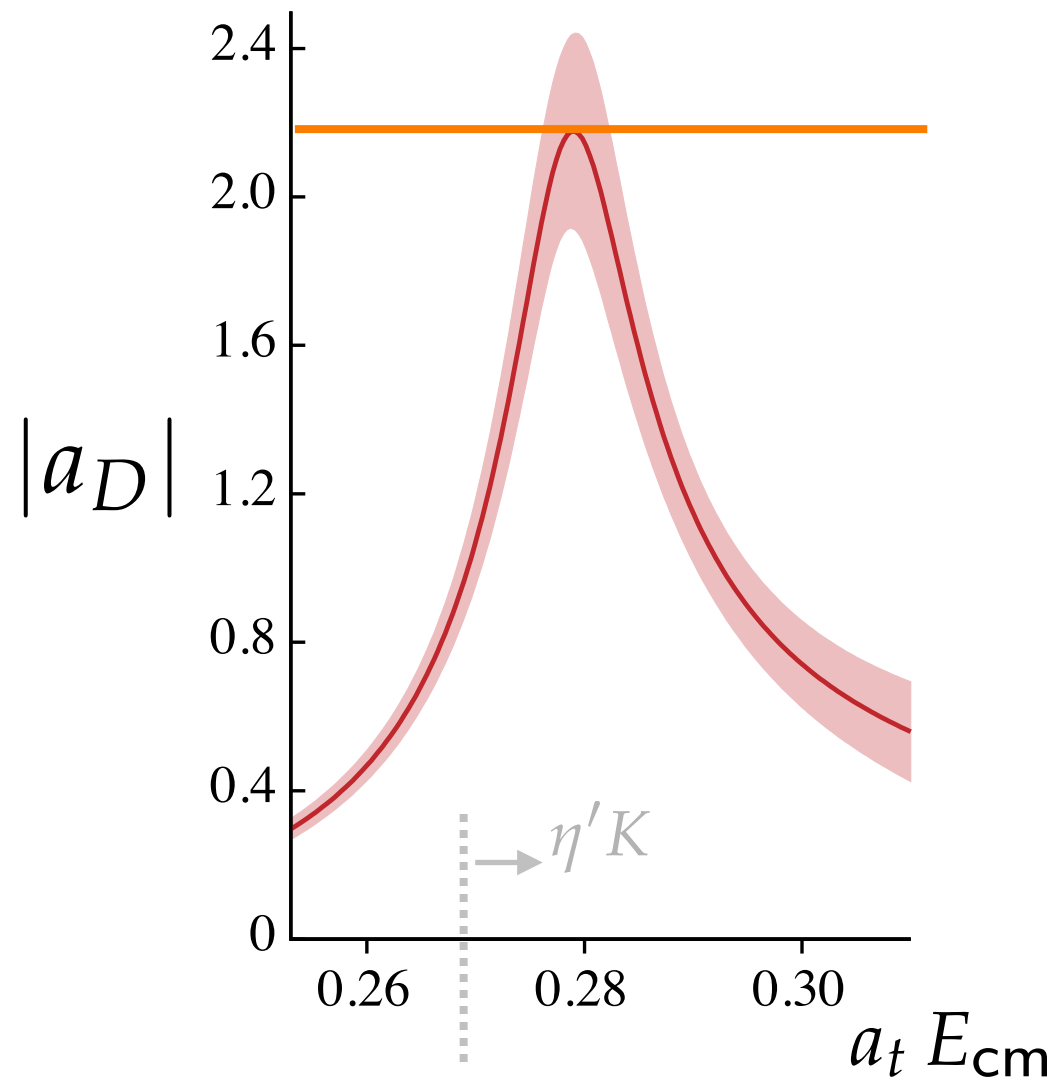
$m_\pi \sim 391 \text{ MeV}$

LASS D-WAVE



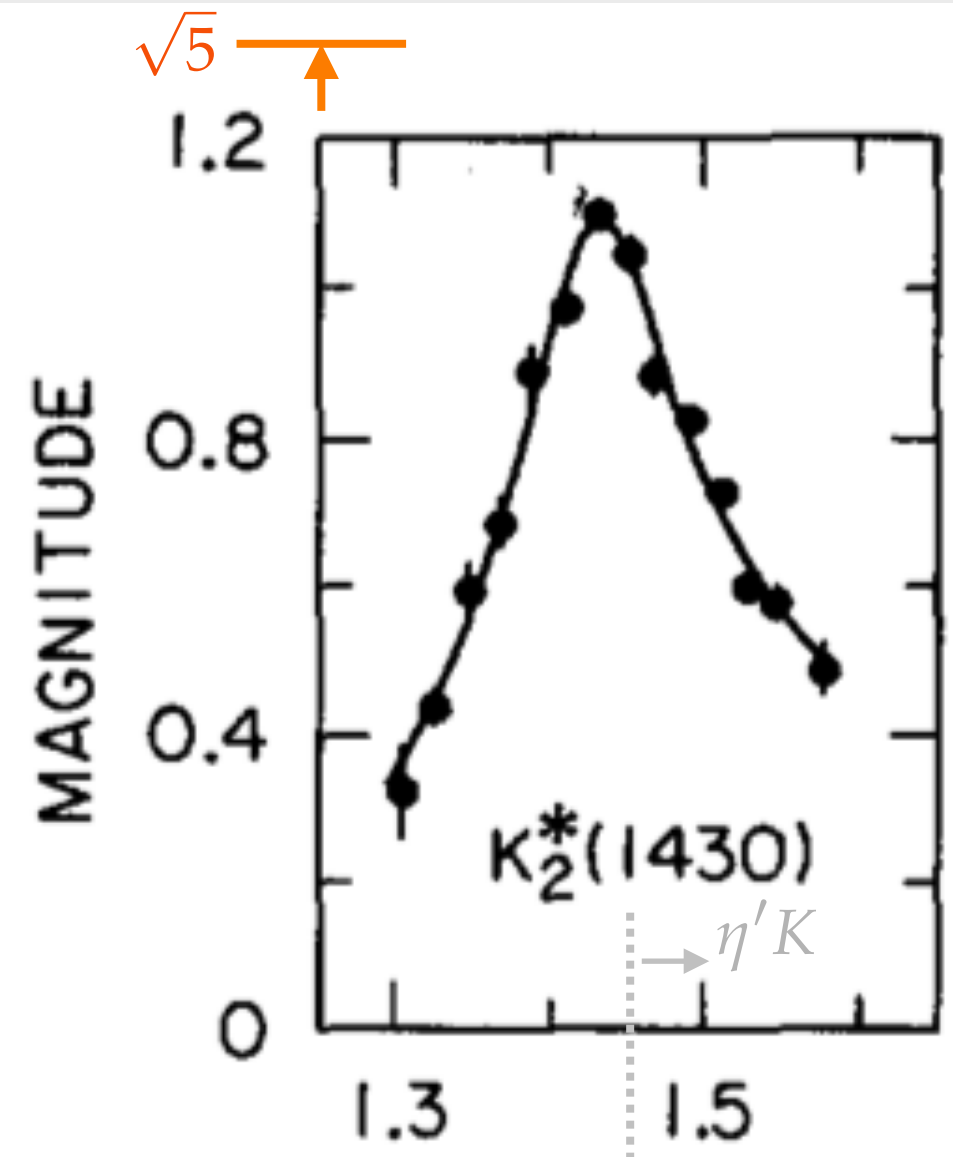
LASS, NPB296 493

D-WAVE $\pi K \rightarrow \pi K$ AMPLITUDE



$m_\pi \sim 391 \text{ MeV}$

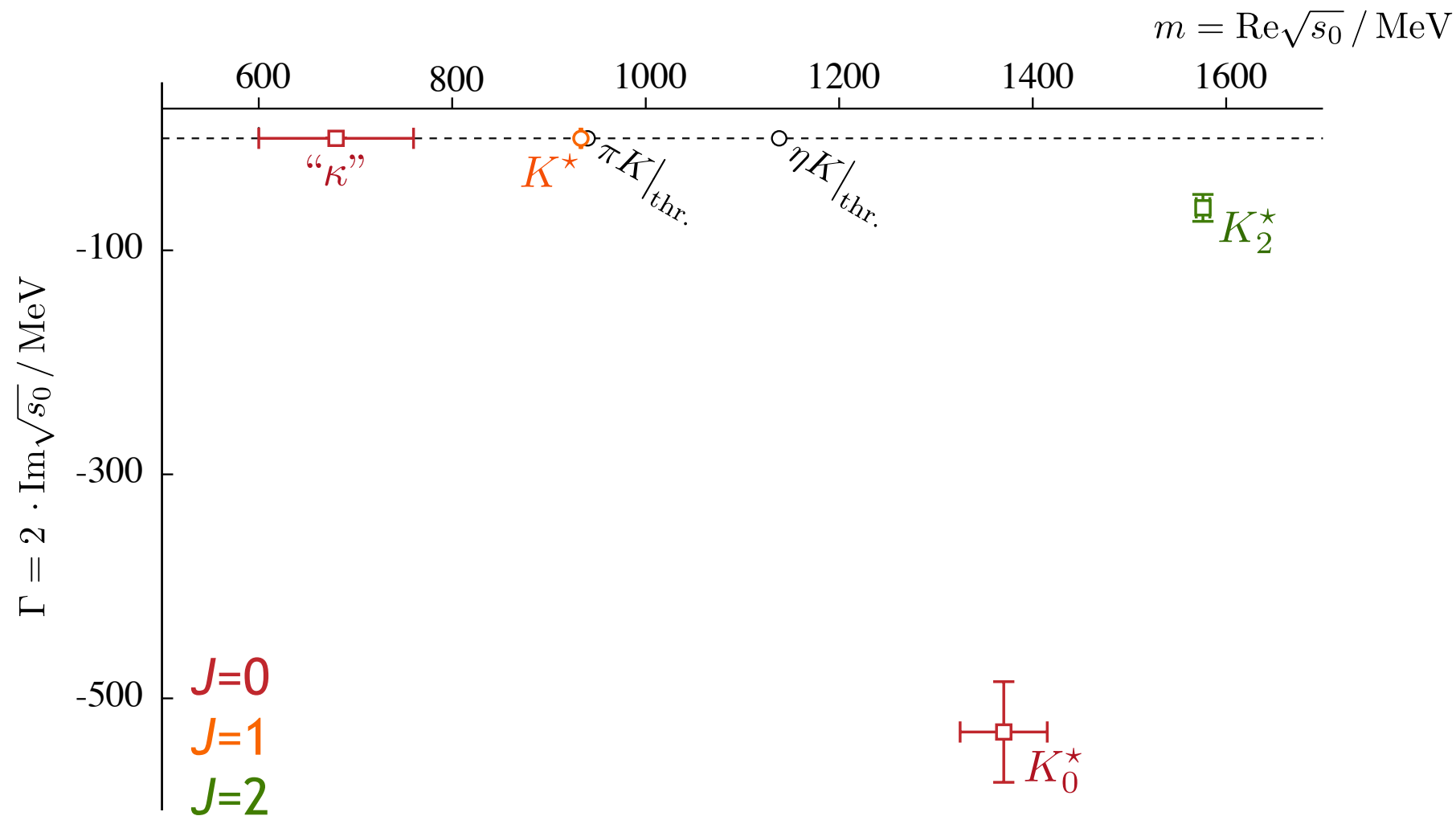
LASS D-WAVE



LASS, NPB296 493

- S-matrix poles as least model-dependent characterization of resonances

COMPLEX ENERGY PLANE

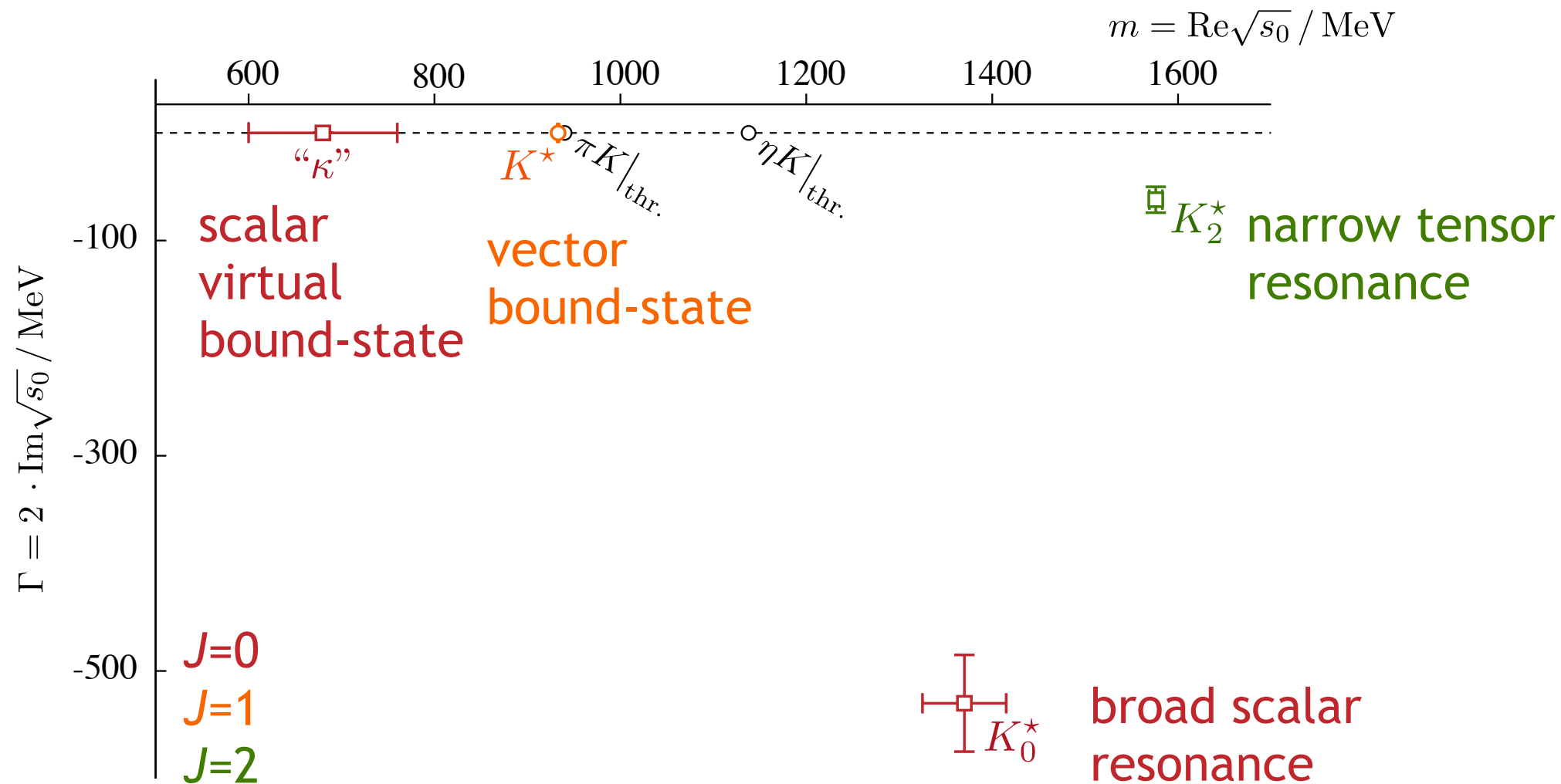


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PRL 113 182001
PRD 91 054008

- S-matrix poles as least model-dependent characterization of resonances

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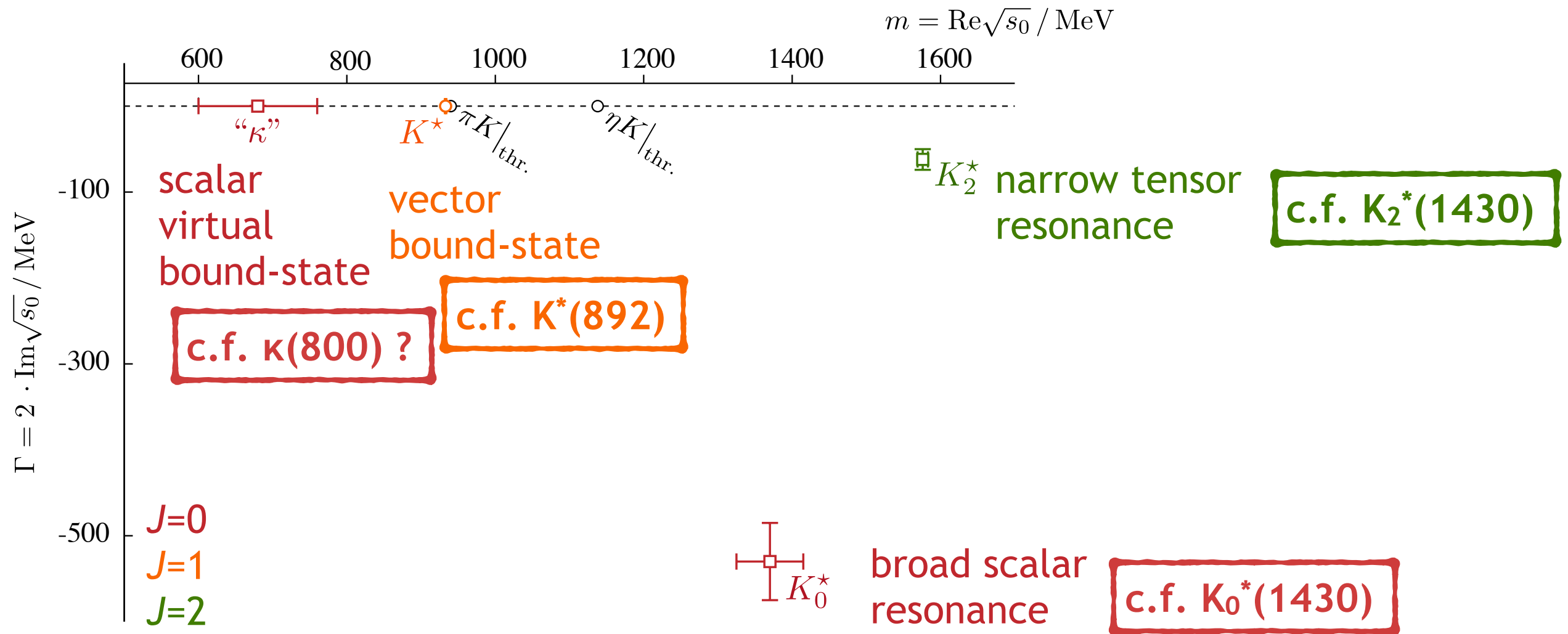


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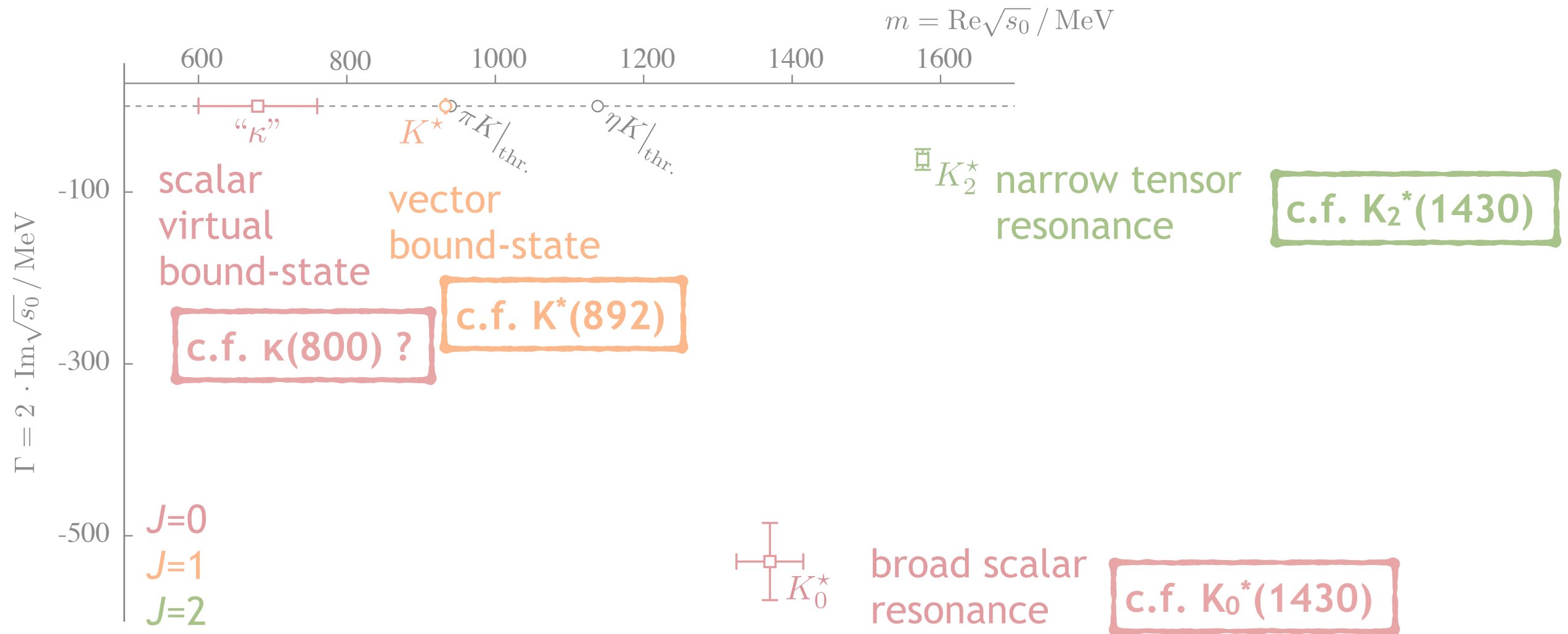


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PRD 91 054008

- S -matrix poles as least model-dependent characterization of resonances

COMPLEX ENERGY PLANE



$m_\pi \sim 391 \text{ MeV}$

PRL 113 182001
PRD 91 054008

... but no strong channel-coupling here ...

$\pi\eta/K\bar{K}$ scattering and the $a_0(980)$

45

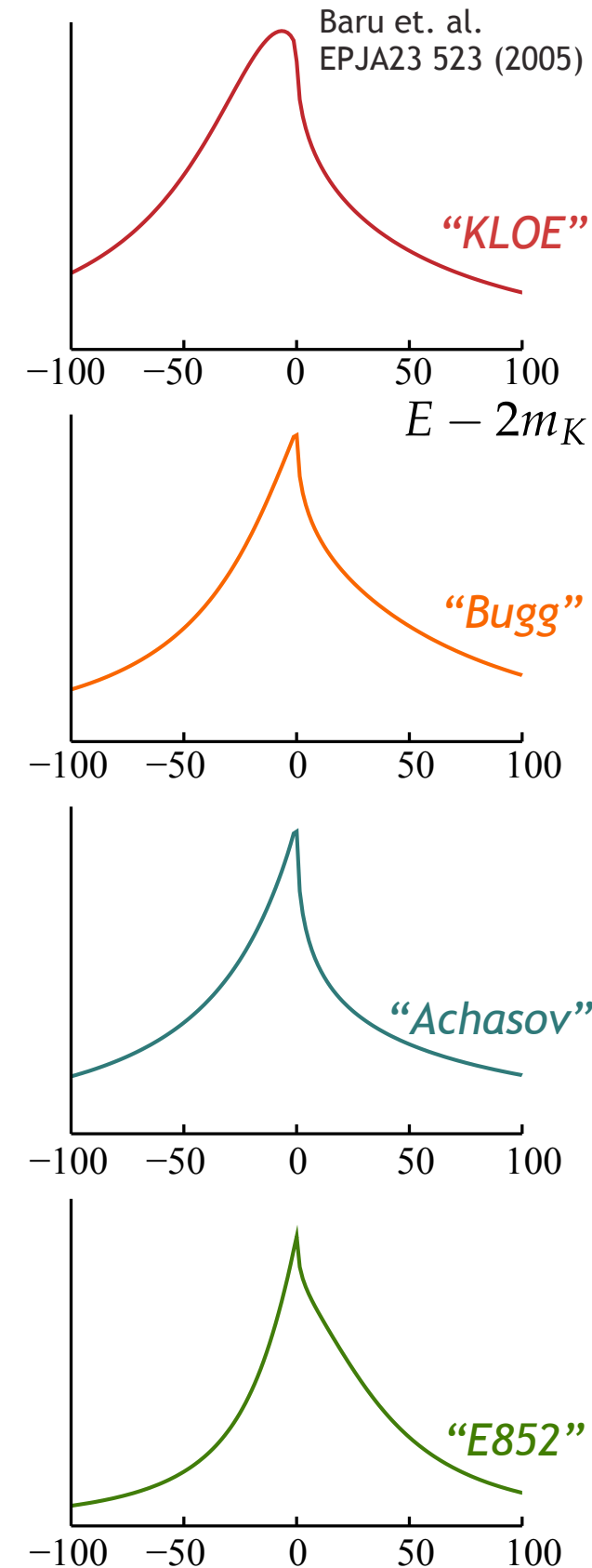
- sharp experimental enhancement at $K\bar{K}$ threshold

- usually in ‘less-simple’ production processes

$$\text{e.g. } p\bar{p} \rightarrow \pi\pi\eta$$

$$\phi \rightarrow \gamma\pi\eta$$

- amplitude models typically give $\frac{g^2(K\bar{K})}{g^2(\pi\eta)} \sim 1$



$\pi\eta/K\bar{K}$ scattering and the $a_0(980)$

46

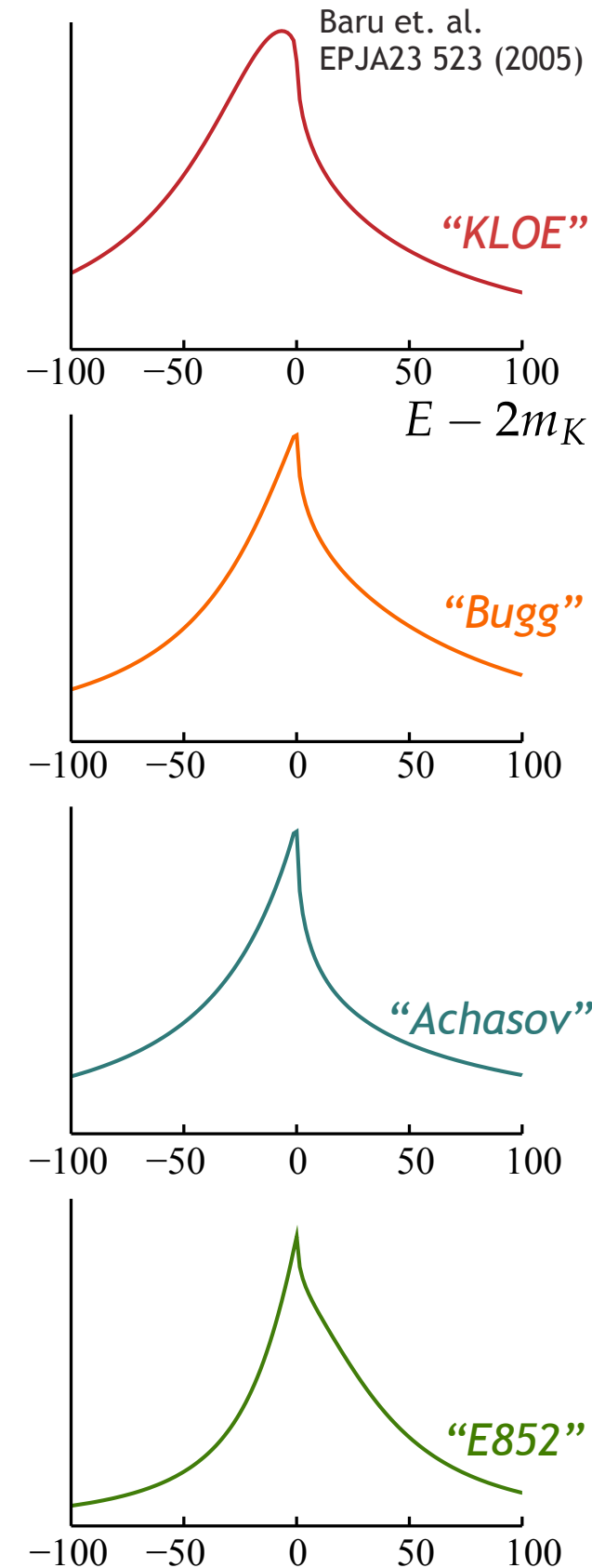
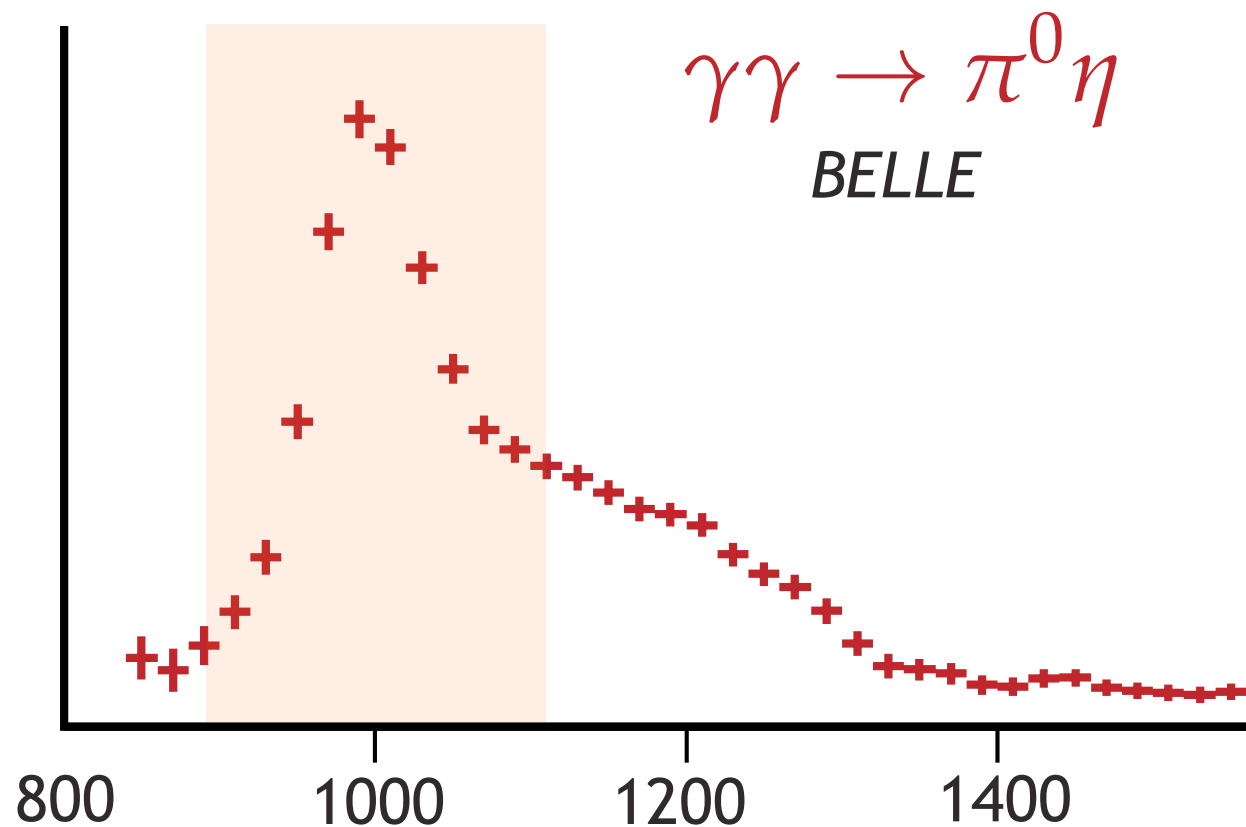
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has a ‘partner’ resonance, $f_0(980)$, in $\pi\pi$ at $K\bar{K}$ threshold

- often argued that these states don’t fit into the $q\bar{q}$ picture

hidden strange tetraquarks ?

$K\bar{K}$ molecules ?

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Progress of Theoretical Physics Supplement No. 168, 2007

Can Experiment Distinguish Tetraquark Scalars, Molecules and $\bar{q}q$ Mesons?

Michael PENNINGTON*)

*Institute for Particle Physics Phenomenology,
Durham University, Durham DH1 3LE, UK*

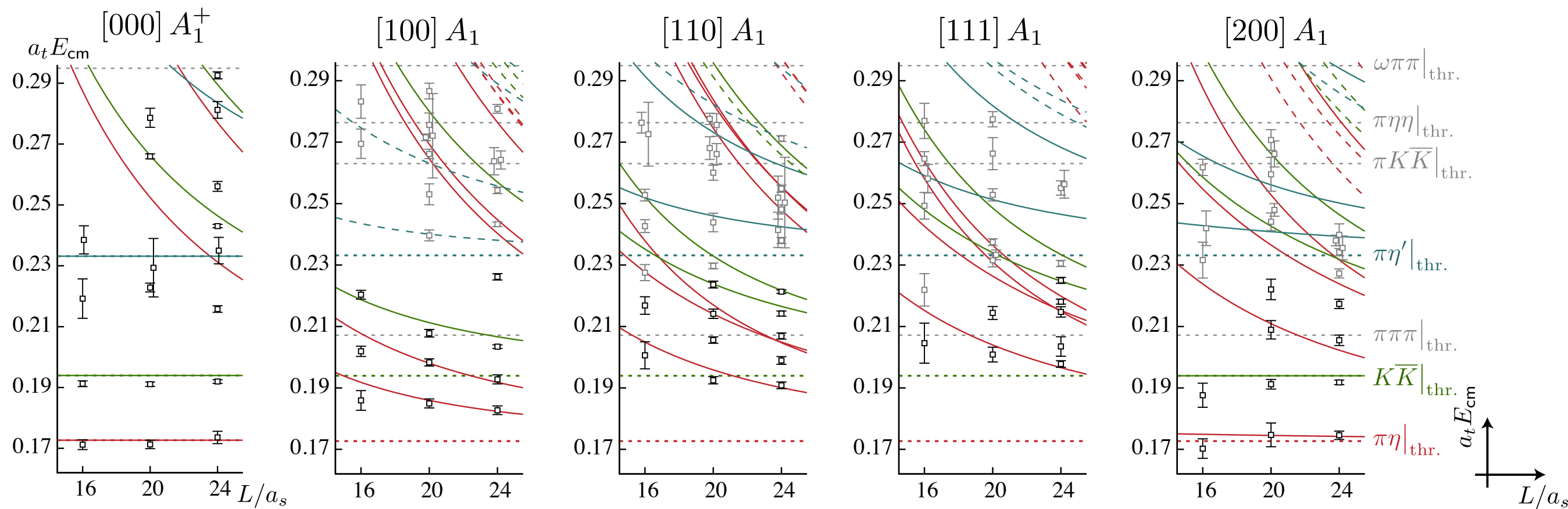
... and can theory ?

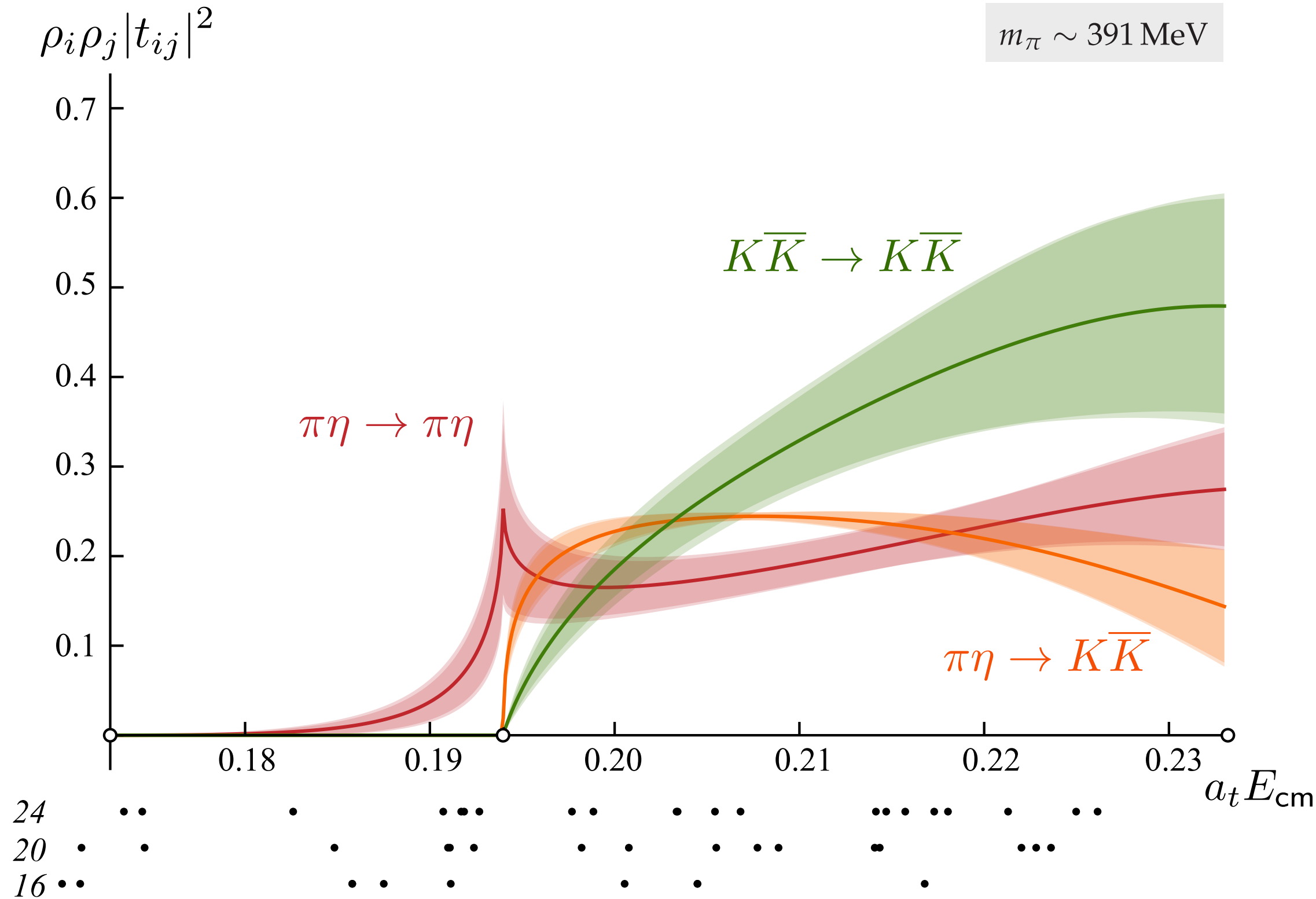
$\pi\eta/K\bar{K}/\pi\eta'$ lattice QCD spectra

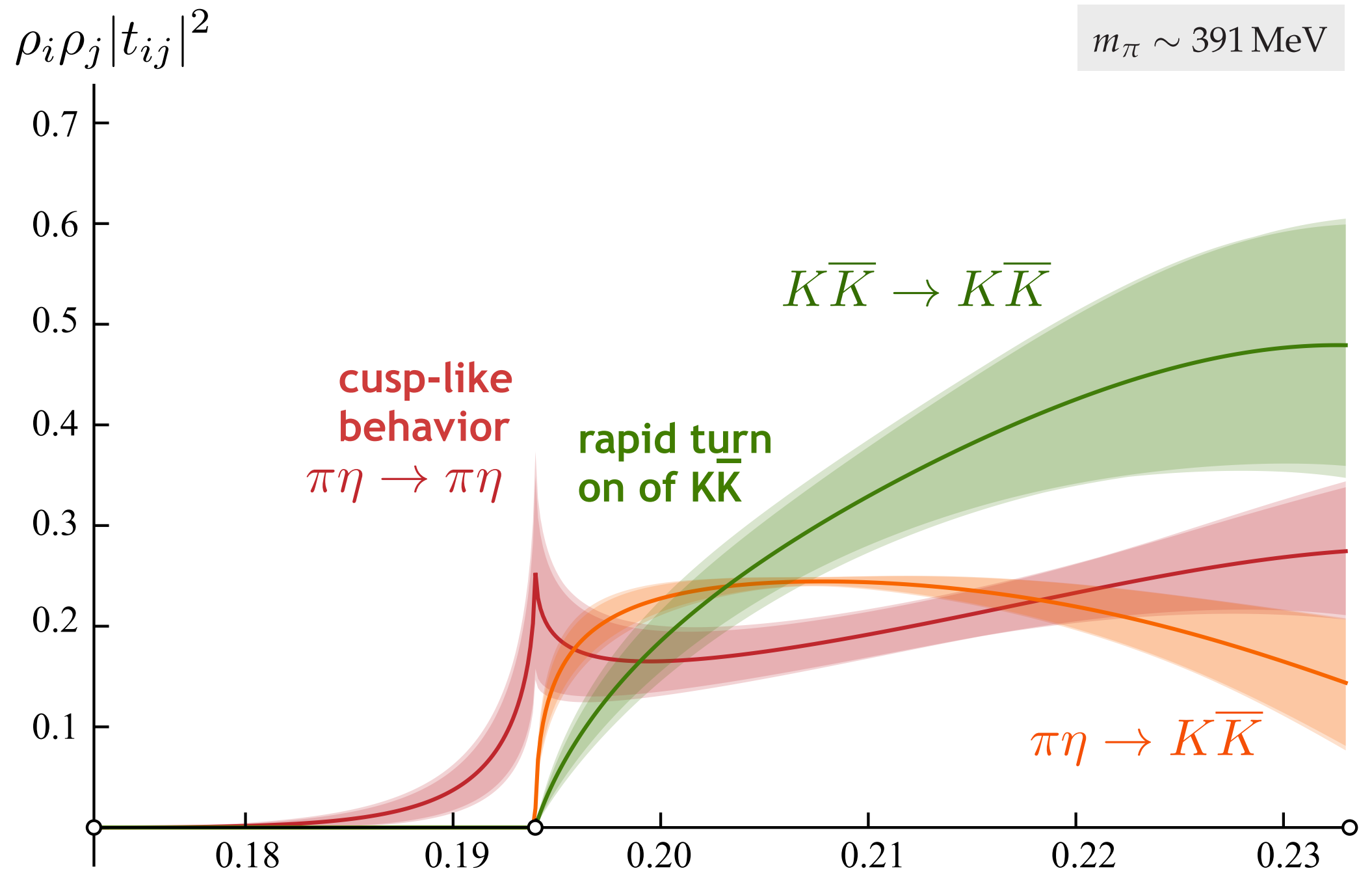
$m_\pi \sim 391$ MeV

48

- used a basis of $\pi\eta/K\bar{K}/\pi\eta'$ like operators and $\bar{\psi}\Gamma\overleftrightarrow{D}\dots\overleftrightarrow{D}\psi$

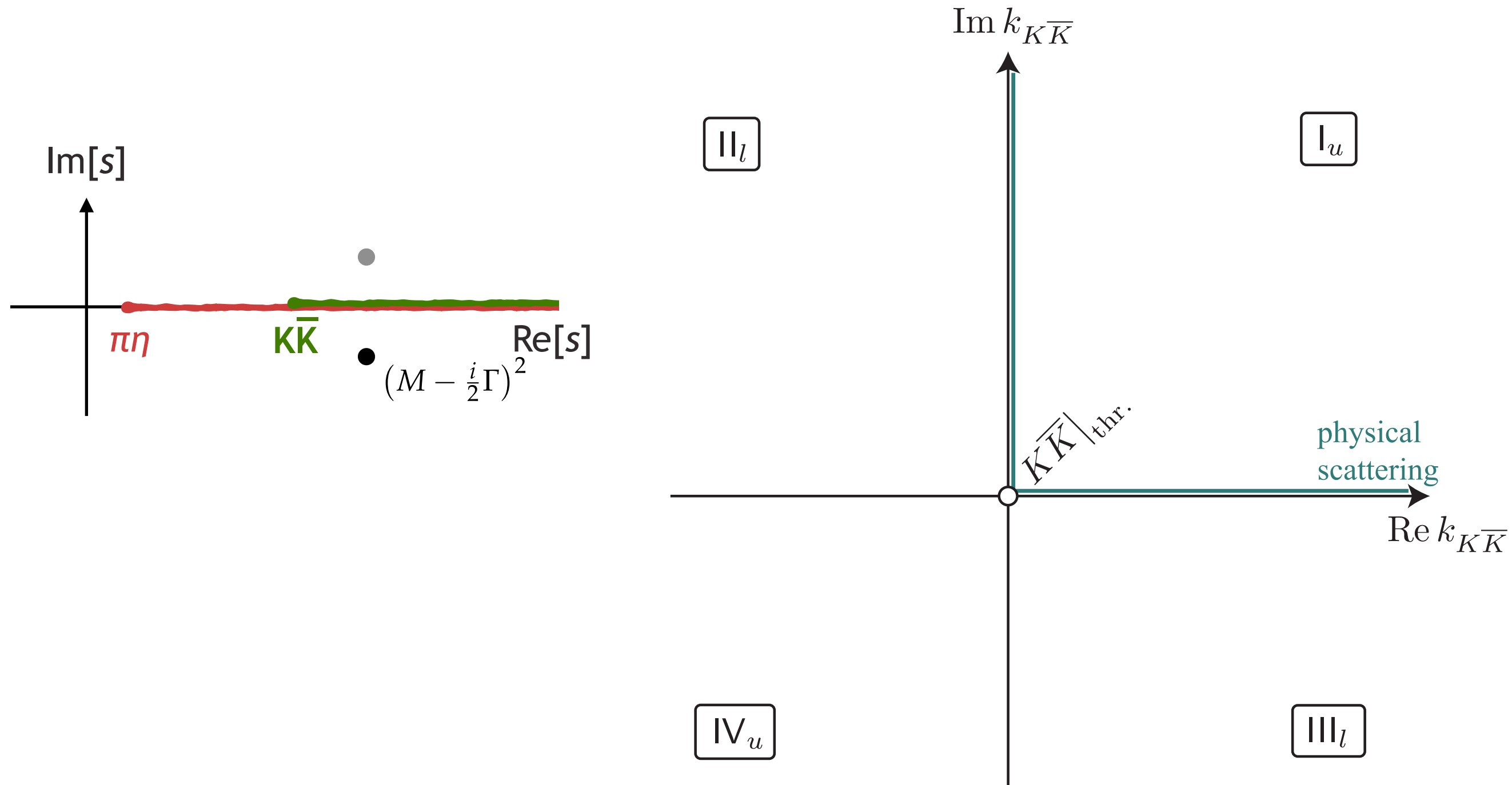






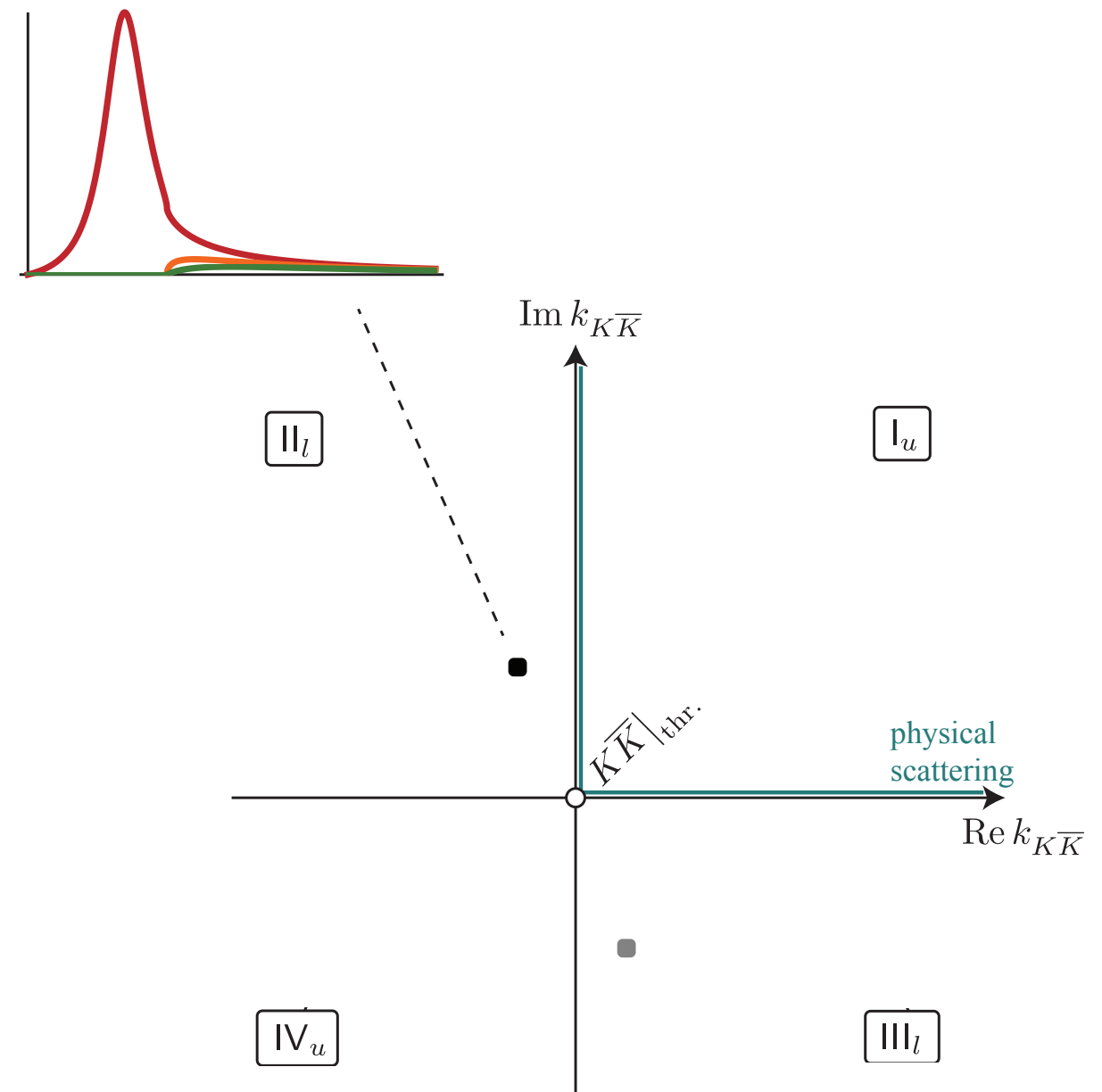
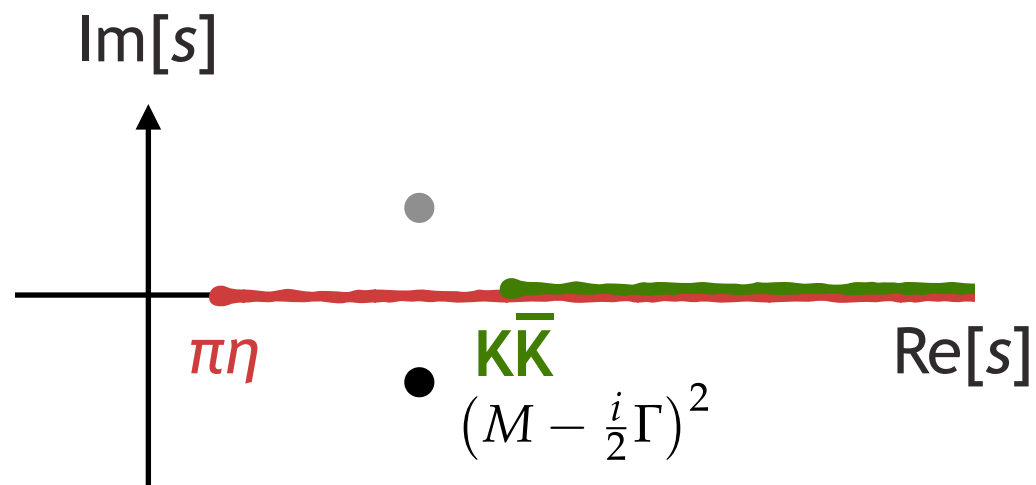
- such a strong effect at threshold likely indicates a nearby singularity

- unitarity implies four Riemann sheets in this case



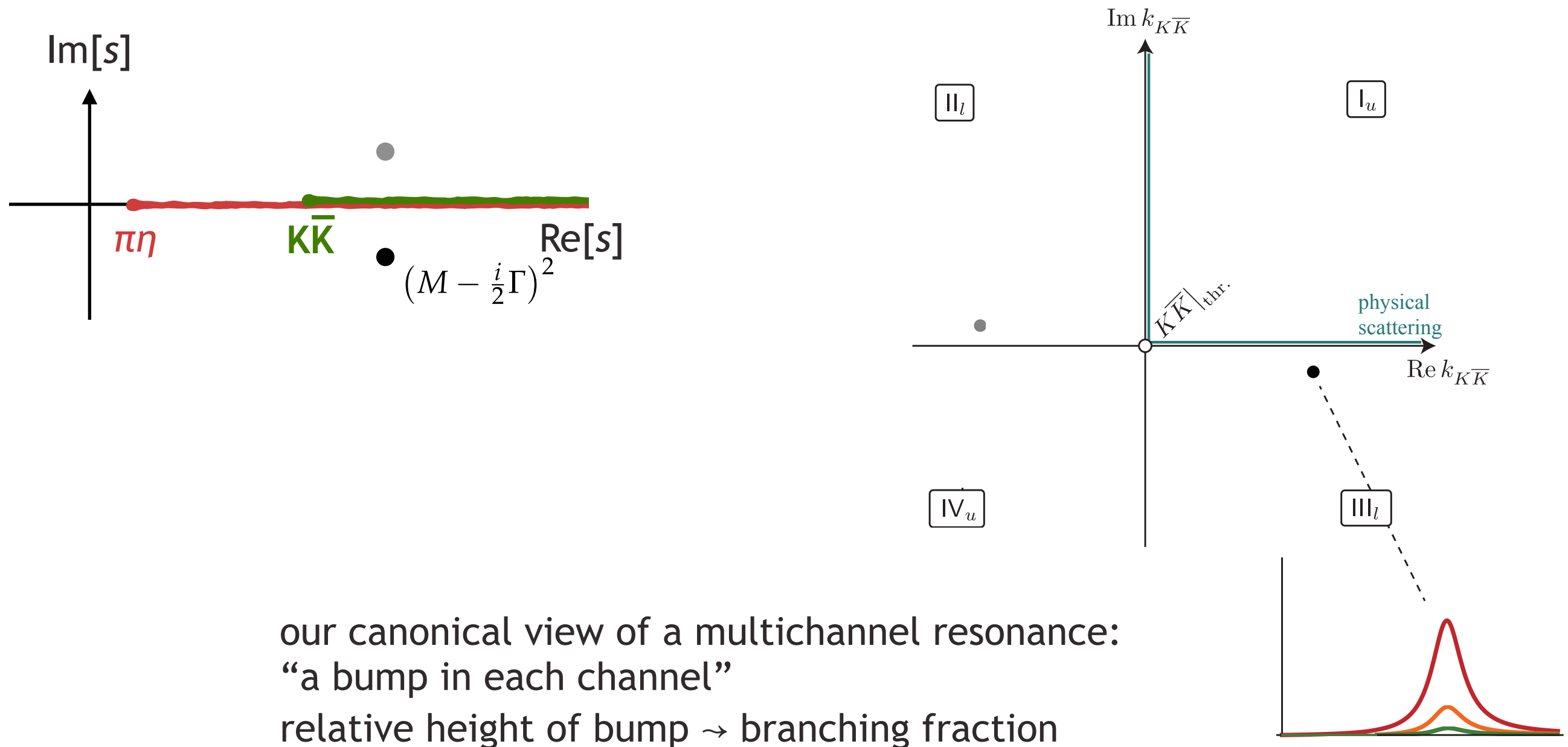
‘Flatté form’

$$t_{ij}(s) = \frac{g_i g_j}{m^2 - s - i g_1^2 \rho_1(s) - i g_2^2 \rho_2(s)}$$

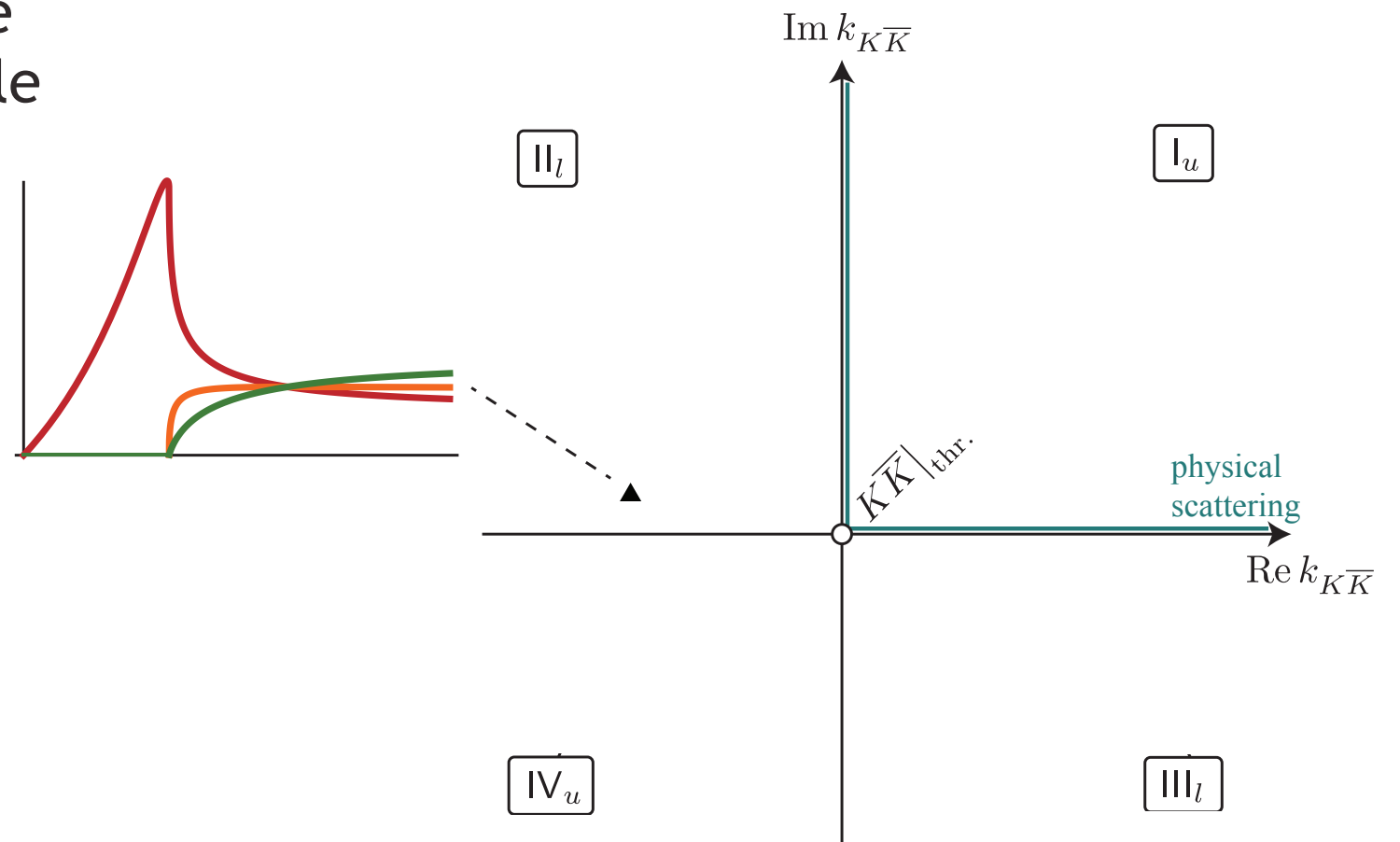


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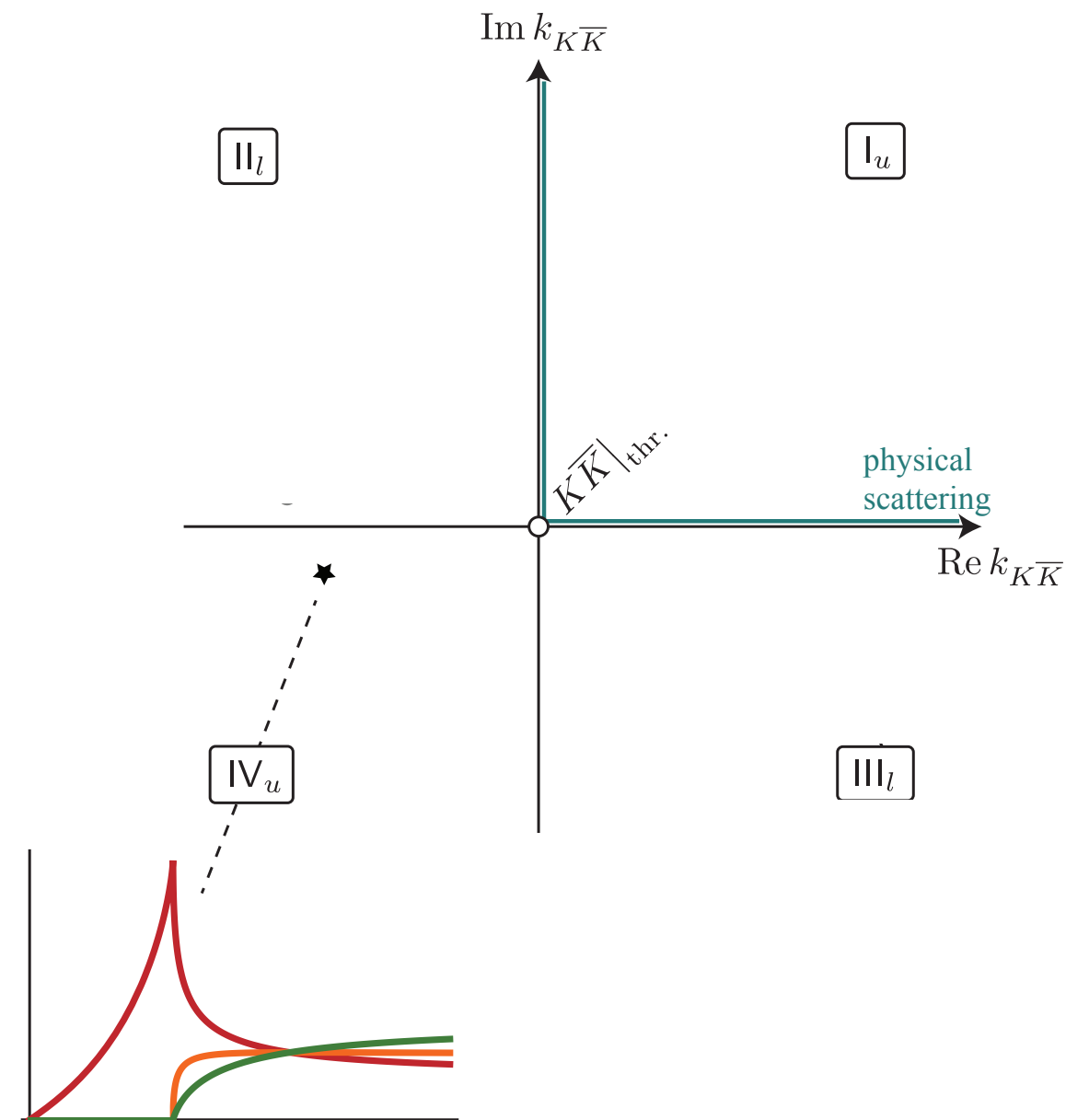
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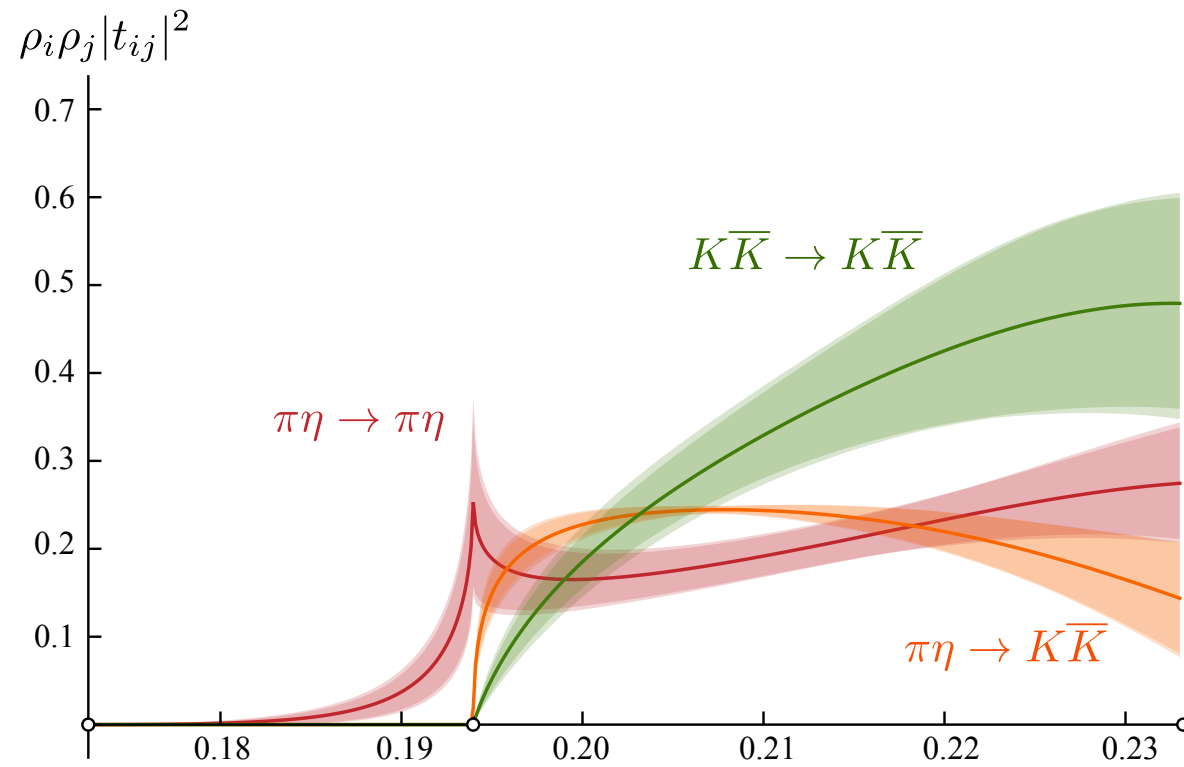


nothing forbids an amplitude with only a single nearby pole



fits to experimental data
tend to exhibit this structure

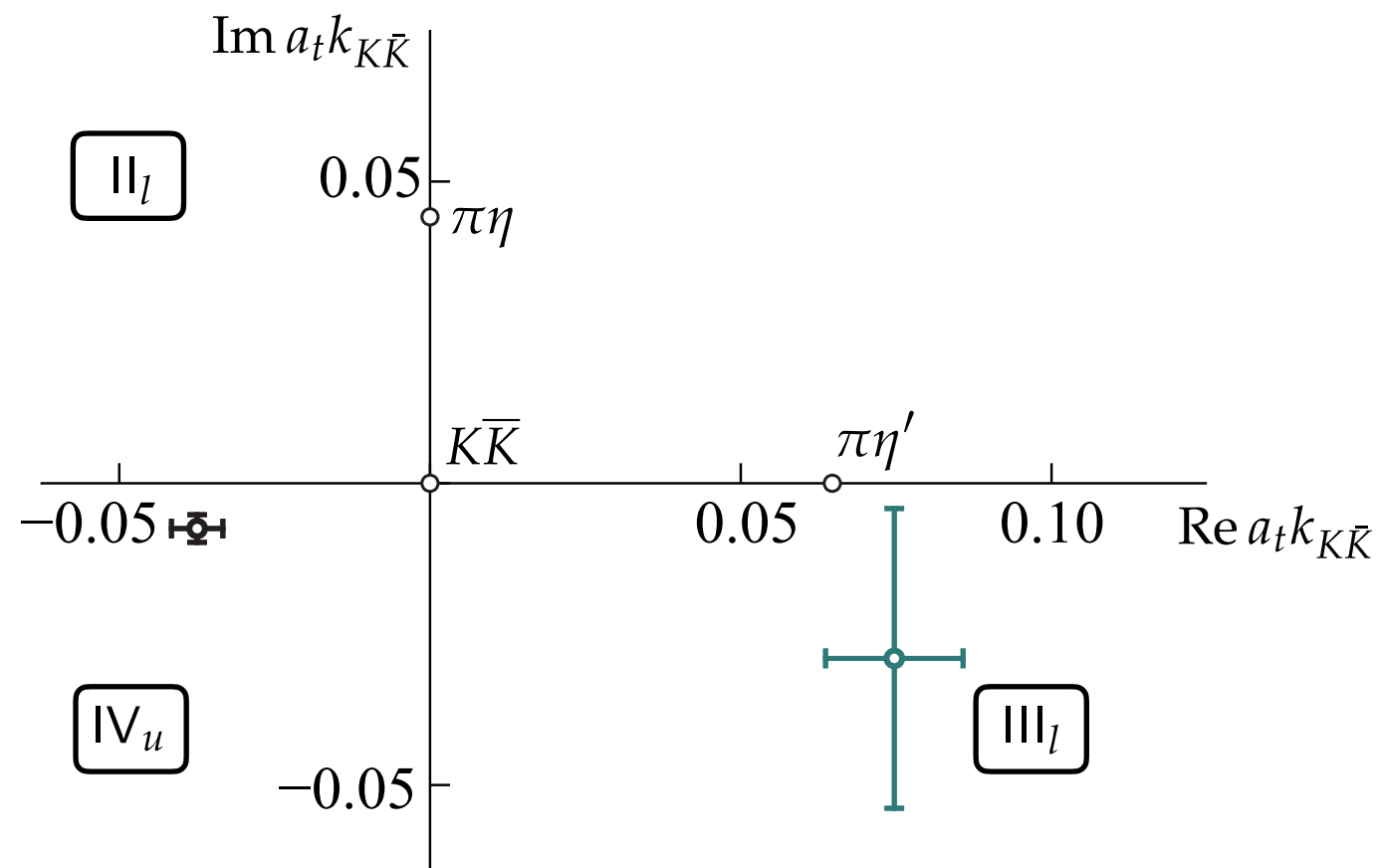
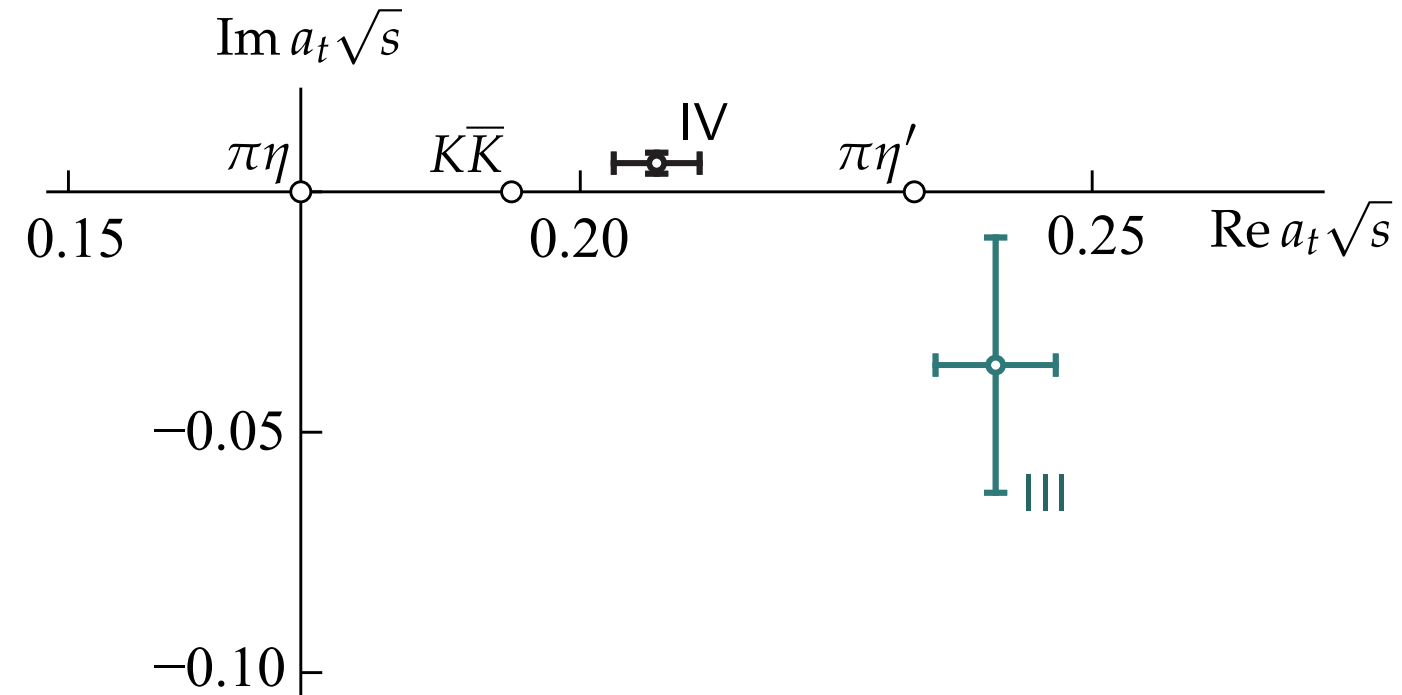
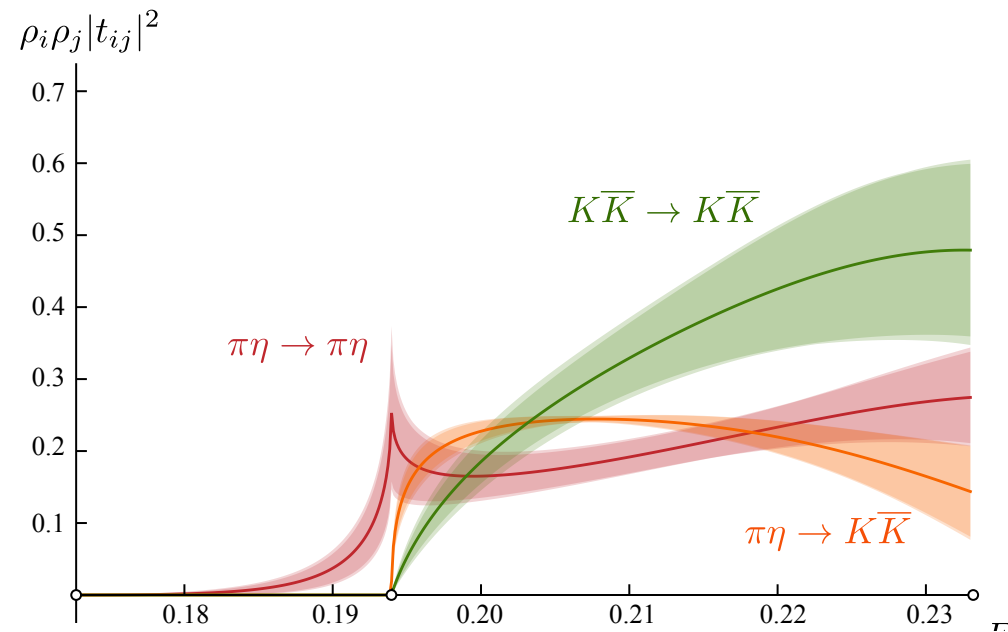




this fit from a K -matrix parameterization

$$t_{ij}^{-1}(E) = K_{ij}^{-1}(E) + \delta_{ij} I_i(E)$$

$$K_{ij}(E) = \frac{g_i g_j}{m^2 - E^2} + \gamma_{ij}$$



bit esoteric isn't it ... ?

Pole counting and resonance classification

D. Morgan

Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, UK

Received 14 January 1992

‘confined’ state coupled to decay continuum → Breit-Wigner like (two poles)

molecular state from long-range potential → one pole

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other molecule diagnostics ?

couple to an external current e.g. $\phi \rightarrow \gamma(\pi\eta, K\bar{K})$
or $(\pi\eta, KK) \rightarrow \gamma(\pi\eta, K\bar{K})$ or other currents ...

and extract form-factors from the residue of the pole

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*examples of the
interesting convergence of
lattice QCD,
S-matrix ideas,
and phenomenology*

- application of lattice methods has **reinvigorated theoretical hadron spectroscopy**
 - have a tool for **phenomenology directly connected to QCD**
 - put ideas of **gluonic excitations** / **hybrid hadrons** on a firmer footing
 - **finite-volume methods** give access to **scattering amplitudes**
 - can now really calculate the physical observables
- the **Hadron Spectrum Collaboration** is the world-leader in these efforts
 - nobody else is doing ‘spectrum’ calculations of this scope

scattering

scalar mesons at lower m_π

axial and other
mesons in e.g. $\pi\rho$, $\pi\omega$

hybrid meson decays at heavier
quark masses

simple baryon resonances

more photon transitions

three-body scattering

⋮

phenomenology

tetraquark operator basis

molecules ? pole distributions ?
form-factors ?

⋮

and haven't mentioned **charmonium** sector ...
XYZ's ...

- 2009 dynamical anisotropic lattices, distillation
- 2010 highly excited isovector meson spectrum
- 2011 highly excited isoscalar meson spectrum
highly excited baryon spectrum
phenomenology of hybrid mesons
- 2012 hybrid baryon spectrum
 $\pi\pi$ scattering, isospin=2
highly excited charmonium spectrum
- 2013 $\pi\pi$ scattering, isospin=1, ρ resonance
coupled-channel formalism
- 2014 coupled-channel $\pi K, \eta K$ scattering
- 2015 excited meson radiative transitions
 $\gamma\pi \rightarrow \pi\pi$ and the $\rho \rightarrow \pi\gamma$ transition
- 2016 (to appear) coupled-channel $\pi\eta, K\bar{K}$ scattering

JEFFERSON LAB

Jozef Dudek
Robert Edwards
Balint Joo
David Richards

TRINITY COLLEGE, DUBLIN

Mike Peardon
Sinead Ryan

TATA, MUMBAI

Nilmani Mathur

CAMBRIDGE UNIVERSITY

Christopher Thomas

U. OF MARYLAND

Steve Wallace

MESON SPECTRUM

PRL103 262001 (2009) $I = 1$
PRD82 034508 (2010) $I = 1, K^*$
PRD83 111502 (2011) $I = 0$
JHEP07 126 (2011) $c\bar{c}$
PRD88 094505 (2013) $I = 0$
JHEP05 021 (2013) D, D_s

BARYON SPECTRUM

PRD84 074508 (2011) $(N, \Delta)^*$
PRD85 054016 (2012) $(N, \Delta)_{\text{hyb}}$
PRD87 054506 (2013) $(N \dots \Xi)^*$
PRD90 074504 (2014) Ω_{ccc}^*
PRD91 094502 (2015) Ξ_{cc}^*

“TECHNOLOGY”

PRD79 034502 (2009) lattices
PRD80 054506 (2009) distillation
PRD85 014507 (2012) $\vec{p} > 0$

HADRON SCATTERING

PRD83 071504 (2011) $\pi\pi I = 2$
PRD86 034031 (2012) $\pi\pi I = 2$
PRD87 034505 (2013) $\pi\pi I = 1, \rho$
PRL113 182001 (2014) $\pi K, \eta K$
PRD91 054008 (2015) $\pi K, \eta K$
PRD92 094502 (2015) $\pi\pi, K\bar{K}$

MATRIX ELEMENTS

PRD91 114501 (2015) $M' \rightarrow \gamma M$
PRD90 014511 (2014) f_{π^*}
PRL115 242001 (2015) $\gamma^* \pi \rightarrow \pi\pi$