UNIVERSITA' DEGLI STUDI DI ROMA TOR VERGATA

Differential cross section calculation for $ep \to e'K^+\Lambda$ electroproduction process adding a hybrid contribution at amplitude level

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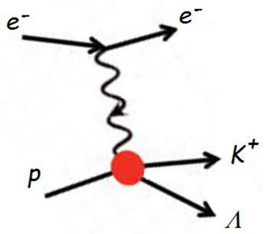
Contents

1 Calculation of $\langle\lambda_f|\,T_r\,|\lambda_\gamma\lambda_p\rangle$

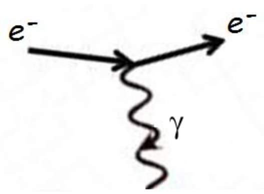
Our task is to achieve a full description of the $\langle\lambda_f|\,T_r\,|\lambda_\gamma\lambda_p\rangle$ resonant amplitude of a single hybrid resonance, since this is the term that we want to add to the $M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$ of Eq. 15 of article [1] to simulate the presence of an hybrid resonance in an electroproduction process.
In Sec. 1.1 the kinematics of $ep \to K^+\Lambda e'$ electroproduction process is described and then in Sec.1.2 the $\langle\lambda_f|\,T_r\,|\lambda_\gamma\lambda_p\rangle$ is calculated for $J^P = \frac{1}{2}^+$ and for $J^P = \frac{3}{2}^+$.

1.1 Kinematics of electroproduction

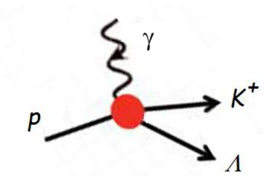
We are considering the electroproduction process



It can be divided into two parts, a leptonic one, with the original electron that scatters producing a photon,



and in an hadronic one, with the same photon being absorbed by a proton in the target, and with this composite state decaying into a $K^+\Lambda$ final state,



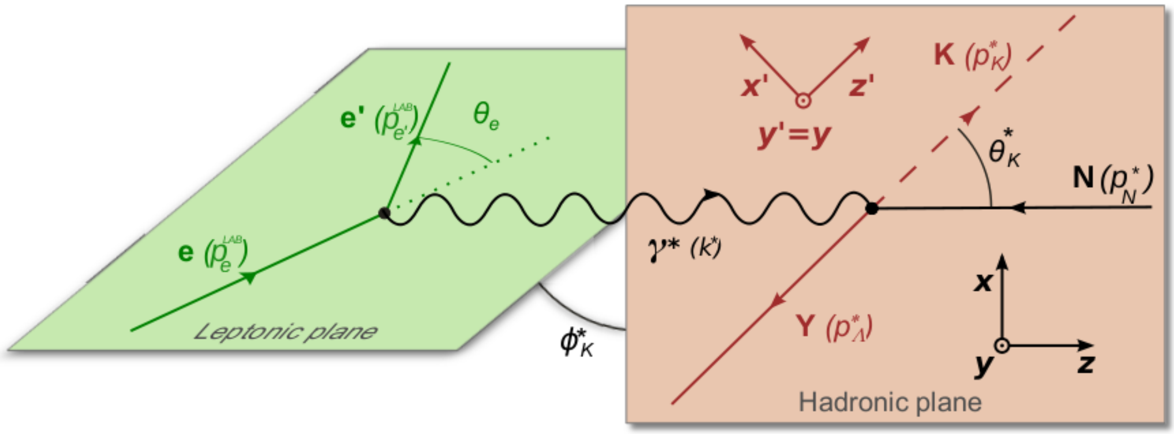
The common way to describe this double system is to represent the leptonic part in the laboratory frame and the hadronic one in the center of mass (CMS) frame (where the proton-photon composite state is at rest), as it is possible to see in Fig. 1.1.
In CMS frame we have:

$$k^* = \begin{pmatrix} \omega^* \\ \mathbf{k}^* \end{pmatrix}, p_N^* = \begin{pmatrix} E_P^* \\ \mathbf{p}_P^* \end{pmatrix}, p_K^* = \begin{pmatrix} E_K^* \\ \mathbf{p}_K^* \end{pmatrix}, p_\Lambda^* = \begin{pmatrix} E_\Lambda^* \\ \mathbf{p}_\Lambda^* \end{pmatrix} = \begin{pmatrix} E_\Lambda^* \\ -\mathbf{p}_K^* \end{pmatrix} \tag{1}$$

And in laboratory frame we have:

$$p_e^{LAB} = \begin{pmatrix} E_e^{LAB} \\ \mathbf{p}_e^{LAB} \end{pmatrix}, p_{e'}^{LAB} = \begin{pmatrix} E_{e'}^{LAB} \\ \mathbf{p}_{e'}^{LAB} \end{pmatrix}, k^{LAB} = \begin{pmatrix} \omega_{LAB} \\ \mathbf{k}_{LAB} \end{pmatrix}, p_N^{LAB} = \begin{pmatrix} m_P \\ 0 \end{pmatrix}, p_K^{LAB} = \begin{pmatrix} E_K^{LAB} \\ \mathbf{p}_K^{LAB} \end{pmatrix}, p_\Lambda^{LAB} = \begin{pmatrix} E_\Lambda^{LAB} \\ \mathbf{p}_\Lambda^{LAB} \end{pmatrix} \tag{2}$$

Several considerations are necessary. First of all the number of degrees of freedom of the system. The initial state is completely determined, with the virtual photon going in the z-axis direction. The final state has all masses



fixed so we have 3x3=9 (electron, lambda and kaon tri-momenta) free degrees of freedom. But we have four constraints, from the conservation of four-momenta, so 9-4=5 d.o.f to describe the system. In order to have the z-axis coincident with the photon direction, we have to rotate our system of an angle Φ that fixes the orientation of the leptonic plane. With this condition the two frames are connected with a Lorentz transformation (a translation) along z-axis. The other 4 d.o.f. can be: Q^2, W, θ_K^* and ϕ_K^*. In the following we will demonstrate this statement.

1.1.1 Lorentz transformation

A Lorentz transformation is a coordinate transformation between two coordinate frames that move at constant velocity relative to each other.
For a transformation in a generic direction between two systems with parallel axis, without rotations, we have

$$\begin{bmatrix} ct' \\ \mathbf{r} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\boldsymbol{\beta}^T \\ -\gamma\boldsymbol{\beta} & \mathbf{I} + (\gamma-1)\boldsymbol{\beta}\boldsymbol{\beta}^T/\beta^2 \end{bmatrix} \begin{bmatrix} ct \\ \mathbf{r} \end{bmatrix} \tag{3}$$

with $\mathbf{I}$ the identity matrix, $\beta = \dfrac{\boldsymbol{v}}{c}$ is

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c} = \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix} = \frac{1}{c}\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{4}$$

and $\beta^T = \dfrac{\boldsymbol{v}^T}{c}$ is its traspost, a row vector:

$$\boldsymbol{\beta}^T = \frac{\boldsymbol{v}^T}{c} = \begin{bmatrix} \beta_x & \beta_y & \beta_z \end{bmatrix} = \frac{1}{c}\begin{bmatrix} v_x & v_y & v_z \end{bmatrix} \tag{5}$$

In our case the translation is for construction along z-axis, so the Lorentz transformations reduces to:

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} \tag{6}$$

i.e.

$$\begin{aligned} t' &= \gamma(t - \frac{v}{c^2}z) \\ x' &= x \\ y' &= y \\ z' &= \gamma(z - vt) \end{aligned} \tag{7}$$

The first component of the four-momentum is the energy, and the other three components are the three-momentum, so the Lorentz transformation from the laboratory ($\boldsymbol{P}_{tot} = [\sum_k E_k, \sum_k \vec{p_k}]$) to the CMS ($\boldsymbol{P}^*_{tot} = [\sum_k E^*_k, \vec{0}] = [W, \vec{0}]$, with W the invariant mass of the $K^+\Lambda$ system) frame is:

$$\begin{bmatrix} W \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma_{CMS} & 0 & 0 & -\beta_{CMS}\gamma_{CMS} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta_{CMS}\gamma_{CMS} & 0 & 0 & \gamma_{CMS} \end{bmatrix} \begin{bmatrix} \sum_k E_k \\ 0 \\ 0 \\ |\sum_k \vec{p_k}| \end{bmatrix} \tag{8}$$

i.e.

$$\begin{aligned} W =& \gamma_{CMS} \sum_k E_k - \beta_{CMS}\gamma_{CMS} |\sum_k \vec{p_k}| \\ 0 =& -\beta_{CMS}\gamma_{CMS} \sum_k E_k + \gamma_{CMS} |\sum_k \vec{p_k}| \end{aligned} \tag{9}$$

From the second equation we obtain

$$\beta_{CMS} = \frac{|\sum_k \vec{p_k}|}{\sum_k E_k} = \frac{|\vec{p}^{lab}_{tot}|}{E^{lab}_{tot}} = \frac{|\mathbf{k}_{LAB}|}{\omega_{LAB} + m_P} \tag{10}$$

from which we also obtain γ_{CMS}

$$\gamma_{CMS} = \frac{1}{\sqrt{1-\beta^2_{CMS}}} = \frac{1}{\sqrt{1 - \dfrac{\mathbf{k}^2_{LAB}}{(\omega_{LAB}+m_P)^2}}} = \frac{\omega_{LAB} + m_P}{\sqrt{(\omega_{LAB}+m_P)^2 - \mathbf{k}^2_{LAB}}} = \frac{E^{LAB}_{TOT}}{W} \tag{11}$$

And we finally achieve the relationship between the laboratory and CMS photon three-vector:

$$\begin{aligned} \mathbf{k}^* =& \gamma_{CMS}(\mathbf{k}_{LAB} - \boldsymbol{\beta}_{CMS}\omega_{LAB}) = \\ =& \frac{\omega_{LAB} + m_P}{\sqrt{(\omega_{LAB}+m_P)^2 - \mathbf{k}^2_{LAB}}} (\mathbf{k}_{LAB} - \frac{\mathbf{k}_{LAB}}{\omega_{LAB} + m_P}\omega_{LAB}) = \mathbf{k}_{LAB}(\frac{m_P}{W}) \end{aligned} \tag{12}$$

and energy

$$\omega^* = \frac{W^2 - m_P^2 - Q^2}{2W} \tag{13}$$

where $Q^2 = -(k^{LAB})^2 = -q^2$.

1.1.2 Leptonic plane

As already said the leptonic plane, that contains the electron scattering in $\gamma e'$, is described in the laboratory frame. We use the approximation

$$m_e << |\mathbf{p}_e| \Rightarrow E_e = |\mathbf{p}_e| \tag{14}$$

The relationship between the quadri-momenta of e, e' and γ is:

$$p_\gamma = p_e - p_{e'} \tag{15}$$

From this equation we can obtain the formula that links the Q^2, to the energies of the electron before and after the scattering, and to the scattering angle:

$$\begin{aligned} q^2 =& \omega^2_{LAB} - \mathbf{k}^2_{LAB} = (E_e - E_{e'})^2 - (\mathbf{p}_e - \mathbf{p}'_e)^2 = -2E_eE_{e'} + 2E_eE_{e'}cos\theta_e = \\ =& -2E_eE_{e'}(1-cos\theta_e) = -4E_eE_{e'}sin^2\frac{\theta_e}{2} \Rightarrow Q^2 = -q^2 = 4E_eE_{e'}sin^2\frac{\theta_e}{2} \end{aligned} \tag{16}$$

But Q^2 can also be expressed as

$$Q^2 = -q^2 = -(\omega_{LAB}^2 - \mathbf{k}_{LAB}^2) = |\mathbf{k}_{LAB}|^2 - \omega_{LAB}^2 \tag{17}$$

In the laboratory frame the relationship holds:

$$\begin{aligned} W^2 &= (m_P + \omega_{LAB})^2 - (\mathbf{k}_{LAB})^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - |\mathbf{k}_{LAB}|^2 \\ &\Rightarrow |\mathbf{k}_{LAB}|^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - W^2 \end{aligned} \tag{18}$$

So, substituting 18 in 17, we have:

$$\begin{aligned} Q^2 &= |\mathbf{k}_{LAB}|^2 - \omega_{LAB}^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - W^2 - \omega_{LAB}^2 \\ &= m_P^2 + 2m_P\omega_{LAB} - W^2 \end{aligned} \tag{19}$$

and we can use this result to obtain the energy of the photon:

$$\omega_{LAB} = \frac{W^2 + Q^2 - m_P^2}{2m_P} \tag{20}$$

In this way we also fixed the value of $E_{e'}$ since

$$p_\gamma = p_e - p_{e'} \Rightarrow \omega_{LAB} = E_e - E_{e'} \tag{21}$$

So, knowing W and Q^2, we also know $E_{e'}$ and the scattering angle θ_e. At this point we know all in the leptonic plane, as the components of the final electron are

$$p_{e'} = \begin{pmatrix} E_{e'} \\ |\boldsymbol{p_{e'}}|sin\theta_e cos\Phi \\ |\boldsymbol{p_{e'}}|sin\theta_e sin\Phi \\ |\boldsymbol{p_{e'}}|cos\theta_e \end{pmatrix} \tag{22}$$

and from the quadri-momentum conservation we also know k^{LAB}.

1.1.3 Hadronic plane

In the hadronic plane, which contains the photon, the proton, K^+ and Λ, we use the CMS frame, which presents a particular geometry with

$$\begin{aligned} \theta_\Lambda^* &= \pi - \theta_K^* \\ \phi_\Lambda^* &= \pi + \phi_K^* \end{aligned} \tag{23}$$

Both θ_K^* and ϕ_K^* are independent variables of the system. It is important to note that ϕ_K^* is, for construction, the angle between the leptonic and the hadronic plane.
We obtain the modulus of the three-momentum for K^+ and Λ using the formula for the two-body decay in the CMS frame:

$$|\mathbf{p}_K^*| = |\mathbf{p}_\Lambda^*| = \frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W} \tag{24}$$

where m_K is 0.4937 GeV/c^2 and m_Λ is 1.116 GeV/c^2.
At this point we have completely defined the kinematics of all the system and we can go on with the determination of $\langle\lambda_f|\, T_r \,|\lambda_\gamma\lambda_p\rangle$.

1.2 Results: $\langle\lambda_f|\, T_r \,|\lambda_\gamma\lambda_p\rangle$

From the Letter of Intent [2], we know that a relativistic Breit-Wigner (BW) ansatz gives the following expression for the $\langle\lambda_f|\, T_r \,|\lambda_\gamma\lambda_p\rangle$ resonant amplitude of a single hybrid resonance in the helicity representation:

$$M_{\lambda_\gamma}^{\lambda_p\lambda_Y} = \langle\lambda_f|\, T_r \,|\lambda_\gamma\lambda_p\rangle = \frac{\langle\lambda_f|\, T_{dec} \,|\lambda_R\rangle \langle\lambda_R|\, T_{em} \,|\lambda_\gamma\lambda_p\rangle}{M_r^2 - W^2 - i\Gamma_r M_r}, \tag{25}$$

where M_r and Γ_r are the resonance mass and total width respectively.
The matrix elements $\langle\lambda_f|\, T_{dec}\, |\lambda_R\rangle$ and $\langle\lambda_R|\, T_{em}\, |\lambda_\gamma\lambda_p\rangle$ are the electromagnetic production and hadronic decay amplitudes of the N^* with helicity $\lambda_R = \lambda_\gamma - \lambda_p$, in which λ_γ and λ_p stand for the helicities of the photon and proton in the initial state, and λ_f represents the helicity of final-state hadron in the N^* decays. The hadronic decay amplitudes $\langle\lambda_f|\, T_{dec}\, |\lambda_R\rangle$ are related to the Γ_{λ_f} partial hadronic decay widths of the N^* to KY final states f of helicity $\lambda_f = \lambda_Y$ by:

$$\langle\lambda_f|\, T_{dec}\, |\lambda_R\rangle = \langle\lambda_f|\, T^{J_r}_{dec}\, |\lambda_R\rangle\, d^{J_r}_{\mu\nu}(cos\theta^*_K)e^{i\mu\phi^*_K}, \tag{26}$$

with $\mu = \lambda_R$ and $\nu = -\lambda_Y$, and

$$\langle\lambda_f|\, T^{J_r}_{dec}\, |\lambda_R\rangle = \frac{2\sqrt{2\pi}\sqrt{2J_r+1}M_r\sqrt{\Gamma_{\lambda_f}}}{\sqrt{p^r_i}}\sqrt{\frac{p^r_i}{p_i}}. \tag{27}$$

p^r_i and p_i are the magnitudes of the three-momenta of the final state K for the $N^* \to K\Lambda$ decay (i=1) or for the $N^* \to K\Sigma$ decay (i=2), evaluated at $W = M_r$ and at the running W , respectively. The variables θ_K, ϕ^*_K are the CMS polar and azimuthal angles for the final kaon (already defined in Eq. 23), and J_r stands for the N^* spin. The final state Λ or Σ baryons can only be in the helicity states $\lambda_f = \pm\frac{1}{2}$. The hadronic decay amplitudes $\langle\lambda_f|\, T^{J_r}_{dec}\, |\lambda_R\rangle$ with $\lambda_f = \pm\frac{1}{2}$ are related by P-invariance, which imposes the absolute values for both amplitudes to be the same. Therefore, the hybrid state partial decay widths to the $K\Lambda$ and $K\Sigma$ final states Γ_{λ_f} can be estimated as:

$$\Gamma_{\lambda_f} = \frac{1}{2}\Gamma_r 0.05, \tag{28}$$

where the factor 0.05 reflects the adopted 5% BF for hybrid baryon decays to the KY final state. The following relationships between the transition amplitudes $\langle\lambda_R|\, T_{em}\, |\lambda_\gamma\lambda_p\rangle$ and the $\gamma_\nu NN^*$ electrocouplings were obtained in the paper [3]:

$$\begin{aligned} \langle\lambda_R|\, T_{em}\, |\lambda_\gamma\lambda_p\rangle &= \frac{W}{M_r}\sqrt{\frac{8M_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}}A_{1/2,3/2}(Q^2), \\ &\text{with } |\lambda_\gamma - \lambda_p| = \frac{1}{2}, \frac{3}{2} \text{ for transverse photons, and} \\ \langle\lambda_R|\, T_{em}\, |\lambda_\gamma\lambda_p\rangle &= \frac{W}{M_r}\sqrt{\frac{16M_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}}S_{1/2}(Q^2), \\ &\text{for longitudinal photons,} \end{aligned} \tag{29}$$

where q_γ is the absolute value of the initial photon three-momentum of virtuality $Q^2 > 0$ with $q_\gamma = \sqrt{Q^2 + \omega^{*2}}$ and ω^* the photon energy in the CMS frame at the running W

$$\omega^* = \frac{W^2 - Q^2 - M_N^2}{2W}. \tag{30}$$

as already saw in Eq. 13.
The $q_{\gamma,r}$ value is then computed from Eq. 30 with W=M_r. We will investigate hybrid baryon states with spin-parities $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$. Electroexcitation of the former state can be described by two electrocouplings $A_{1/2}$ and $S_{1/2}$, while the latter should be described by three electrocouplings, $A_{1/2}$, $S_{1/2}$ and $A_{3/2}$. Information on the expected Q^2-evolution of the aforementioned electrocouplings for hybrid states is, to the best of our knowledge, currently not available. We will vary the hybrid baryon electrocouplings to determine their minimal absolute values above which the signal from the hybrid baryon can be observed in the difference between the angular distributions with and without hybrid baryon contributions. These studies will be independently done in each Q^2 bin of the proposed experiment.
The following restrictions will be imposed in the variation of the hybrid baryon electrocouplings, assuming positive values of all electrocouplings.

- **the hybrid baryon of $\frac{3}{2}^+$ spin-parity:** Three electrocouplings $A_{1/2}$, $S_{1/2}$ and $A_{3/2}$ will be computed varying the positive parameter A as:

$$\begin{aligned} A_{1/2} &= A, \\ S_{1/2} &= AQ, \\ A_{3/2} &= A/Q^2, \\ Q &= \sqrt{Q^2} \end{aligned} \tag{31}$$

- **Hybrid baryon of $\frac{1}{2}^+$ spin-parity:** Electrocouplings will be varied under two assumptions: a) $S_{1/2} = 0$ $GeV^{-1/2}$ as predicted by model for the hybrid N(1440)$1/2^+$ resonance, and b) the relations 31 with $A_{3/2} = 0$ $GeV^{-1/2}$ will be employed.

If we substitute Eq. 26 and Eq. 29 in Eq. 25, we have

$$\begin{aligned} \langle\lambda_f| T_r |\lambda_\gamma\lambda_p\rangle &= \frac{\langle\lambda_f| T_{dec} |\lambda_\gamma\lambda_R\rangle\langle\lambda_R| T_{em} |\lambda_\gamma\lambda_p\rangle}{M_r^2 - W^2 - i\Gamma_r M_r} = \\ &= \frac{\langle\lambda_f| T_{dec}^{J_r} |\lambda_R\rangle}{M_r^2 - W^2 - i\Gamma_r M_r} d_{\mu\nu}^{J_r}(cos\theta_K^*)e^{i\mu\phi_K^*}\frac{W}{M_r}\sqrt{\frac{cM_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}}\begin{cases} A_{1/2,3/2}(Q^2),\ c = 8 \\ S_{1/2}(Q^2),\ c = 16 \end{cases} \end{aligned} \tag{32}$$

and if we also substitute Eq. 27,

$$= \frac{\dfrac{2\sqrt{2\pi}\sqrt{2J_r+1}M_r\sqrt{\Gamma_{\lambda_f}}}{\cancel{\sqrt{p_i^r}}}\sqrt{\dfrac{\cancel{p_i^r}}{p_i}}}{M_r^2 - W^2 - i\Gamma_r M_r} d_{\mu\nu}^{J_r}(cos\theta_K^*)e^{i\mu\phi_K^*}\frac{W}{M_r}\sqrt{\frac{cM_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}}\begin{cases} A_{1/2,3/2}(Q^2),\ c = 8 \\ S_{1/2}(Q^2),\ c = 16 \end{cases} \tag{33}$$

and Eq. 28,

$$= \frac{\dfrac{2\sqrt{2\pi}\sqrt{2J_r+1}M_r\sqrt{\frac{1}{2}\Gamma_r 0.05}}{\sqrt{p_i}}}{M_r^2 - W^2 - i\Gamma_r M_r} d_{\mu\nu}^{J_r}(cos\theta_K^*)e^{i\mu\phi_K^*}\frac{W}{M_r}\sqrt{\frac{cM_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}}\begin{cases} A_{1/2,3/2}(Q^2),\ c = 8 \\ S_{1/2}(Q^2),\ c = 16 \end{cases} \tag{34}$$

As already said, we are interested at the cases with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$, so J_r can be $\frac{1}{2}$ or $\frac{3}{2}$.

1.2.1 $J^P = \frac{1}{2}^+$

In this case we have

$$\begin{aligned} &\langle\lambda_f| T_r |\lambda_\gamma\lambda_p\rangle = \\ &= \frac{\dfrac{2\sqrt{2\pi}\sqrt{2}M_r\sqrt{\Gamma_r 0.025}}{\sqrt{p_i}}}{M_r^2 - W^2 - i\Gamma_r M_r} d_{\mu\nu}^{1/2}(cos\theta_K^*)e^{i\mu\phi_K^*}\frac{W}{M_r}\sqrt{\frac{cM_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}}\begin{cases} A_{1/2}(Q^2) = A,\ c = 8 \\ S_{1/2}(Q^2) = AQ,\ c = 16 \end{cases} \end{aligned} \tag{35}$$

Now we need to calculate $\lambda_R = \lambda_\gamma - \lambda_p = \mu$ and $\nu = -\lambda_Y$ for all the possible values of λ_P, λ_Y and λ_γ. The results are reported in 1.
At this point we can express the $M_{\lambda_\gamma}^{\lambda_p\lambda_Y}$ in terms of a factor that doesn't change with λ_P, λ_Y and λ_γ,

$$F_{J=1/2} = \frac{\dfrac{2\sqrt{2\pi}\sqrt{2}M_r\sqrt{\Gamma_r 0.025}}{\sqrt{p_i}}}{M_r^2 - W^2 - i\Gamma_r M_r}\frac{W}{M_r}\sqrt{\frac{cM_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}}\begin{cases} A_{1/2}(Q^2) = A,\ c = 8 \\ S_{1/2}(Q^2) = AQ,\ c = 16 \end{cases} \tag{36}$$

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$\mu = 1/2$ $\nu = -1/2$	$\mu = 1/2$ $\nu = 1/2$	$\mu = 3/2$ $\nu = -1/2$	$\mu = 3/2$ $\nu = 1/2$
0	$\mu = -1/2$ $\nu = -1/2$	$\mu = -1/2$ $\nu = 1/2$	$\mu = 1/2$ $\nu = -1/2$	$\mu = 1/2$ $\nu = 1/2$
-1	$\mu = -3/2$ $\nu = -1/2$	$\mu = -3/2$ $\nu = 1/2$	$\mu = -1/2$ $\nu = -1/2$	$\mu = -1/2$ $\nu = 1/2$

Table 1: Values of $\lambda_R = \lambda_\gamma - \lambda_p = \mu$ and $\nu = -\lambda_Y$ for all the possible values of λ_P, λ_Y and λ_γ.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{1/2}A_{1/2}d^{1/2}_{1/2\ -1/2}$	$F_{1/2}A_{1/2}d^{1/2}_{1/2\ 1/2}$	0	0
0	$F_{1/2}S_{1/2}d^{1/2}_{-1/2\ -1/2}$	$F_{1/2}S_{1/2}d^{1/2}_{-1/2\ 1/2}$	$F_{1/2}S_{1/2}d^{1/2}_{1/2\ -1/2}$	$F_{1/2}S_{1/2}d^{1/2}_{1/2\ 1/2}$
-1	0	0	$F_{1/2}A_{1/2}d^{1/2}_{-1/2\ -1/2}$	$F_{1/2}A_{1/2}d^{1/2}_{-1/2\ 1/2}$

Table 2: Values of $M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$ for $J^P = \frac{1}{2}^+$ for all the possible values of λ_P, λ_Y and λ_γ. $\lambda_\gamma = \pm 1$ for transverse photons and $\lambda_\gamma = 0$ for longitudinal photons. Note that the terms with $\mu = 3/2$ are not present.

multiplied for terms that present a dependence.
Using $\phi^*_K = 0$, we obtain the results presented in Tab. 2.
The $d^{1/2}_{\mu\nu}(cos\theta^*_K)$ coefficients are Wigner rotation matrix elements, for whom the relationship $d^J_{m',m} = (-1)^{m-m'} d^J_{m,m'} = d^J_{-m,-m'}$ is valid and that are:

$$\begin{aligned} d^{1/2}_{1/2\ 1/2}(cos\theta^*_K) &= cos\theta^*_K/2 \\ d^{1/2}_{1/2\ -1/2}(cos\theta^*_K) &= -sin\theta^*_K/2 \\ d^{1/2}_{-1/2\ -1/2}(cos\theta^*_K) &= d^{1/2}_{1/2\ 1/2}(cos\theta^*_K) = cos\theta^*_K/2 \\ -d^{1/2}_{-1/2\ 1/2}(cos\theta^*_K) &= d^{1/2}_{1/2\ -1/2}(cos\theta^*_K) = -sin\theta^*_K/2 \end{aligned} \tag{37}$$

We can substitute these values in Table 2, obtaining the results in Tab. 3.
It is convenient to express our $M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$ as $M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$=Re($M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$) + i Im($M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$). So, first of all, we isolate the complex part in the $M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$:

$$\frac{1}{M_r^2 - W^2 - i\Gamma_r M_r} = \frac{(M_r^2 - W^2) + i(\Gamma_r M_r)}{(M_r^2 - W^2)^2 + (\Gamma_r M_r)^2} \tag{38}$$

Now we call:

$$F'_{J=1/2} = \frac{\sqrt{4\pi\alpha}\, 2\sqrt{2\pi}\sqrt{2}M_r\sqrt{\Gamma_r 0.025}}{\sqrt{p_i}((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2)} \frac{W}{M_r}\sqrt{\frac{cM_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}} \begin{cases} A_{1/2}(Q^2) = A,\ c = 8 \\ S_{1/2}(Q^2) = AQ,\ c = 16 \end{cases} \tag{39}$$

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{1/2}A_{1/2}(-sin\theta_K^*/2)$	$F_{1/2}A_{1/2}(cos\theta_K^*/2)$	0	0
0	$F_{1/2}S_{1/2}(cos\theta_K^*/2)$	$F_{1/2}S_{1/2}(sin\theta_K^*/2)$	$F_{1/2}S_{1/2}(-sin\theta_K^*/2)$	$F_{1/2}S_{1/2}(cos\theta_K^*/2)$
-1	0	0	$F_{1/2}A_{1/2}(cos\theta_K^*/2)$	$F_{1/2}A_{1/2}(sin\theta_K^*/2)$

Table 3: Values of $M_{\lambda_\gamma}^{\lambda_p\lambda_Y}$ for $J^P = \frac{1}{2}^+$ for all the possible values of λ_P, λ_Y and λ_γ. The exact values of Wigner rotation matrix elements have been inserted.

where $\sqrt{4\pi\alpha}$ is the transformation factor for the amplitude from Ghent to JM convention, and we obtain values for $Re(M_{\lambda_\gamma}^{\lambda_p\lambda_Y})$ reported in table 4 and for $Im(M_{\lambda_\gamma}^{\lambda_p\lambda_Y})$ in table 5 for $J^P = \frac{1}{2}^+$.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F'_{1/2}A(-sin\theta_K^*/2)(M_r^2 - W^2)$	$F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2)$	0	0
0	$F'_{1/2}AQ(cos\theta_K^*/2)(M_r^2 - W^2)$	$F'_{1/2}AQ(sin\theta_K^*/2)(M_r^2 - W^2)$	$F'_{1/2}AQ(-sin\theta_K^*/2)(M_r^2 - W^2)$	$F'_{1/2}AQ(cos\theta_K^*/2)(M_r^2 - W^2)$
-1	0	0	$F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2)$	$F'_{1/2}A(sin\theta_K^*/2)(M_r^2 - W^2)$

Table 4: Values of $Re(M_{\lambda_\gamma}^{\lambda_p\lambda_Y})$ for $J^P = \frac{1}{2}^+$ for all the possible values of λ_P, λ_Y and λ_γ. The exact values of Wigner rotation matrix elements have been inserted.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F'_{1/2}A(-sin\theta^*_K/2)\Gamma_r M_r$	$F'_{1/2}A(cos\theta^*_K/2)\Gamma_r M_r$	0	0
0	$F'_{1/2}AQ(cos\theta^*_K/2)\Gamma_r M_r$	$F'_{1/2}AQ(sin\theta^*_K/2)\Gamma_r M_r$	$F'_{1/2}AQ(-sin\theta^*_K/2)\Gamma_r M_r$	$F'_{1/2}AQ(cos\theta^*_K/2)\Gamma_r M_r$
-1	0	0	$F'_{1/2}A(cos\theta^*_K/2)\Gamma_r M_r$	$F'_{1/2}A(sin\theta^*_K/2)\Gamma_r M_r$

Table 5: Values of $Im(M_{\lambda_\gamma}^{\lambda_p \lambda_Y})$ for $J^P = \frac{1}{2}^+$ for all the possible values of λ_P, λ_Y and λ_γ. The exact values of Wigner rotation matrix elements have been inserted.

1.2.2 $J^P = \frac{3}{2}^+$

In this case we have

$$
\begin{aligned}
&\langle \lambda_f | \, T_r \, | \lambda_\gamma \lambda_p \rangle = \\
&= \frac{\frac{2\sqrt{2\pi}2M_r\sqrt{\Gamma_r 0.025}}{\sqrt{p_i}}}{M_r^2 - W^2 - i\Gamma_r M_r} d^{3/2}_{\mu\nu}(cos\theta^*_K) e^{i\mu\phi^*_K} \frac{W}{M_r}\sqrt{\frac{cM_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}} \begin{cases} A_{1/2}(Q^2) = A, \; c = 8 \\ A_{3/2}(Q^2) = A/Q^2, \; c = 8 \\ S_{1/2}(Q^2) = AQ, \; c = 16 \end{cases}
\end{aligned} \tag{40}
$$

Exploiting the results in Table 1 for $\lambda_R = \lambda_\gamma - \lambda_p = \mu$ and $\nu = -\lambda_Y$ with respect to all the possible values of λ_P, λ_Y and λ_γ, we obtain the different values of the $d^{3/2}_{\mu\nu}(cos\theta^*_K)$ Wigner rotation matrix elements with respect to all the possible values of λ_P, λ_Y and λ_γ (Table 6).

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$d^{3/2}_{1/2\ -1/2}(cos\theta^*_K)$	$d^{3/2}_{1/2\ 1/2}(cos\theta^*_K)$	$d^{3/2}_{3/2\ -1/2}(cos\theta^*_K)$	$d^{3/2}_{3/2\ 1/2}(cos\theta^*_K)$
0	$d^{3/2}_{-1/2\ -1/2}(cos\theta^*_K)$	$d^{3/2}_{-1/2\ 1/2}(cos\theta^*_K)$	$d^{3/2}_{1/2\ -1/2}(cos\theta^*_K)$	$d^{3/2}_{1/2\ 1/2}(cos\theta^*_K)$
-1	$d^{3/2}_{-3/2\ -1/2}(cos\theta^*_K)$	$d^{3/2}_{-3/2\ 1/2}(cos\theta^*_K)$	$d^{3/2}_{-1/2\ -1/2}(cos\theta^*_K)$	$d^{3/2}_{-1/2\ 1/2}(cos\theta^*_K)$

Table 6: Values of $d^{3/2}_{\mu\nu}(cos\theta^*_K)$ for all the possible values of λ_P, λ_Y and λ_γ.

The $d^{3/2}_{\mu\nu}(cos\theta^*_K)$ Wigner rotation matrix elements, for whom the relationship
$d^J_{m',m} = (-1)^{m-m'} d^J_{m,m'} = d^J_{-m,-m'}$ is valid, are:

$$
\begin{aligned}
d^{3/2}_{3/2\ 3/2}(cos\theta^*_K) &= \frac{1+cos\theta^*_K}{2} cos\theta^*_K/2 = d^{3/2}_{-3/2\ -3/2}(cos\theta^*_K) \\
d^{3/2}_{3/2\ 1/2}(cos\theta^*_K) &= -\sqrt{3}\frac{1+cos\theta^*_K}{2} sin\theta^*_K/2 = -d^{3/2}_{-3/2\ -1/2}(cos\theta^*_K) \\
d^{3/2}_{3/2\ -1/2}(cos\theta^*_K) &= \sqrt{3}\frac{1-cos\theta^*_K}{2} cos\theta^*_K/2 = (-1)^{-3/2-1/2} d^{3/2}_{-3/2\ 1/2}(cos\theta^*_K) = d^{3/2}_{-3/2\ 1/2}(cos\theta^*_K) \\
d^{3/2}_{3/2\ -3/2}(cos\theta^*_K) &= -\frac{1-cos\theta^*_K}{2} sin\theta^*_K/2 \\
d^{3/2}_{1/2\ 1/2}(cos\theta^*_K) &= \frac{3cos\theta^*_K - 1}{2} cos\theta^*_K/2 = d^{3/2}_{-1/2\ -1/2}(cos\theta^*_K) \\
d^{3/2}_{1/2\ -1/2}(cos\theta^*_K) &= -\frac{3cos\theta^*_K + 1}{2} sin\theta^*_K/2 = -d^{3/2}_{-1/2\ 1/2}(cos\theta^*_K)
\end{aligned} \tag{41}
$$

Also in this case we can express the $M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$ in terms of a factor that doesn't change with λ_P, λ_Y and λ_γ,

$$
F_{J=3/2} = \frac{\frac{2\sqrt{2\pi}2M_r\sqrt{\Gamma_r 0.025}}{\sqrt{p_i}}}{M_r^2 - W^2 - i\Gamma_r M_r} \frac{W}{M_r}\sqrt{\frac{cM_N M_r q_{\gamma_r}}{4\pi\alpha}}\sqrt{\frac{q_{\gamma_r}}{q_\gamma}} \begin{cases} A_{1/2}(Q^2) = A, \; c = 8 \\ A_{3/2}(Q^2) = A/Q^2, \; c = 8 \\ S_{1/2}(Q^2) = AQ, \; c = 16 \end{cases} \tag{42}
$$

multiplied for terms that present a dependence. Using $\phi^*_K = 0$, we obtain the results presented in Table 7.
Again it is convenient to express our $M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$ as $M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$=Re($M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$) + i Im($M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$). So, first of all, we isolate the complex part in the $M^{\lambda_p\lambda_Y}_{\lambda_\gamma}$:

$$
\frac{1}{M_r^2 - W^2 - i\Gamma_r M_r} = \frac{(M_r^2 - W^2) + i(\Gamma_r M_r)}{(M_r^2 - W^2)^2 + (\Gamma_r M_r)^2} \tag{43}
$$

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{3/2}Ad^{3/2}_{1/2\ -1/2}$	$F_{3/2}Ad^{3/2}_{1/2\ 1/2}$	$F_{3/2}A/Q^2d^{3/2}_{3/2\ -1/2}$	$F_{3/2}A/Q^2d^{3/2}_{3/2\ 1/2}$
0	$F_{3/2}AQd^{3/2}_{-1/2\ -1/2}$	$F_{3/2}AQd^{3/2}_{-1/2\ 1/2}$	$F_{3/2}AQd^{3/2}_{1/2\ -1/2}$	$F_{3/2}AQd^{3/2}_{1/2\ 1/2}$
-1	$F_{3/2}A/Q^2d^{3/2}_{-3/2\ -1/2}$	$F_{3/2}A/Q^2d^{3/2}_{-3/2\ 1/2}$	$F_{3/2}Ad^{3/2}_{-1/2\ -1/2}$	$F_{3/2}Ad^{3/2}_{-1/2\ 1/2}$

Table 7: Values of $M^{\lambda_p \lambda_Y}_{\lambda_\gamma}$ for $J^P = \frac{3}{2}^+$ for all the possible values of λ_P, λ_Y and λ_γ. Note that now the terms with $\mu = 3/2$ are present, and that the $A_{1/2}(Q^2)$, $A_{3/2}(Q^2)$ and $S_{1/2}(Q^2)$ terms have been replaced with A, A/Q^2 and AQ respectively.

Now we call:

$$F'_{J=3/2} = \frac{\sqrt{4\pi\alpha}2\sqrt{2\pi}2M_r\sqrt{\Gamma_r 0.025}}{\sqrt{p_i}((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2)} \frac{W}{M_r} \sqrt{\frac{cM_N M_r q_{\gamma_r}}{4\pi\alpha}} \sqrt{\frac{q_{\gamma_r}}{q_\gamma}} \begin{cases} A_{1/2}(Q^2) = A,\ c = 8 \\ A_{3/2}(Q^2) = A/Q^2,\ c = 8 \\ S_{1/2}(Q^2) = AQ,\ c = 16 \end{cases} \tag{44}$$

where $\sqrt{4\pi\alpha}$ is the transformation factor for the amplitude from Ghent to JM convention, and we finally obtain values for $Re(M^{\lambda_p \lambda_Y}_{\lambda_\gamma})$ reported in table 8 and for $Im(M^{\lambda_p \lambda_Y}_{\lambda_\gamma})$ in table 9.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F'_{3/2}A(-\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)$	$F'_{3/2}A(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)$	$F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)$	$F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)$
0	$F'_{3/2}AQ(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)$	$F'_{3/2}AQ(\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)$	$F'_{3/2}AQ(-\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)$	$F'_{3/2}AQ(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)$
-1	$F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)$	$F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)$	$F'_{3/2}A(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)$	$F'_{3/2}A(\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)$

Table 8: Values of $Re(M^{\lambda_p\lambda_Y}_{\lambda_\gamma})$ for $J^P = \frac{3}{2}^+$ for all the possible values of λ_P, λ_Y and λ_γ. The exact values of Wigner rotation matrix elements have been inserted.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F'_{3/2}A(-\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})\Gamma_r M_r$	$F'_{3/2}A(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})\Gamma_r M_r$	$F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2})\Gamma_r M_r$	$F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2})\Gamma_r M_r$
0	$F'_{3/2}AQ(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})\Gamma_r M_r$	$F'_{3/2}AQ(\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})\Gamma_r M_r$	$F'_{3/2}AQ(-\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})\Gamma_r M_r$	$F'_{3/2}AQ(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})\Gamma_r M_r$
-1	$F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2})\Gamma_r M_r$	$F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2})\Gamma_r M_r$	$F'_{3/2}A(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})\Gamma_r M_r$	$F'_{3/2}A(\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})\Gamma_r M_r$

Table 9: Values of $Im(M^{\lambda_p\lambda_Y}_{\lambda_\gamma})$ for $J^P = \frac{3}{2}^+$ for all the possible values of λ_P, λ_Y and λ_γ. The exact values of Wigner rotation matrix elements have been inserted.

2 Calculation of $H_{\lambda,\lambda'}$

2.1 $J^P = \frac{1}{2}^+$

2.1.1 $H_{0,0}$

We want to calculate the term $H_{0,0}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N\lambda_Y} M_\lambda^{'\lambda_N\lambda_Y} (M_\lambda^{'\lambda_N\lambda_Y})^\dagger \tag{45}$$

and using

$$M_\lambda^{'\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y} + \mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{46}$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Table 3 for $J^P = \frac{1}{2}^+$. Specifically, the real parts are reported in Table 4, and the imaginary parts are reported in Table 5. We have for $H_{0,0}$:

$$\begin{aligned}
H_{0,0} =\ & (M_0^{++} + \mathscr{M}_0^{++})(M_0^{++} + \mathscr{M}_0^{++})^\dagger + (M_0^{--} + \mathscr{M}_0^{--})(M_0^{--} + \mathscr{M}_0^{--})^\dagger + \\
& + (M_0^{+-} + \mathscr{M}_0^{+-})(M_0^{+-} + \mathscr{M}_0^{+-})^\dagger + (M_0^{-+} + \mathscr{M}_0^{-+})(M_0^{-+} + \mathscr{M}_0^{-+})^\dagger = \\
& = (ReM_0^{++} + iImM_0^{++} + Re\mathscr{M}_0^{++} + iIm\mathscr{M}_0^{++})(ReM_0^{++} - iImM_0^{++} + Re\mathscr{M}_0^{++} - iIm\mathscr{M}_0^{++}) + \\
& + (ReM_0^{--} + iImM_0^{--} + Re\mathscr{M}_0^{--} + iIm\mathscr{M}_0^{--})(ReM_0^{--} - iImM_0^{--} + Re\mathscr{M}_0^{--} - iIm\mathscr{M}_0^{--}) + \\
& + (ReM_0^{+-} + iImM_0^{+-} + Re\mathscr{M}_0^{+-} + iIm\mathscr{M}_0^{+-})(ReM_0^{+-} - iImM_0^{+-} + Re\mathscr{M}_0^{+-} - iIm\mathscr{M}_0^{+-}) + \\
& + (ReM_0^{-+} + iImM_0^{-+} + Re\mathscr{M}_0^{-+} + iIm\mathscr{M}_0^{-+})(ReM_0^{-+} - iImM_0^{-+} + Re\mathscr{M}_0^{-+} - iIm\mathscr{M}_0^{-+}) = \\
& = ReM_0^{++}ReM_0^{++} \cancel{-iReM_0^{++}ImM_0^{++}} + ReM_0^{++}Re\mathscr{M}_0^{++} \cancel{-iReM_0^{++}Im\mathscr{M}_0^{++}} + \\
& \cancel{+iImM_0^{++}ReM_0^{++}} + ImM_0^{++}ImM_0^{++} + \cancel{iImM_0^{++}Re\mathscr{M}_0^{++}} + ImM_0^{++}Im\mathscr{M}_0^{++} + \\
& + Re\mathscr{M}_0^{++}ReM_0^{++} \cancel{-iRe\mathscr{M}_0^{++}ImM_0^{++}} + Re\mathscr{M}_0^{++}Re\mathscr{M}_0^{++} \cancel{-iRe\mathscr{M}_0^{++}Im\mathscr{M}_0^{++}} + \\
& \cancel{+iIm\mathscr{M}_0^{++}ReM_0^{++}} + Im\mathscr{M}_0^{++}ImM_0^{++} + \cancel{iIm\mathscr{M}_0^{++}Re\mathscr{M}_0^{++}} + Im\mathscr{M}_0^{++}Im\mathscr{M}_0^{++} + \\
& + ReM_0^{--}ReM_0^{--} \cancel{-iReM_0^{--}ImM_0^{--}} + ReM_0^{--}Re\mathscr{M}_0^{--} \cancel{-iReM_0^{--}Im\mathscr{M}_0^{--}} + \\
& \cancel{+iImM_0^{--}ReM_0^{--}} + ImM_0^{--}ImM_0^{--} + \cancel{iImM_0^{--}Re\mathscr{M}_0^{--}} + ImM_0^{--}Im\mathscr{M}_0^{--} + \\
& + Re\mathscr{M}_0^{--}ReM_0^{--} \cancel{-iRe\mathscr{M}_0^{--}ImM_0^{--}} + Re\mathscr{M}_0^{--}Re\mathscr{M}_0^{--} \cancel{-iRe\mathscr{M}_0^{--}Im\mathscr{M}_0^{--}} + \\
& \cancel{+iIm\mathscr{M}_0^{--}ReM_0^{--}} + Im\mathscr{M}_0^{--}ImM_0^{--} + \cancel{iIm\mathscr{M}_0^{--}Re\mathscr{M}_0^{--}} + Im\mathscr{M}_0^{--}Im\mathscr{M}_0^{--} + \\
& + ReM_0^{+-}ReM_0^{+-} \cancel{-iReM_0^{+-}ImM_0^{+-}} + ReM_0^{+-}Re\mathscr{M}_0^{+-} \cancel{-iReM_0^{+-}Im\mathscr{M}_0^{+-}} + \\
& \cancel{+iImM_0^{+-}ReM_0^{+-}} + ImM_0^{+-}ImM_0^{+-} + \cancel{iImM_0^{+-}Re\mathscr{M}_0^{+-}} + ImM_0^{+-}Im\mathscr{M}_0^{+-} + \\
& + Re\mathscr{M}_0^{+-}ReM_0^{+-} \cancel{-iRe\mathscr{M}_0^{+-}ImM_0^{+-}} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_0^{+-} \cancel{-iRe\mathscr{M}_0^{+-}Im\mathscr{M}_0^{+-}} + \\
& \cancel{+iIm\mathscr{M}_0^{+-}ReM_0^{+-}} + Im\mathscr{M}_0^{+-}ImM_0^{+-} + \cancel{iIm\mathscr{M}_0^{+-}Re\mathscr{M}_0^{+-}} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_0^{+-} + \\
& + ReM_0^{-+}ReM_0^{-+} \cancel{-iReM_0^{-+}ImM_0^{-+}} + ReM_0^{-+}Re\mathscr{M}_0^{-+} \cancel{-iReM_0^{-+}Im\mathscr{M}_0^{-+}} + \\
& \cancel{+iImM_0^{-+}ReM_0^{-+}} + ImM_0^{-+}ImM_0^{-+} + \cancel{iImM_0^{-+}Re\mathscr{M}_0^{-+}} + ImM_0^{-+}Im\mathscr{M}_0^{-+} + \\
& + Re\mathscr{M}_0^{-+}ReM_0^{-+} \cancel{-iRe\mathscr{M}_0^{-+}ImM_0^{-+}} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_0^{-+} \cancel{-iRe\mathscr{M}_0^{-+}Im\mathscr{M}_0^{-+}} + \\
& \cancel{+iIm\mathscr{M}_0^{-+}ReM_0^{-+}} + Im\mathscr{M}_0^{-+}ImM_0^{-+} + \cancel{iIm\mathscr{M}_0^{-+}Re\mathscr{M}_0^{-+}} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_0^{-+} = \\
& = (ReM_0^{++})^2 + \cancel{2ReM_0^{++}Re\mathscr{M}_0^{++}} + (ImM_0^{++})^2 + \cancel{2ImM_0^{++}Im\mathscr{M}_0^{++}} + (Im\mathscr{M}_0^{++})^2 + (Re\mathscr{M}_0^{++})^2 + \\
& + (ReM_0^{--})^2 + \cancel{2ReM_0^{--}Re\mathscr{M}_0^{--}} + (ImM_0^{--})^2 + \cancel{2ImM_0^{--}Im\mathscr{M}_0^{--}} + (Im\mathscr{M}_0^{--})^2 + (Re\mathscr{M}_0^{--})^2 + \\
& + (ReM_0^{+-})^2 + 2ReM_0^{+-}Re\mathscr{M}_0^{+-} + (ImM_0^{+-})^2 + 2ImM_0^{+-}Im\mathscr{M}_0^{+-} + (Im\mathscr{M}_0^{+-})^2 + (Re\mathscr{M}_0^{+-})^2 + \\
& + (ReM_0^{-+})^2 + 2ReM_0^{-+}Re\mathscr{M}_0^{-+} + (ImM_0^{-+})^2 + 2ImM_0^{-+}Im\mathscr{M}_0^{-+} + (Im\mathscr{M}_0^{-+})^2 + (Re\mathscr{M}_0^{-+})^2
\end{aligned} \tag{47}$$

Considering that for $J^P = \frac{1}{2}^+$:

$$
\begin{aligned}
&\mathscr{M}_1^{++} = \mathscr{M}_{-1}^{++} \;\; \mathscr{M}_0^{++} = -\mathscr{M}_0^{--} \;\; \mathscr{M}_1^{-+} = -\mathscr{M}_{-1}^{-+} \;\; \mathscr{M}_1^{+-} = -\mathscr{M}_{-1}^{+-} \;\; \mathscr{M}_1^{--} = \mathscr{M}_{-1}^{--} \\
&M_1^{++} = -M_{-1}^{--} \;\; M_1^{+-} = M_{-1}^{-+} \;\; M_0^{+-} = -M_0^{-+} \;\; M_0^{++} = M_0^{--} \\
&M_{-1}^{+-} = M_{-1}^{++} = M_1^{-+} = M_1^{--} = 0
\end{aligned}
\tag{48}
$$

and substituting values from Tabs. 4 and 5, we can write:

$$
\begin{aligned}
H_{0,0} = {} & 2(F'_{1/2}AQ)^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + 2(ReM_0^{++2} + ImM_0^{++2}) + 2(ReM_0^{+-2} + ImM_0^{+-2}) + 2ReM_0^{-+2} + \\
& + 2ImM_0^{-+2} + 2F'_{1/2}AQ(sin\frac{\theta_K^*}{2})[Re\mathscr{M}_0^{+-}(M_r^2 - W^2) + Im\mathscr{M}_0^{+-}\Gamma_r M_r - Re\mathscr{M}_0^{-+}(M_r^2 - W^2) - Im\mathscr{M}_0^{-+}\Gamma_r M_r]
\end{aligned}
\tag{49}
$$

Or

$$
\begin{aligned}
H_{0,0} = {} & 2(F'_{1/2}AQ)^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + 2 \mid \mathscr{M}_0^{++} \mid^2 + \mid \mathscr{M}_0^{+-} \mid^2 + \mid \mathscr{M}_0^{-+} \mid^2 + \\
& + 2F'_{1/2}AQ(sin\frac{\theta_K^*}{2})[Re\mathscr{M}_0^{+-}(M_r^2 - W^2) + Im\mathscr{M}_0^{+-}\Gamma_r M_r - Re\mathscr{M}_0^{-+}(M_r^2 - W^2) - Im\mathscr{M}_0^{-+}\Gamma_r M_r]
\end{aligned}
\tag{50}
$$

2.1.2 $H_{1,1}$

$$
\begin{aligned}
H_{1,1} = {} & (M_1^{++} + \mathscr{M}_1^{++})(M_1^{++} + \mathscr{M}_1^{++})^\dagger + (M_1^{--} + \mathscr{M}_1^{--})(M_1^{--} + \mathscr{M}_1^{--})^\dagger + \\
& + (M_1^{+-} + \mathscr{M}_1^{+-})(M_1^{+-} + \mathscr{M}_1^{+-})^\dagger + (M_1^{-+} + \mathscr{M}_1^{-+})(M_1^{-+} + \mathscr{M}_1^{-+})^\dagger = \\
& = (ReM_1^{++} + iImM_1^{++} + Re\mathscr{M}_1^{++} + iIm\mathscr{M}_1^{++})(ReM_1^{++} - iImM_1^{++} + Re\mathscr{M}_1^{++} - iIm\mathscr{M}_1^{++}) + \\
& + (ReM_1^{--} + iImM_1^{--} + Re\mathscr{M}_1^{--} + iIm\mathscr{M}_1^{--})(ReM_1^{--} - iImM_1^{--} + Re\mathscr{M}_1^{--} - iIm\mathscr{M}_1^{--}) + \\
& + (ReM_1^{+-} + iImM_1^{+-} + Re\mathscr{M}_1^{+-} + iIm\mathscr{M}_1^{+-})(ReM_1^{+-} - iImM_1^{+-} + Re\mathscr{M}_1^{+-} - iIm\mathscr{M}_1^{+-}) + \\
& + (ReM_1^{-+} + iImM_1^{-+} + Re\mathscr{M}_1^{-+} + iIm\mathscr{M}_1^{-+})(ReM_1^{-+} - iImM_1^{-+} + Re\mathscr{M}_1^{-+} - iIm\mathscr{M}_1^{-+}) = \\
& = ReM_1^{++} ReM_1^{++} \cancel{-iReM_1^{++} ImM_1^{++}} + ReM_1^{++} Re\mathscr{M}_1^{++} \cancel{-iReM_1^{++} Im\mathscr{M}_1^{++}} + \\
& \cancel{+iImM_1^{++} ReM_1^{++}} + ImM_1^{++} ImM_1^{++} + \cancel{iImM_1^{++} Re\mathscr{M}_1^{++}} + ImM_1^{++} Im\mathscr{M}_1^{++} + \\
& + Re\mathscr{M}_1^{++} ReM_1^{++} \cancel{-iRe\mathscr{M}_1^{++} ImM_1^{++}} + Re\mathscr{M}_1^{++} Re\mathscr{M}_1^{++} \cancel{-iRe\mathscr{M}_1^{++} Im\mathscr{M}_1^{++}} + \\
& \cancel{+iIm\mathscr{M}_1^{++} ReM_1^{++}} + Im\mathscr{M}_1^{++} ImM_1^{++} + \cancel{iIm\mathscr{M}_1^{++} Re\mathscr{M}_1^{++}} + Im\mathscr{M}_1^{++} Im\mathscr{M}_1^{++} + \\
& + ReM_1^{--} ReM_1^{--} \cancel{-iReM_1^{--} ImM_1^{--}} + ReM_1^{--} Re\mathscr{M}_1^{--} \cancel{-iReM_1^{--} Im\mathscr{M}_1^{--}} + \\
& \cancel{+iImM_1^{--} ReM_1^{--}} + ImM_1^{--} ImM_1^{--} + \cancel{iImM_1^{--} Re\mathscr{M}_1^{--}} + ImM_1^{--} Im\mathscr{M}_1^{--} + \\
& + Re\mathscr{M}_1^{--} ReM_1^{--} \cancel{-iRe\mathscr{M}_1^{--} ImM_1^{--}} + Re\mathscr{M}_1^{--} Re\mathscr{M}_1^{--} \cancel{-iRe\mathscr{M}_1^{--} Im\mathscr{M}_1^{--}} + \\
& \cancel{+iIm\mathscr{M}_1^{--} ReM_1^{--}} + Im\mathscr{M}_1^{--} ImM_1^{--} + \cancel{iIm\mathscr{M}_1^{--} Re\mathscr{M}_1^{--}} + Im\mathscr{M}_1^{--} Im\mathscr{M}_1^{--} + \\
& + ReM_1^{+-} ReM_1^{+-} \cancel{-iReM_1^{+-} ImM_1^{+-}} + ReM_1^{+-} Re\mathscr{M}_1^{+-} \cancel{-iReM_1^{+-} Im\mathscr{M}_1^{+-}} + \\
& \cancel{+iImM_1^{+-} ReM_1^{+-}} + ImM_1^{+-} ImM_1^{+-} + \cancel{iImM_1^{+-} Re\mathscr{M}_1^{+-}} + ImM_1^{+-} Im\mathscr{M}_1^{+-} + \\
& + Re\mathscr{M}_1^{+-} ReM_1^{+-} \cancel{-iRe\mathscr{M}_1^{+-} ImM_1^{+-}} + Re\mathscr{M}_1^{+-} Re\mathscr{M}_1^{+-} \cancel{-iRe\mathscr{M}_1^{+-} Im\mathscr{M}_1^{+-}} + \\
& \cancel{+iIm\mathscr{M}_1^{+-} ReM_1^{+-}} + Im\mathscr{M}_1^{+-} ImM_1^{+-} + \cancel{iIm\mathscr{M}_1^{+-} Re\mathscr{M}_1^{+-}} + Im\mathscr{M}_1^{+-} Im\mathscr{M}_1^{+-} + \\
& + ReM_1^{-+} ReM_1^{-+} \cancel{-iReM_1^{-+} ImM_1^{-+}} + ReM_1^{-+} Re\mathscr{M}_1^{-+} \cancel{-iReM_1^{-+} Im\mathscr{M}_1^{-+}} + \\
& \cancel{+iImM_1^{-+} ReM_1^{-+}} + ImM_1^{-+} ImM_1^{-+} + \cancel{iImM_1^{-+} Re\mathscr{M}_1^{-+}} + ImM_1^{-+} Im\mathscr{M}_1^{-+} + \\
& + Re\mathscr{M}_1^{-+} ReM_1^{-+} \cancel{-iRe\mathscr{M}_1^{-+} ImM_1^{-+}} + Re\mathscr{M}_1^{-+} Re\mathscr{M}_1^{-+} \cancel{-iRe\mathscr{M}_1^{-+} Im\mathscr{M}_1^{-+}} + \\
& \cancel{+iIm\mathscr{M}_1^{-+} ReM_1^{-+}} + Im\mathscr{M}_1^{-+} ImM_1^{-+} + \cancel{iIm\mathscr{M}_1^{-+} Re\mathscr{M}_1^{-+}} + Im\mathscr{M}_1^{-+} Im\mathscr{M}_1^{-+} = \\
& = (ReM_1^{++})^2 + 2ReM_1^{++} Re\mathscr{M}_1^{++} + (ImM_1^{++})^2 + 2ImM_1^{++} Im\mathscr{M}_1^{++} + (Im\mathscr{M}_1^{++})^2 + (Re\mathscr{M}_1^{++})^2 + \\
& + (ReM_1^{--})^2 + 2ReM_1^{--} Re\mathscr{M}_1^{--} + (ImM_1^{--})^2 + 2ImM_1^{--} Im\mathscr{M}_1^{--} + (Im\mathscr{M}_1^{--})^2 + (Re\mathscr{M}_1^{--})^2 + \\
& + (ReM_1^{+-})^2 + 2ReM_1^{+-} Re\mathscr{M}_1^{+-} + (ImM_1^{+-})^2 + 2ImM_1^{+-} Im\mathscr{M}_1^{+-} + (Im\mathscr{M}_1^{+-})^2 + (Re\mathscr{M}_1^{+-})^2 + \\
& + (ReM_1^{-+})^2 + 2ReM_1^{-+} Re\mathscr{M}_1^{-+} + (ImM_1^{-+})^2 + 2ImM_1^{-+} Im\mathscr{M}_1^{-+} + (Im\mathscr{M}_1^{-+})^2 + (Re\mathscr{M}_1^{-+})^2
\end{aligned}
\tag{51}
$$

Considering that for $J^P = \frac{1}{2}^+$:

$$
\begin{aligned}
& \mathscr{M}_1^{++} = \mathscr{M}_{-1}^{++} \;\; \mathscr{M}_0^{++} = -\mathscr{M}_0^{--} \;\; \mathscr{M}_1^{-+} = -\mathscr{M}_{-1}^{-+} \;\; \mathscr{M}_1^{+-} = -\mathscr{M}_{-1}^{+-} \;\; \mathscr{M}_1^{--} = \mathscr{M}_{-1}^{--} \\
& M_1^{++} = -M_{-1}^{--} \;\; M_1^{+-} = M_{-1}^{-+} \;\; M_0^{+-} = -M_0^{-+} \;\; M_0^{++} = M_0^{--} \\
& M_{-1}^{+-} = M_{-1}^{++} = M_1^{-+} = M_1^{--} = 0
\end{aligned}
\tag{52}
$$

And substituting values from Tabs. 4 and 5 we can write:

$$
\begin{aligned}
H_{1,1} = {} & (F'_{1/2}A(sin\theta_K^*/2))^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + (Im\mathscr{M}_1^{++})^2 + (Re\mathscr{M}_1^{++})^2 + \\
& + (Im\mathscr{M}_1^{--})^2 + (Re\mathscr{M}_1^{--})^2 + \\
& + (F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2))^2 + (F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r))^2 + (Im\mathscr{M}_1^{+-})^2 + (Re\mathscr{M}_1^{+-})^2 + \\
& + (Im\mathscr{M}_1^{-+})^2 + (Re\mathscr{M}_1^{-+})^2 + \\
& + 2F'_{1/2}A(-sin\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_1^{++} + 2F'_{1/2}A(-sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{++} + \\
& + 2F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_1^{+-} + 2F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{+-}
\end{aligned}
\tag{53}
$$

In conclusion:

$$
\begin{aligned}
H_{1,1} = & (F'_{1/2}A)^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + |\mathscr{M}_1^{++}|^2 + |\mathscr{M}_1^{--}|^2 + |\mathscr{M}_1^{+-}|^2 + |\mathscr{M}_1^{-+}|^2 + \\
& + 2F'_{1/2}A(-sin\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_1^{++} + 2F'_{1/2}A(-sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{++} + \\
& + 2F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_1^{+-} + 2F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{+-}
\end{aligned} \tag{54}
$$

2.1.3 $H_{-1,-1}$

$$
\begin{aligned}
H_{-1,-1} = & (M_{-1}^{++} + \mathscr{M}_{-1}^{++})(M_{-1}^{++} + \mathscr{M}_{-1}^{++})^\dagger + (M_{-1}^{--} + \mathscr{M}_{-1}^{--})(M_{-1}^{--} + \mathscr{M}_{-1}^{--})^\dagger + \\
& + (M_{-1}^{+-} + \mathscr{M}_{-1}^{+-})(M_{-1}^{+-} + \mathscr{M}_{-1}^{+-})^\dagger + (M_{-1}^{-+} + \mathscr{M}_{-1}^{-+})(M_{-1}^{-+} + \mathscr{M}_{-1}^{-+})^\dagger = \\
& = (ReM_{-1}^{++})^2 + 2ReM_{-1}^{++}Re\mathscr{M}_{-1}^{++} + (ImM_{-1}^{++})^2 + 2ImM_{-1}^{++}Im\mathscr{M}_{-1}^{++} + (Im\mathscr{M}_{-1}^{++})^2 + (Re\mathscr{M}_{-1}^{++})^2 + \\
& + (ReM_{-1}^{--})^2 + 2ReM_{-1}^{--}Re\mathscr{M}_{-1}^{--} + (ImM_{-1}^{--})^2 + 2ImM_{-1}^{--}Im\mathscr{M}_{-1}^{--} + (Im\mathscr{M}_{-1}^{--})^2 + (Re\mathscr{M}_{-1}^{--})^2 + \\
& + (ReM_{-1}^{+-})^2 + 2ReM_{-1}^{+-}Re\mathscr{M}_{-1}^{+-} + (ImM_{-1}^{+-})^2 + 2ImM_{-1}^{+-}Im\mathscr{M}_{-1}^{+-} + (Im\mathscr{M}_{-1}^{+-})^2 + (Re\mathscr{M}_{-1}^{+-})^2 + \\
& + (ReM_{-1}^{-+})^2 + 2ReM_{-1}^{-+}Re\mathscr{M}_{-1}^{-+} + (ImM_{-1}^{-+})^2 + 2ImM_{-1}^{-+}Im\mathscr{M}_{-1}^{-+} + (Im\mathscr{M}_{-1}^{-+})^2 + (Re\mathscr{M}_{-1}^{-+})^2
\end{aligned} \tag{55}
$$

Considering that for $J^P = \frac{1}{2}^+$:

$$
\begin{aligned}
& \mathscr{M}_1^{++} = \mathscr{M}_{-1}^{++} \;\; \mathscr{M}_0^{++} = -\mathscr{M}_0^{--} \;\; \mathscr{M}_1^{-+} = -\mathscr{M}_{-1}^{-+} \;\; \mathscr{M}_1^{+-} = -\mathscr{M}_{-1}^{+-} \;\; \mathscr{M}_1^{--} = \mathscr{M}_{-1}^{--} \\
& M_1^{++} = -M_{-1}^{--} \;\; M_1^{+-} = M_{-1}^{-+} \;\; M_0^{+-} = -M_0^{-+} \;\; M_0^{++} = M_0^{--} \\
& M_{-1}^{+-} = M_{-1}^{++} = M_1^{-+} = M_1^{--} = 0
\end{aligned} \tag{56}
$$

And substituting values from Tabs. 4 and 5 we can write:

$$
\begin{aligned}
H_{-1,-1} = & (Im\mathscr{M}_{-1}^{++})^2 + (Re\mathscr{M}_{-1}^{++})^2 + \\
& + (F'_{1/2}A(sin\theta_K^*/2))^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + (Im\mathscr{M}_{-1}^{--})^2 + (Re\mathscr{M}_{-1}^{--})^2 + \\
& + (Im\mathscr{M}_{-1}^{+-})^2 + (Re\mathscr{M}_{-1}^{+-})^2 + \\
& + (F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2))^2 + (F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r))^2 + (Im\mathscr{M}_{-1}^{-+})^2 + (Re\mathscr{M}_{-1}^{-+})^2 + \\
& + 2F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_{-1}^{-+} + 2F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+} + \\
& + 2F'_{1/2}A(sin\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_{-1}^{--} + 2F'_{1/2}A(sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--}
\end{aligned} \tag{57}
$$

In conclusion

$$
\begin{aligned}
H_{-1,-1} = & (F'_{1/2}A)^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + |\mathscr{M}_{-1}^{++}|^2 + |\mathscr{M}_{-1}^{--}|^2 + |\mathscr{M}_{-1}^{+-}|^2 + |\mathscr{M}_{-1}^{-+}|^2 + \\
& + 2F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_{-1}^{-+} + 2F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+} + \\
& + 2F'_{1/2}A(sin\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_{-1}^{--} + 2F'_{1/2}A(sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--}
\end{aligned} \tag{58}
$$

2.1.4 $H_{1,-1}$

We want to calculate the term $H_{1,-1}$.
Considering the definition

$$
H_{\lambda\lambda'} = \sum_{\lambda_N \lambda_Y} M_\lambda^{'\lambda_N\lambda_Y}(M_\lambda^{'\lambda_N\lambda_Y})^\dagger \tag{59}
$$

and using

$$
M_\lambda^{'\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y} + \mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{60}
$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Table 3, for $J^P = \frac{1}{2}^+$. Specifically, the real parts are reported in Table 4, and the imaginary parts are reported in Table 5.
We have for $H_{1,-1}$ for $J^P = \frac{1}{2}^+$:

$$
\begin{aligned}
H_{1,-1} = {} & (M_1^{++} + \mathscr{M}_1^{++})(M_{-1}^{++} + \mathscr{M}_{-1}^{++})^\dagger + (M_1^{--} + \mathscr{M}_1^{--})(M_{-1}^{--} + \mathscr{M}_{-1}^{--})^\dagger + \\
& + (M_1^{+-} + \mathscr{M}_1^{+-})(M_{-1}^{+-} + \mathscr{M}_{-1}^{+-})^\dagger + (M_1^{-+} + \mathscr{M}_1^{-+})(M_{-1}^{-+} + \mathscr{M}_{-1}^{-+})^\dagger = \\
& = (ReM_1^{++} + iImM_1^{++} + Re\mathscr{M}_1^{++} + iIm\mathscr{M}_1^{++})(ReM_{-1}^{++} - iImM_{-1}^{++} + Re\mathscr{M}_{-1}^{++} - iIm\mathscr{M}_{-1}^{++}) + \\
& + (ReM_1^{--} + iImM_1^{--} + Re\mathscr{M}_1^{--} + iIm\mathscr{M}_1^{--})(ReM_{-1}^{--} - iImM_{-1}^{--} + Re\mathscr{M}_{-1}^{--} - iIm\mathscr{M}_{-1}^{--}) + \\
& + (ReM_1^{+-} + iImM_1^{+-} + Re\mathscr{M}_1^{+-} + iIm\mathscr{M}_1^{+-})(ReM_{-1}^{+-} - iImM_{-1}^{+-} + Re\mathscr{M}_{-1}^{+-} - iIm\mathscr{M}_{-1}^{+-}) + \\
& + (ReM_1^{-+} + iImM_1^{-+} + Re\mathscr{M}_1^{-+} + iIm\mathscr{M}_1^{-+})(ReM_{-1}^{-+} - iImM_{-1}^{-+} + Re\mathscr{M}_{-1}^{-+} - iIm\mathscr{M}_{-1}^{-+}) = \\
& = ReM_1^{++}ReM_{-1}^{++} - iReM_1^{++}ImM_{-1}^{++} + ReM_1^{++}Re\mathscr{M}_{-1}^{++} - iReM_1^{++}Im\mathscr{M}_{-1}^{++} + \\
& + iImM_1^{++}ReM_{-1}^{++} + ImM_1^{++}ImM_{-1}^{++} + iImM_1^{++}Re\mathscr{M}_{-1}^{++} + ImM_1^{++}Im\mathscr{M}_{-1}^{++} + \\
& + Re\mathscr{M}_1^{++}ReM_{-1}^{++} - iRe\mathscr{M}_1^{++}ImM_{-1}^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + \\
& + iIm\mathscr{M}_1^{++}ReM_{-1}^{++} + Im\mathscr{M}_1^{++}ImM_{-1}^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + \\
& + ReM_1^{--}ReM_{-1}^{--} - iReM_1^{--}ImM_{-1}^{--} + ReM_1^{--}Re\mathscr{M}_{-1}^{--} - iReM_1^{--}Im\mathscr{M}_{-1}^{--} + \\
& + iImM_1^{--}ReM_{-1}^{--} + ImM_1^{--}ImM_{-1}^{--} + iImM_1^{--}Re\mathscr{M}_{-1}^{--} + ImM_1^{--}Im\mathscr{M}_{-1}^{--} + \\
& + Re\mathscr{M}_1^{--}ReM_{-1}^{--} - iRe\mathscr{M}_1^{--}ImM_{-1}^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
& + iIm\mathscr{M}_1^{--}ReM_{-1}^{--} + Im\mathscr{M}_1^{--}ImM_{-1}^{--} + iIm\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
& + ReM_1^{+-}ReM_{-1}^{+-} - iReM_1^{+-}ImM_{-1}^{+-} + ReM_1^{+-}Re\mathscr{M}_{-1}^{+-} - iReM_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
& + iImM_1^{+-}ReM_{-1}^{+-} + ImM_1^{+-}ImM_{-1}^{+-} + iImM_1^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
& + Re\mathscr{M}_1^{+-}ReM_{-1}^{+-} - iRe\mathscr{M}_1^{+-}ImM_{-1}^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
& + iIm\mathscr{M}_1^{+-}ReM_{-1}^{+-} + Im\mathscr{M}_1^{+-}ImM_{-1}^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
& + ReM_1^{-+}ReM_{-1}^{-+} - iReM_1^{-+}ImM_{-1}^{-+} + ReM_1^{-+}Re\mathscr{M}_{-1}^{-+} - iReM_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
& + iImM_1^{-+}ReM_{-1}^{-+} + ImM_1^{-+}ImM_{-1}^{-+} + iImM_1^{-+}Re\mathscr{M}_{-1}^{-+} + ImM_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
& + Re\mathscr{M}_1^{-+}ReM_{-1}^{-+} - iRe\mathscr{M}_1^{-+}ImM_{-1}^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
& + iIm\mathscr{M}_1^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_1^{-+}ImM_{-1}^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+}
\end{aligned}
\tag{61}
$$

Considering that for $J^P = \frac{1}{2}^+$:

$$
M_{-1}^{+-} = M_{-1}^{++} = M_1^{-+} = M_1^{--} = 0 \tag{62}
$$

we can simplify:

$$
\begin{aligned}
H_{1,-1} = {} & ReM_1^{++}Re\mathscr{M}_{-1}^{++} - iReM_1^{++}Im\mathscr{M}_{-1}^{++} + \\
& + iImM_1^{++}Re\mathscr{M}_{-1}^{++} + ImM_1^{++}Im\mathscr{M}_{-1}^{++} + \\
& + Re\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + \\
& + iIm\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + \\
& + Re\mathscr{M}_1^{--}ReM_{-1}^{--} - iRe\mathscr{M}_1^{--}ImM_{-1}^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
& + iIm\mathscr{M}_1^{--}ReM_{-1}^{--} + Im\mathscr{M}_1^{--}ImM_{-1}^{--} + iIm\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
& + ReM_1^{+-}Re\mathscr{M}_{-1}^{+-} - iReM_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
& + iImM_1^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
& + Re\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
& + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
& + Re\mathscr{M}_1^{-+}ReM_{-1}^{-+} - iRe\mathscr{M}_1^{-+}ImM_{-1}^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
& + iIm\mathscr{M}_1^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_1^{-+}ImM_{-1}^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+}
\end{aligned}
\tag{63}
$$

2.1.5 $H_{-1,1}$

We want to calculate the term $H_{-1,1}$.

Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N\lambda_Y} M'^{\lambda_N\lambda_Y}_{\lambda}(M'^{\lambda_N\lambda_Y}_{\lambda})^{\dagger} \tag{64}$$

and using

$$M'^{\lambda_N\lambda_Y}_{\lambda} = M^{\lambda_N\lambda_Y}_{\lambda} + \mathscr{M}^{\lambda_N\lambda_Y}_{\lambda} \tag{65}$$

where the $\mathscr{M}^{\lambda_N\lambda_Y}_{\lambda}$ are the values from the grids, and the $M^{\lambda_N\lambda_Y}_{\lambda}$ are the values reported in Tables 3 and 7, for $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ respectively, we have for $H_{-1,1}$ for $J^P = \frac{1}{2}^+$:

$$\begin{aligned}
H_{-1,1} &= (M^{++}_{-1} + \mathscr{M}^{++}_{-1})(M^{++}_{1} + \mathscr{M}^{++}_{1})^{\dagger} + (M^{--}_{-1} + \mathscr{M}^{--}_{-1})(M^{--}_{1} + \mathscr{M}^{--}_{1})^{\dagger} + \\
&+ (M^{+-}_{-1} + \mathscr{M}^{+-}_{-1})(M^{+-}_{1} + \mathscr{M}^{+-}_{1})^{\dagger} + (M^{-+}_{-1} + \mathscr{M}^{-+}_{-1})(M^{-+}_{1} + \mathscr{M}^{-+}_{1})^{\dagger} = \\
&= (ReM^{++}_{-1} + iImM^{++}_{-1} + Re\mathscr{M}^{++}_{-1} + iIm\mathscr{M}^{++}_{-1})(ReM^{++}_{1} - iImM^{++}_{1} + Re\mathscr{M}^{++}_{1} - iIm\mathscr{M}^{++}_{1}) + \\
&+ (ReM^{--}_{-1} + iImM^{--}_{-1} + Re\mathscr{M}^{--}_{-1} + iIm\mathscr{M}^{--}_{-1})(ReM^{--}_{1} - iImM^{--}_{1} + Re\mathscr{M}^{--}_{1} - iIm\mathscr{M}^{--}_{1}) + \\
&+ (ReM^{+-}_{-1} + iImM^{+-}_{-1} + Re\mathscr{M}^{+-}_{-1} + iIm\mathscr{M}^{+-}_{-1})(ReM^{+-}_{1} - iImM^{+-}_{1} + Re\mathscr{M}^{+-}_{1} - iIm\mathscr{M}^{+-}_{1}) + \\
&+ (ReM^{-+}_{-1} + iImM^{-+}_{-1} + Re\mathscr{M}^{-+}_{-1} + iIm\mathscr{M}^{-+}_{-1})(ReM^{-+}_{1} - iImM^{-+}_{1} + Re\mathscr{M}^{-+}_{1} - iIm\mathscr{M}^{-+}_{1}) = \\
&= ReM^{++}_{-1}ReM^{++}_{1} - iReM^{++}_{-1}ImM^{++}_{1} + ReM^{++}_{-1}Re\mathscr{M}^{++}_{1} - iReM^{++}_{-1}Im\mathscr{M}^{++}_{1} + \\
&+ iImM^{++}_{-1}ReM^{++}_{1} + ImM^{++}_{-1}ImM^{++}_{1} + iImM^{++}_{-1}Re\mathscr{M}^{++}_{1} + ImM^{++}_{-1}Im\mathscr{M}^{++}_{1} + \\
&+ Re\mathscr{M}^{++}_{-1}ReM^{++}_{1} - iRe\mathscr{M}^{++}_{-1}ImM^{++}_{1} + Re\mathscr{M}^{++}_{-1}Re\mathscr{M}^{++}_{1} - iRe\mathscr{M}^{++}_{-1}Im\mathscr{M}^{++}_{1} + \\
&+ iIm\mathscr{M}^{++}_{-1}ReM^{++}_{1} + Im\mathscr{M}^{++}_{-1}ImM^{++}_{1} + iIm\mathscr{M}^{++}_{-1}Re\mathscr{M}^{++}_{1} + Im\mathscr{M}^{++}_{-1}Im\mathscr{M}^{++}_{1} + \\
&+ ReM^{--}_{-1}ReM^{--}_{1} - iReM^{--}_{-1}ImM^{--}_{1} + ReM^{--}_{-1}Re\mathscr{M}^{--}_{1} - iReM^{--}_{-1}Im\mathscr{M}^{--}_{1} + \\
&+ iImM^{--}_{-1}ReM^{--}_{1} + ImM^{--}_{-1}ImM^{--}_{1} + iImM^{--}_{-1}Re\mathscr{M}^{--}_{1} + ImM^{--}_{-1}Im\mathscr{M}^{--}_{1} + \\
&+ Re\mathscr{M}^{--}_{-1}ReM^{--}_{1} - iRe\mathscr{M}^{--}_{-1}ImM^{--}_{1} + Re\mathscr{M}^{--}_{-1}Re\mathscr{M}^{--}_{1} - iRe\mathscr{M}^{--}_{-1}Im\mathscr{M}^{--}_{1} + \\
&+ iIm\mathscr{M}^{--}_{-1}ReM^{--}_{1} + Im\mathscr{M}^{--}_{-1}ImM^{--}_{1} + iIm\mathscr{M}^{--}_{-1}Re\mathscr{M}^{--}_{1} + Im\mathscr{M}^{--}_{-1}Im\mathscr{M}^{--}_{1} + \\
&+ ReM^{+-}_{-1}ReM^{+-}_{1} - iReM^{+-}_{-1}ImM^{+-}_{1} + ReM^{+-}_{-1}Re\mathscr{M}^{+-}_{1} - iReM^{+-}_{-1}Im\mathscr{M}^{+-}_{1} + \\
&+ iImM^{+-}_{-1}ReM^{+-}_{1} + ImM^{+-}_{-1}ImM^{+-}_{1} + iImM^{+-}_{-1}Re\mathscr{M}^{+-}_{1} + ImM^{+-}_{-1}Im\mathscr{M}^{+-}_{1} + \\
&+ Re\mathscr{M}^{+-}_{-1}ReM^{+-}_{1} - iRe\mathscr{M}^{+-}_{-1}ImM^{+-}_{1} + Re\mathscr{M}^{+-}_{-1}Re\mathscr{M}^{+-}_{1} - iRe\mathscr{M}^{+-}_{-1}Im\mathscr{M}^{+-}_{1} + \\
&+ iIm\mathscr{M}^{+-}_{-1}ReM^{+-}_{1} + Im\mathscr{M}^{+-}_{-1}ImM^{+-}_{1} + iIm\mathscr{M}^{+-}_{-1}Re\mathscr{M}^{+-}_{1} + Im\mathscr{M}^{+-}_{-1}Im\mathscr{M}^{+-}_{1} + \\
&+ ReM^{-+}_{-1}ReM^{-+}_{1} - iReM^{-+}_{-1}ImM^{-+}_{1} + ReM^{-+}_{-1}Re\mathscr{M}^{-+}_{1} - iReM^{-+}_{-1}Im\mathscr{M}^{-+}_{1} + \\
&+ iImM^{-+}_{-1}ReM^{-+}_{1} + ImM^{-+}_{-1}ImM^{-+}_{1} + iImM^{-+}_{-1}Re\mathscr{M}^{-+}_{1} + ImM^{-+}_{-1}Im\mathscr{M}^{-+}_{1} + \\
&+ Re\mathscr{M}^{-+}_{-1}ReM^{-+}_{1} - iRe\mathscr{M}^{-+}_{-1}ImM^{-+}_{1} + Re\mathscr{M}^{-+}_{-1}Re\mathscr{M}^{-+}_{1} - iRe\mathscr{M}^{-+}_{-1}Im\mathscr{M}^{-+}_{1} + \\
&+ iIm\mathscr{M}^{-+}_{-1}ReM^{-+}_{1} + Im\mathscr{M}^{-+}_{-1}ImM^{-+}_{1} + iIm\mathscr{M}^{-+}_{-1}Re\mathscr{M}^{-+}_{1} + Im\mathscr{M}^{-+}_{-1}Im\mathscr{M}^{-+}_{1}
\end{aligned} \tag{66}$$

Considering that for $J^P = \frac{1}{2}^+$:

$$M^{+-}_{-1} = M^{++}_{-1} = M^{-+}_{1} = M^{--}_{1} = 0 \tag{67}$$

we can simplify:

$$
\begin{aligned}
H_{-1,1} = &+Re\mathscr{M}_{-1}^{++}ReM_1^{++} - iRe\mathscr{M}_{-1}^{++}ImM_1^{++} + Re\mathscr{M}_{-1}^{++}Re\mathscr{M}_1^{++} - iRe\mathscr{M}_{-1}^{++}Im\mathscr{M}_1^{++}+\\
&+iIm\mathscr{M}_{-1}^{++}ReM_1^{++} + Im\mathscr{M}_{-1}^{++}ImM_1^{++} + iIm\mathscr{M}_{-1}^{++}Re\mathscr{M}_1^{++} + Im\mathscr{M}_{-1}^{++}Im\mathscr{M}_1^{++}+\\
&+ReM_{-1}^{--}Re\mathscr{M}_1^{--} - iReM_{-1}^{--}Im\mathscr{M}_1^{--}+\\
&+iImM_{-1}^{--}Re\mathscr{M}_1^{--} + ImM_{-1}^{--}Im\mathscr{M}_1^{--}+\\
&+Re\mathscr{M}_{-1}^{--}Re\mathscr{M}_1^{--} - iRe\mathscr{M}_{-1}^{--}Im\mathscr{M}_1^{--}+\\
&+iIm\mathscr{M}_{-1}^{--}Re\mathscr{M}_1^{--} + Im\mathscr{M}_{-1}^{--}Im\mathscr{M}_1^{--}+\\
&+Re\mathscr{M}_{-1}^{+-}ReM_1^{+-} - iRe\mathscr{M}_{-1}^{+-}ImM_1^{+-} + Re\mathscr{M}_{-1}^{+-}Re\mathscr{M}_1^{+-} - iRe\mathscr{M}_{-1}^{+-}Im\mathscr{M}_1^{+-}+\\
&+iIm\mathscr{M}_{-1}^{+-}ReM_1^{+-} + Im\mathscr{M}_{-1}^{+-}ImM_1^{+-} + iIm\mathscr{M}_{-1}^{+-}Re\mathscr{M}_1^{+-} + Im\mathscr{M}_{-1}^{+-}Im\mathscr{M}_1^{+-}+\\
&+ReM_{-1}^{-+}Re\mathscr{M}_1^{-+} - iReM_{-1}^{-+}Im\mathscr{M}_1^{-+}+\\
&+iImM_{-1}^{-+}Re\mathscr{M}_1^{-+} + ImM_{-1}^{-+}Im\mathscr{M}_1^{-+}+\\
&+Re\mathscr{M}_{-1}^{-+}Re\mathscr{M}_1^{-+} - iRe\mathscr{M}_{-1}^{-+}Im\mathscr{M}_1^{-+}+\\
&+iIm\mathscr{M}_{-1}^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_{-1}^{-+}Im\mathscr{M}_1^{-+}
\end{aligned} \tag{68}
$$

2.1.6 $H_{0,1}$

We want to calculate the term $H_{0,1}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N\lambda_Y} M_\lambda^{'\lambda_N\lambda_Y}(M_\lambda^{'\lambda_N\lambda_Y})^\dagger \tag{69}$$

and using

$$M_\lambda^{'\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y} + \mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{70}$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Tables 3 and 7, for $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ respectively, we have for $H_{0,1}$ for $J^P = \frac{1}{2}^+$:

$$
\begin{aligned}
H_{0,1} = {} & (M_0^{++} + \mathscr{M}_0^{++})(M_1^{++} + \mathscr{M}_1^{++})^\dagger + (M_0^{--} + \mathscr{M}_0^{--})(M_1^{--} + \mathscr{M}_1^{--})^\dagger + \\
& + (M_0^{+-} + \mathscr{M}_0^{+-})(M_1^{+-} + \mathscr{M}_1^{+-})^\dagger + (M_0^{-+} + \mathscr{M}_0^{-+})(M_1^{-+} + \mathscr{M}_1^{-+})^\dagger = \\
& = (ReM_0^{++} + iImM_0^{++} + Re\mathscr{M}_0^{++} + iIm\mathscr{M}_0^{++})(ReM_1^{++} - iImM_1^{++} + Re\mathscr{M}_1^{++} - iIm\mathscr{M}_1^{++}) + \\
& + (ReM_0^{--} + iImM_0^{--} + Re\mathscr{M}_0^{--} + iIm\mathscr{M}_0^{--})(ReM_1^{--} - iImM_1^{--} + Re\mathscr{M}_1^{--} - iIm\mathscr{M}_1^{--}) + \\
& + (ReM_0^{+-} + iImM_0^{+-} + Re\mathscr{M}_0^{+-} + iIm\mathscr{M}_0^{+-})(ReM_1^{+-} - iImM_1^{+-} + Re\mathscr{M}_1^{+-} - iIm\mathscr{M}_1^{+-}) + \\
& + (ReM_0^{-+} + iImM_0^{-+} + Re\mathscr{M}_0^{-+} + iIm\mathscr{M}_0^{-+})(ReM_1^{-+} - iImM_1^{-+} + Re\mathscr{M}_1^{-+} - iIm\mathscr{M}_1^{-+}) = \\
& = ReM_0^{++}ReM_1^{++} - iReM_0^{++}ImM_1^{++} + ReM_0^{++}Re\mathscr{M}_1^{++} - iReM_0^{++}Im\mathscr{M}_1^{++} + \\
& + iImM_0^{++}ReM_1^{++} + ImM_0^{++}ImM_1^{++} + iImM_0^{++}Re\mathscr{M}_1^{++} + ImM_0^{++}Im\mathscr{M}_1^{++} + \\
& + Re\mathscr{M}_0^{++}ReM_1^{++} - iRe\mathscr{M}_0^{++}ImM_1^{++} + Re\mathscr{M}_0^{++}Re\mathscr{M}_1^{++} - iRe\mathscr{M}_0^{++}Im\mathscr{M}_1^{++} + \\
& + iIm\mathscr{M}_0^{++}ReM_1^{++} + Im\mathscr{M}_0^{++}ImM_1^{++} + iIm\mathscr{M}_0^{++}Re\mathscr{M}_1^{++} + Im\mathscr{M}_0^{++}Im\mathscr{M}_1^{++} + \\
& + ReM_0^{--}ReM_1^{--} - iReM_0^{--}ImM_1^{--} + ReM_0^{--}Re\mathscr{M}_1^{--} - iReM_0^{--}Im\mathscr{M}_1^{--} + \\
& + iImM_0^{--}ReM_1^{--} + ImM_0^{--}ImM_1^{--} + iImM_0^{--}Re\mathscr{M}_1^{--} + ImM_0^{--}Im\mathscr{M}_1^{--} + \\
& + Re\mathscr{M}_0^{--}ReM_1^{--} - iRe\mathscr{M}_0^{--}ImM_1^{--} + Re\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} - iRe\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + \\
& + iIm\mathscr{M}_0^{--}ReM_1^{--} + Im\mathscr{M}_0^{--}ImM_1^{--} + iIm\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} + Im\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + \\
& + ReM_0^{+-}ReM_1^{+-} - iReM_0^{+-}ImM_1^{+-} + ReM_0^{+-}Re\mathscr{M}_1^{+-} - iReM_0^{+-}Im\mathscr{M}_1^{+-} + \\
& + iImM_0^{+-}ReM_1^{+-} + ImM_0^{+-}ImM_1^{+-} + iImM_0^{+-}Re\mathscr{M}_1^{+-} + ImM_0^{+-}Im\mathscr{M}_1^{+-} + \\
& + Re\mathscr{M}_0^{+-}ReM_1^{+-} - iRe\mathscr{M}_0^{+-}ImM_1^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} - iRe\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + \\
& + iIm\mathscr{M}_0^{+-}ReM_1^{+-} + Im\mathscr{M}_0^{+-}ImM_1^{+-} + iIm\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + \\
& + ReM_0^{-+}ReM_1^{-+} - iReM_0^{-+}ImM_1^{-+} + ReM_0^{-+}Re\mathscr{M}_1^{-+} - iReM_0^{-+}Im\mathscr{M}_1^{-+} + \\
& + iImM_0^{-+}ReM_1^{-+} + ImM_0^{-+}ImM_1^{-+} + iImM_0^{-+}Re\mathscr{M}_1^{-+} + ImM_0^{-+}Im\mathscr{M}_1^{-+} + \\
& + Re\mathscr{M}_0^{-+}ReM_1^{-+} - iRe\mathscr{M}_0^{-+}ImM_1^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} - iRe\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+} + \\
& + iIm\mathscr{M}_0^{-+}ReM_1^{-+} + Im\mathscr{M}_0^{-+}ImM_1^{-+} + iIm\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+}
\end{aligned}
\tag{71}
$$

Considering that for $J^P = \frac{1}{2}^+$:

$$
M_{-1}^{+-} = M_{-1}^{++} = M_1^{-+} = M_1^{--} = 0 \tag{72}
$$

we can simplify:

$$
\begin{aligned}
H_{0,1} = {} & ReM_0^{++}ReM_1^{++} - iReM_0^{++}ImM_1^{++} + ReM_0^{++}Re\mathscr{M}_1^{++} - iReM_0^{++}Im\mathscr{M}_1^{++} + \\
& + iImM_0^{++}ReM_1^{++} + ImM_0^{++}ImM_1^{++} + iImM_0^{++}Re\mathscr{M}_1^{++} + ImM_0^{++}Im\mathscr{M}_1^{++} + \\
& + Re\mathscr{M}_0^{++}ReM_1^{++} - iRe\mathscr{M}_0^{++}ImM_1^{++} + Re\mathscr{M}_0^{++}Re\mathscr{M}_1^{++} - iRe\mathscr{M}_0^{++}Im\mathscr{M}_1^{++} + \\
& + iIm\mathscr{M}_0^{++}ReM_1^{++} + Im\mathscr{M}_0^{++}ImM_1^{++} + iIm\mathscr{M}_0^{++}Re\mathscr{M}_1^{++} + Im\mathscr{M}_0^{++}Im\mathscr{M}_1^{++} + \\
& + ReM_0^{--}Re\mathscr{M}_1^{--} - iReM_0^{--}Im\mathscr{M}_1^{--} + iImM_0^{--}Re\mathscr{M}_1^{--} + ImM_0^{--}Im\mathscr{M}_1^{--} + \\
& + Re\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} - iRe\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + iIm\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} + Im\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + \\
& + ReM_0^{+-}ReM_1^{+-} - iReM_0^{+-}ImM_1^{+-} + ReM_0^{+-}Re\mathscr{M}_1^{+-} - iReM_0^{+-}Im\mathscr{M}_1^{+-} + \\
& + iImM_0^{+-}ReM_1^{+-} + ImM_0^{+-}ImM_1^{+-} + iImM_0^{+-}Re\mathscr{M}_1^{+-} + ImM_0^{+-}Im\mathscr{M}_1^{+-} + \\
& + Re\mathscr{M}_0^{+-}ReM_1^{+-} - iRe\mathscr{M}_0^{+-}ImM_1^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} - iRe\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + \\
& + iIm\mathscr{M}_0^{+-}ReM_1^{+-} + Im\mathscr{M}_0^{+-}ImM_1^{+-} + iIm\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + \\
& + ReM_0^{-+}Re\mathscr{M}_1^{-+} - iReM_0^{-+}Im\mathscr{M}_1^{-+} + iImM_0^{-+}Re\mathscr{M}_1^{-+} + ImM_0^{-+}Im\mathscr{M}_1^{-+} + \\
& + Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} - iRe\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+} + iIm\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+}
\end{aligned}
\tag{73}
$$

2.1.7 $H_{1,0}$

We want to calculate the term $H_{1,0}$.

Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N \lambda_Y} M'^{\lambda_N \lambda_Y}_{\lambda} (M'^{\lambda_N \lambda_Y}_{\lambda})^\dagger \tag{74}$$

and using

$$M'^{\lambda_N \lambda_Y}_{\lambda} = M^{\lambda_N \lambda_Y}_{\lambda} + \mathscr{M}^{\lambda_N \lambda_Y}_{\lambda} \tag{75}$$

where the $\mathscr{M}^{\lambda_N \lambda_Y}_{\lambda}$ are the values from the grids, and the $M^{\lambda_N \lambda_Y}_{\lambda}$ are the values reported in Tables 3 and 7, for $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ respectively, we have for $H_{1,0}$ for $J^P = \frac{1}{2}^+$:

$$\begin{aligned}
H_{1,0} &= (M_1^{++} + \mathscr{M}_1^{++})(M_0^{++} + \mathscr{M}_0^{++})^\dagger + (M_1^{--} + \mathscr{M}_1^{--})(M_0^{--} + \mathscr{M}_0^{--})^\dagger + \\
&+ (M_1^{+-} + \mathscr{M}_1^{+-})(M_0^{+-} + \mathscr{M}_0^{+-})^\dagger + (M_1^{-+} + \mathscr{M}_1^{-+})(M_0^{-+} + \mathscr{M}_0^{-+})^\dagger = \\
&= (ReM_1^{++} + iImM_1^{++} + Re\mathscr{M}_1^{++} + iIm\mathscr{M}_1^{++})(ReM_0^{++} - iImM_0^{++} + Re\mathscr{M}_0^{++} - iIm\mathscr{M}_0^{++}) + \\
&+ (ReM_1^{--} + iImM_1^{--} + Re\mathscr{M}_1^{--} + iIm\mathscr{M}_1^{--})(ReM_0^{--} - iImM_0^{--} + Re\mathscr{M}_0^{--} - iIm\mathscr{M}_0^{--}) + \\
&+ (ReM_1^{+-} + iImM_1^{+-} + Re\mathscr{M}_1^{+-} + iIm\mathscr{M}_1^{+-})(ReM_0^{+-} - iImM_0^{+-} + Re\mathscr{M}_0^{+-} - iIm\mathscr{M}_0^{+-}) + \\
&+ (ReM_1^{-+} + iImM_1^{-+} + Re\mathscr{M}_1^{-+} + iIm\mathscr{M}_1^{-+})(ReM_0^{-+} - iImM_0^{-+} + Re\mathscr{M}_0^{-+} - iIm\mathscr{M}_0^{-+}) = \\
&= ReM_1^{++}ReM_0^{++} - iReM_1^{++}ImM_0^{++} + ReM_1^{++}Re\mathscr{M}_0^{++} - iReM_1^{++}Im\mathscr{M}_0^{++} + \\
&+ iImM_1^{++}ReM_0^{++} + ImM_1^{++}ImM_0^{++} + iImM_1^{++}Re\mathscr{M}_0^{++} + ImM_1^{++}Im\mathscr{M}_0^{++} + \\
&+ Re\mathscr{M}_1^{++}ReM_0^{++} - iRe\mathscr{M}_1^{++}ImM_0^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_0^{++} + \\
&+ iIm\mathscr{M}_1^{++}ReM_0^{++} + Im\mathscr{M}_1^{++}ImM_0^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_0^{++} + \\
&+ ReM_1^{--}ReM_0^{--} - iReM_1^{--}ImM_0^{--} + ReM_1^{--}Re\mathscr{M}_0^{--} - iReM_1^{--}Im\mathscr{M}_0^{--} + \\
&+ iImM_1^{--}ReM_0^{--} + ImM_1^{--}ImM_0^{--} + iImM_1^{--}Re\mathscr{M}_0^{--} + ImM_1^{--}Im\mathscr{M}_0^{--} + \\
&+ Re\mathscr{M}_1^{--}ReM_0^{--} - iRe\mathscr{M}_1^{--}ImM_0^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + \\
&+ iIm\mathscr{M}_1^{--}ReM_0^{--} + Im\mathscr{M}_1^{--}ImM_0^{--} + iIm\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + \\
&+ ReM_1^{+-}ReM_0^{+-} - iReM_1^{+-}ImM_0^{+-} + ReM_1^{+-}Re\mathscr{M}_0^{+-} - iReM_1^{+-}Im\mathscr{M}_0^{+-} + \\
&+ iImM_1^{+-}ReM_0^{+-} + ImM_1^{+-}ImM_0^{+-} + iImM_1^{+-}Re\mathscr{M}_0^{+-} + ImM_1^{+-}Im\mathscr{M}_0^{+-} + \\
&+ Re\mathscr{M}_1^{+-}ReM_0^{+-} - iRe\mathscr{M}_1^{+-}ImM_0^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + \\
&+ iIm\mathscr{M}_1^{+-}ReM_0^{+-} + Im\mathscr{M}_1^{+-}ImM_0^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + \\
&+ ReM_1^{-+}ReM_0^{-+} - iReM_1^{-+}ImM_0^{-+} + ReM_1^{-+}Re\mathscr{M}_0^{-+} - iReM_1^{-+}Im\mathscr{M}_0^{-+} + \\
&+ iImM_1^{-+}ReM_0^{-+} + ImM_1^{-+}ImM_0^{-+} + iImM_1^{-+}Re\mathscr{M}_0^{-+} + ImM_1^{-+}Im\mathscr{M}_0^{-+} + \\
&+ Re\mathscr{M}_1^{-+}ReM_0^{-+} - iRe\mathscr{M}_1^{-+}ImM_0^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+} + \\
&+ iIm\mathscr{M}_1^{-+}ReM_0^{-+} + Im\mathscr{M}_1^{-+}ImM_0^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+}
\end{aligned} \tag{76}$$

Considering that for $J^P = \frac{1}{2}^+$:

$$M_{-1}^{+-} = M_{-1}^{++} = M_1^{-+} = M_1^{--} = 0 \tag{77}$$

we can simplify:

$$\begin{aligned}
H_{1,0} = {} & ReM_1^{++}ReM_0^{++} - iReM_1^{++}ImM_0^{++} + ReM_1^{++}Re\mathscr{M}_0^{++} - iReM_1^{++}Im\mathscr{M}_0^{++} + \\
& + iImM_1^{++}ReM_0^{++} + ImM_1^{++}ImM_0^{++} + iImM_1^{++}Re\mathscr{M}_0^{++} + ImM_1^{++}Im\mathscr{M}_0^{++} + \\
& + Re\mathscr{M}_1^{++}ReM_0^{++} - iRe\mathscr{M}_1^{++}ImM_0^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_0^{++} + \\
& + iIm\mathscr{M}_1^{++}ReM_0^{++} + Im\mathscr{M}_1^{++}ImM_0^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_0^{++} + \\
& + Re\mathscr{M}_1^{--}ReM_0^{--} - iRe\mathscr{M}_1^{--}ImM_0^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + \\
& + iIm\mathscr{M}_1^{--}ReM_0^{--} + Im\mathscr{M}_1^{--}ImM_0^{--} + iIm\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + \\
& + ReM_1^{+-}ReM_0^{+-} - iReM_1^{+-}ImM_0^{+-} + ReM_1^{+-}Re\mathscr{M}_0^{+-} - iReM_1^{+-}Im\mathscr{M}_0^{+-} + \\
& + iImM_1^{+-}ReM_0^{+-} + ImM_1^{+-}ImM_0^{+-} + iImM_1^{+-}Re\mathscr{M}_0^{+-} + ImM_1^{+-}Im\mathscr{M}_0^{+-} + \\
& + Re\mathscr{M}_1^{+-}ReM_0^{+-} - iRe\mathscr{M}_1^{+-}ImM_0^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + \\
& + iIm\mathscr{M}_1^{+-}ReM_0^{+-} + Im\mathscr{M}_1^{+-}ImM_0^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + \\
& + Re\mathscr{M}_1^{-+}ReM_0^{-+} - iRe\mathscr{M}_1^{-+}ImM_0^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+} + \\
& + iIm\mathscr{M}_1^{-+}ReM_0^{-+} + Im\mathscr{M}_1^{-+}ImM_0^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+}
\end{aligned} \tag{78}$$

2.1.8 $H_{0,-1}$

We want to calculate the term $H_{0,-1}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N\lambda_Y} M_\lambda^{'\lambda_N\lambda_Y}(M_\lambda^{'\lambda_N\lambda_Y})^\dagger \tag{79}$$

and using

$$M_\lambda^{'\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y} + \mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{80}$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Tables 3 and 7, for $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ respectively, we have for $H_{0,-1}$ for $J^P = \frac{1}{2}^+$:

$$
\begin{aligned}
H_{0,-1} &= (M_0^{++} + \mathscr{M}_0^{++})(M_{-1}^{++} + \mathscr{M}_{-1}^{++})^\dagger + (M_0^{--} + \mathscr{M}_0^{--})(M_{-1}^{--} + \mathscr{M}_{-1}^{--})^\dagger + \\
&+ (M_0^{+-} + \mathscr{M}_0^{+-})(M_{-1}^{+-} + \mathscr{M}_{-1}^{+-})^\dagger + (M_0^{-+} + \mathscr{M}_0^{-+})(M_{-1}^{-+} + \mathscr{M}_{-1}^{-+})^\dagger = \\
&= (ReM_0^{++} + iImM_0^{++} + Re\mathscr{M}_0^{++} + iIm\mathscr{M}_0^{++})(ReM_{-1}^{++} - iImM_{-1}^{++} + Re\mathscr{M}_{-1}^{++} - iIm\mathscr{M}_{-1}^{++}) + \\
&+ (ReM_0^{--} + iImM_0^{--} + Re\mathscr{M}_0^{--} + iIm\mathscr{M}_0^{--})(ReM_{-1}^{--} - iImM_{-1}^{--} + Re\mathscr{M}_{-1}^{--} - iIm\mathscr{M}_{-1}^{--}) + \\
&+ (ReM_0^{+-} + iImM_0^{+-} + Re\mathscr{M}_0^{+-} + iIm\mathscr{M}_0^{+-})(ReM_{-1}^{+-} - iImM_{-1}^{+-} + Re\mathscr{M}_{-1}^{+-} - iIm\mathscr{M}_{-1}^{+-}) + \\
&+ (ReM_0^{-+} + iImM_0^{-+} + Re\mathscr{M}_0^{-+} + iIm\mathscr{M}_0^{-+})(ReM_{-1}^{-+} - iImM_{-1}^{-+} + Re\mathscr{M}_{-1}^{-+} - iIm\mathscr{M}_{-1}^{-+}) = \\
&= ReM_0^{++}ReM_{-1}^{++} - iReM_0^{++}ImM_{-1}^{++} + ReM_0^{++}Re\mathscr{M}_{-1}^{++} - iReM_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ iImM_0^{++}ReM_{-1}^{++} + ImM_0^{++}ImM_{-1}^{++} + iImM_0^{++}Re\mathscr{M}_{-1}^{++} + ImM_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ Re\mathscr{M}_0^{++}ReM_{-1}^{++} - iRe\mathscr{M}_0^{++}ImM_{-1}^{++} + Re\mathscr{M}_0^{++}Re\mathscr{M}_{-1}^{++} - iRe\mathscr{M}_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ iIm\mathscr{M}_0^{++}ReM_{-1}^{++} + Im\mathscr{M}_0^{++}ImM_{-1}^{++} + iIm\mathscr{M}_0^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ ReM_0^{--}ReM_{-1}^{--} - iReM_0^{--}ImM_{-1}^{--} + ReM_0^{--}Re\mathscr{M}_{-1}^{--} - iReM_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ iImM_0^{--}ReM_{-1}^{--} + ImM_0^{--}ImM_{-1}^{--} + iImM_0^{--}Re\mathscr{M}_{-1}^{--} + ImM_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ Re\mathscr{M}_0^{--}ReM_{-1}^{--} - iRe\mathscr{M}_0^{--}ImM_{-1}^{--} + Re\mathscr{M}_0^{--}Re\mathscr{M}_{-1}^{--} - iRe\mathscr{M}_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ iIm\mathscr{M}_0^{--}ReM_{-1}^{--} + Im\mathscr{M}_0^{--}ImM_{-1}^{--} + iIm\mathscr{M}_0^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ ReM_0^{+-}ReM_{-1}^{+-} - iReM_0^{+-}ImM_{-1}^{+-} + ReM_0^{+-}Re\mathscr{M}_{-1}^{+-} - iReM_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ iImM_0^{+-}ReM_{-1}^{+-} + ImM_0^{+-}ImM_{-1}^{+-} + iImM_0^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ Re\mathscr{M}_0^{+-}ReM_{-1}^{+-} - iRe\mathscr{M}_0^{+-}ImM_{-1}^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_{-1}^{+-} - iRe\mathscr{M}_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ iIm\mathscr{M}_0^{+-}ReM_{-1}^{+-} + Im\mathscr{M}_0^{+-}ImM_{-1}^{+-} + iIm\mathscr{M}_0^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ ReM_0^{-+}ReM_{-1}^{-+} - iReM_0^{-+}ImM_{-1}^{-+} + ReM_0^{-+}Re\mathscr{M}_{-1}^{-+} - iReM_0^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ iImM_0^{-+}ReM_{-1}^{-+} + ImM_0^{-+}ImM_{-1}^{-+} + iImM_0^{-+}Re\mathscr{M}_{-1}^{-+} + ImM_0^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ Re\mathscr{M}_0^{-+}ReM_{-1}^{-+} - iRe\mathscr{M}_0^{-+}ImM_{-1}^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_{-1}^{-+} - iRe\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ iIm\mathscr{M}_0^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_0^{-+}ImM_{-1}^{-+} + iIm\mathscr{M}_0^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+}
\end{aligned} \tag{81}
$$

Considering that for $J^P = \frac{1}{2}^+$:

$$
M_{-1}^{+-} = M_{-1}^{++} = M_{1}^{-+} = M_{1}^{--} = 0 \tag{82}
$$

we can simplify:

$$
\begin{aligned}
H_{0,-1} &= ReM_0^{++}Re\mathscr{M}_{-1}^{++} - iReM_0^{++}Im\mathscr{M}_{-1}^{++} + + iImM_0^{++}Re\mathscr{M}_{-1}^{++} + ImM_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ Re\mathscr{M}_0^{++}Re\mathscr{M}_{-1}^{++} - iRe\mathscr{M}_0^{++}Im\mathscr{M}_{-1}^{++} + + iIm\mathscr{M}_0^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ ReM_0^{--}ReM_{-1}^{--} - iReM_0^{--}ImM_{-1}^{--} + ReM_0^{--}Re\mathscr{M}_{-1}^{--} - iReM_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ iImM_0^{--}ReM_{-1}^{--} + ImM_0^{--}ImM_{-1}^{--} + iImM_0^{--}Re\mathscr{M}_{-1}^{--} + ImM_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ Re\mathscr{M}_0^{--}ReM_{-1}^{--} - iRe\mathscr{M}_0^{--}ImM_{-1}^{--} + Re\mathscr{M}_0^{--}Re\mathscr{M}_{-1}^{--} - iRe\mathscr{M}_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ iIm\mathscr{M}_0^{--}ReM_{-1}^{--} + Im\mathscr{M}_0^{--}ImM_{-1}^{--} + iIm\mathscr{M}_0^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ ReM_0^{+-}Re\mathscr{M}_{-1}^{+-} - iReM_0^{+-}Im\mathscr{M}_{-1}^{+-} + + iImM_0^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ Re\mathscr{M}_0^{+-}Re\mathscr{M}_{-1}^{+-} - iRe\mathscr{M}_0^{+-}Im\mathscr{M}_{-1}^{+-} + + iIm\mathscr{M}_0^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ ReM_0^{-+}ReM_{-1}^{-+} - iReM_0^{-+}ImM_{-1}^{-+} + ReM_0^{-+}Re\mathscr{M}_{-1}^{-+} - iReM_0^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ iImM_0^{-+}ReM_{-1}^{-+} + ImM_0^{-+}ImM_{-1}^{-+} + iImM_0^{-+}Re\mathscr{M}_{-1}^{-+} + ImM_0^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ Re\mathscr{M}_0^{-+}ReM_{-1}^{-+} - iRe\mathscr{M}_0^{-+}ImM_{-1}^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_{-1}^{-+} - iRe\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ iIm\mathscr{M}_0^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_0^{-+}ImM_{-1}^{-+} + iIm\mathscr{M}_0^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+}
\end{aligned} \tag{83}
$$

2.1.9 $H_{-1,0}$

We want to calculate the term $H_{-1,0}$.

Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N \lambda_Y} M'^{\lambda_N \lambda_Y}_{\lambda} (M'^{\lambda_N \lambda_Y}_{\lambda})^{\dagger} \tag{84}$$

and using

$$M'^{\lambda_N \lambda_Y}_{\lambda} = M^{\lambda_N \lambda_Y}_{\lambda} + \mathscr{M}^{\lambda_N \lambda_Y}_{\lambda} \tag{85}$$

where the $\mathscr{M}^{\lambda_N \lambda_Y}_{\lambda}$ are the values from the grids, and the $M^{\lambda_N \lambda_Y}_{\lambda}$ are the values reported in Tables 3 and 7, for $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ respectively, we have for $H_{-1,0}$ for $J^P = \frac{1}{2}^+$:

$$\begin{aligned}
H_{-1,0} &= (M^{++}_{-1} + \mathscr{M}^{++}_{-1})(M^{++}_{0} + \mathscr{M}^{++}_{0})^{\dagger} + (M^{--}_{-1} + \mathscr{M}^{--}_{-1})(M^{--}_{0} + \mathscr{M}^{--}_{0})^{\dagger} + \\
&+ (M^{+-}_{-1} + \mathscr{M}^{+-}_{-1})(M^{+-}_{0} + \mathscr{M}^{+-}_{0})^{\dagger} + (M^{-+}_{-1} + \mathscr{M}^{-+}_{-1})(M^{-+}_{0} + \mathscr{M}^{-+}_{0})^{\dagger} = \\
&= (ReM^{++}_{-1} + iImM^{++}_{-1} + Re\mathscr{M}^{++}_{-1} + iIm\mathscr{M}^{++}_{-1})(ReM^{++}_{0} - iImM^{++}_{0} + Re\mathscr{M}^{++}_{0} - iIm\mathscr{M}^{++}_{0}) + \\
&+ (ReM^{--}_{-1} + iImM^{--}_{-1} + Re\mathscr{M}^{--}_{-1} + iIm\mathscr{M}^{--}_{-1})(ReM^{--}_{0} - iImM^{--}_{0} + Re\mathscr{M}^{--}_{0} - iIm\mathscr{M}^{--}_{0}) + \\
&+ (ReM^{+-}_{-1} + iImM^{+-}_{-1} + Re\mathscr{M}^{+-}_{-1} + iIm\mathscr{M}^{+-}_{-1})(ReM^{+-}_{0} - iImM^{+-}_{0} + Re\mathscr{M}^{+-}_{0} - iIm\mathscr{M}^{+-}_{0}) + \\
&+ (ReM^{-+}_{-1} + iImM^{-+}_{-1} + Re\mathscr{M}^{-+}_{-1} + iIm\mathscr{M}^{-+}_{-1})(ReM^{-+}_{0} - iImM^{-+}_{0} + Re\mathscr{M}^{-+}_{0} - iIm\mathscr{M}^{-+}_{0}) = \\
&= ReM^{++}_{-1}ReM^{++}_{0} - iReM^{++}_{-1}ImM^{++}_{0} + ReM^{++}_{-1}Re\mathscr{M}^{++}_{0} - iReM^{++}_{-1}Im\mathscr{M}^{++}_{0} + \\
&+ iImM^{++}_{-1}ReM^{++}_{0} + ImM^{++}_{-1}ImM^{++}_{0} + iImM^{++}_{-1}Re\mathscr{M}^{++}_{0} + ImM^{++}_{-1}Im\mathscr{M}^{++}_{0} + \\
&+ Re\mathscr{M}^{++}_{-1}ReM^{++}_{0} - iRe\mathscr{M}^{++}_{-1}ImM^{++}_{0} + Re\mathscr{M}^{++}_{-1}Re\mathscr{M}^{++}_{0} - iRe\mathscr{M}^{++}_{-1}Im\mathscr{M}^{++}_{0} + \\
&+ iIm\mathscr{M}^{++}_{-1}ReM^{++}_{0} + Im\mathscr{M}^{++}_{-1}ImM^{++}_{0} + iIm\mathscr{M}^{++}_{-1}Re\mathscr{M}^{++}_{0} + Im\mathscr{M}^{++}_{-1}Im\mathscr{M}^{++}_{0} + \\
&+ ReM^{--}_{-1}ReM^{--}_{0} - iReM^{--}_{-1}ImM^{--}_{0} + ReM^{--}_{-1}Re\mathscr{M}^{--}_{0} - iReM^{--}_{-1}Im\mathscr{M}^{--}_{0} + \\
&+ iImM^{--}_{-1}ReM^{--}_{0} + ImM^{--}_{-1}ImM^{--}_{0} + iImM^{--}_{-1}Re\mathscr{M}^{--}_{0} + ImM^{--}_{-1}Im\mathscr{M}^{--}_{0} + \\
&+ Re\mathscr{M}^{--}_{-1}ReM^{--}_{0} - iRe\mathscr{M}^{--}_{-1}ImM^{--}_{0} + Re\mathscr{M}^{--}_{-1}Re\mathscr{M}^{--}_{0} - iRe\mathscr{M}^{--}_{-1}Im\mathscr{M}^{--}_{0} + \\
&+ iIm\mathscr{M}^{--}_{-1}ReM^{--}_{0} + Im\mathscr{M}^{--}_{-1}ImM^{--}_{0} + iIm\mathscr{M}^{--}_{-1}Re\mathscr{M}^{--}_{0} + Im\mathscr{M}^{--}_{-1}Im\mathscr{M}^{--}_{0} + \\
&+ ReM^{+-}_{-1}ReM^{+-}_{0} - iReM^{+-}_{-1}ImM^{+-}_{0} + ReM^{+-}_{-1}Re\mathscr{M}^{+-}_{0} - iReM^{+-}_{-1}Im\mathscr{M}^{+-}_{0} + \\
&+ iImM^{+-}_{-1}ReM^{+-}_{0} + ImM^{+-}_{-1}ImM^{+-}_{0} + iImM^{+-}_{-1}Re\mathscr{M}^{+-}_{0} + ImM^{+-}_{-1}Im\mathscr{M}^{+-}_{0} + \\
&+ Re\mathscr{M}^{+-}_{-1}ReM^{+-}_{0} - iRe\mathscr{M}^{+-}_{-1}ImM^{+-}_{0} + Re\mathscr{M}^{+-}_{-1}Re\mathscr{M}^{+-}_{0} - iRe\mathscr{M}^{+-}_{-1}Im\mathscr{M}^{+-}_{0} + \\
&+ iIm\mathscr{M}^{+-}_{-1}ReM^{+-}_{0} + Im\mathscr{M}^{+-}_{-1}ImM^{+-}_{0} + iIm\mathscr{M}^{+-}_{-1}Re\mathscr{M}^{+-}_{0} + Im\mathscr{M}^{+-}_{-1}Im\mathscr{M}^{+-}_{0} + \\
&+ ReM^{-+}_{-1}ReM^{-+}_{0} - iReM^{-+}_{-1}ImM^{-+}_{0} + ReM^{-+}_{-1}Re\mathscr{M}^{-+}_{0} - iReM^{-+}_{-1}Im\mathscr{M}^{-+}_{0} + \\
&+ iImM^{-+}_{-1}ReM^{-+}_{0} + ImM^{-+}_{-1}ImM^{-+}_{0} + iImM^{-+}_{-1}Re\mathscr{M}^{-+}_{0} + ImM^{-+}_{-1}Im\mathscr{M}^{-+}_{0} + \\
&+ Re\mathscr{M}^{-+}_{-1}ReM^{-+}_{0} - iRe\mathscr{M}^{-+}_{-1}ImM^{-+}_{0} + Re\mathscr{M}^{-+}_{-1}Re\mathscr{M}^{-+}_{0} - iRe\mathscr{M}^{-+}_{-1}Im\mathscr{M}^{-+}_{0} + \\
&+ iIm\mathscr{M}^{-+}_{-1}ReM^{-+}_{0} + Im\mathscr{M}^{-+}_{-1}ImM^{-+}_{0} + iIm\mathscr{M}^{-+}_{-1}Re\mathscr{M}^{-+}_{0} + Im\mathscr{M}^{-+}_{-1}Im\mathscr{M}^{-+}_{0}
\end{aligned} \tag{86}$$

Considering that for $J^P = \frac{1}{2}^+$:

$$M^{+-}_{-1} = M^{++}_{-1} = M^{-+}_{1} = M^{--}_{1} = 0 \tag{87}$$

we can simplify:

$$
\begin{aligned}
H_{-1,0} = {} & +Re\mathscr{M}_{-1}^{++}ReM_0^{++} - iRe\mathscr{M}_{-1}^{++}ImM_0^{++} + Re\mathscr{M}_{-1}^{++}Re\mathscr{M}_0^{++} - iRe\mathscr{M}_{-1}^{++}Im\mathscr{M}_0^{++}+ \\
& +iIm\mathscr{M}_{-1}^{++}ReM_0^{++} + Im\mathscr{M}_{-1}^{++}ImM_0^{++} + iIm\mathscr{M}_{-1}^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_{-1}^{++}Im\mathscr{M}_0^{++}+ \\
& + ReM_{-1}^{--}ReM_0^{--} - iReM_{-1}^{--}ImM_0^{--} + ReM_{-1}^{--}Re\mathscr{M}_0^{--} - iReM_{-1}^{--}Im\mathscr{M}_0^{--}+ \\
& +iImM_{-1}^{--}ReM_0^{--} + ImM_{-1}^{--}ImM_0^{--} + iImM_{-1}^{--}Re\mathscr{M}_0^{--} + ImM_{-1}^{--}Im\mathscr{M}_0^{--}+ \\
& +Re\mathscr{M}_{-1}^{--}ReM_0^{--} - iRe\mathscr{M}_{-1}^{--}ImM_0^{--} + Re\mathscr{M}_{-1}^{--}Re\mathscr{M}_0^{--} - iRe\mathscr{M}_{-1}^{--}Im\mathscr{M}_0^{--}+ \\
& +iIm\mathscr{M}_{-1}^{--}ReM_0^{--} + Im\mathscr{M}_{-1}^{--}ImM_0^{--} + iIm\mathscr{M}_{-1}^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_{-1}^{--}Im\mathscr{M}_0^{--}+ \\
& +Re\mathscr{M}_{-1}^{+-}ReM_0^{+-} - iRe\mathscr{M}_{-1}^{+-}ImM_0^{+-} + Re\mathscr{M}_{-1}^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_{-1}^{+-}Im\mathscr{M}_0^{+-}+ \\
& +iIm\mathscr{M}_{-1}^{+-}ReM_0^{+-} + Im\mathscr{M}_{-1}^{+-}ImM_0^{+-} + iIm\mathscr{M}_{-1}^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_{-1}^{+-}Im\mathscr{M}_0^{+-}+ \\
& + ReM_{-1}^{-+}ReM_0^{-+} - iReM_{-1}^{-+}ImM_0^{-+} + ReM_{-1}^{-+}Re\mathscr{M}_0^{-+} - iReM_{-1}^{-+}Im\mathscr{M}_0^{-+}+ \\
& +iImM_{-1}^{-+}ReM_0^{-+} + ImM_{-1}^{-+}ImM_0^{-+} + iImM_{-1}^{-+}Re\mathscr{M}_0^{-+} + ImM_{-1}^{-+}Im\mathscr{M}_0^{-+}+ \\
& +Re\mathscr{M}_{-1}^{-+}ReM_0^{-+} - iRe\mathscr{M}_{-1}^{-+}ImM_0^{-+} + Re\mathscr{M}_{-1}^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_{-1}^{-+}Im\mathscr{M}_0^{-+}+ \\
& +iIm\mathscr{M}_{-1}^{-+}ReM_0^{-+} + Im\mathscr{M}_{-1}^{-+}ImM_0^{-+} + iIm\mathscr{M}_{-1}^{-+}Re\mathscr{M}_0^{-+} + Im\mathscr{M}_{-1}^{-+}Im\mathscr{M}_0^{-+}
\end{aligned} \tag{88}
$$

2.2 $J^P = \frac{3}{2}^+$

2.2.1 $H_{0,0}$

We want to calculate the term $H_{0,0}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N\lambda_Y} M_\lambda^{'\lambda_N\lambda_Y}(M_\lambda^{'\lambda_N\lambda_Y})^\dagger \tag{89}$$

and using

$$M_\lambda^{'\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y} + \mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{90}$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Tables 3 and 7, for $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ respectively, we have for $H_{0,0}$:

$$
\begin{aligned}
H_{0,0} = {} & (M_0^{++} + \mathscr{M}_0^{++})(M_0^{++} + \mathscr{M}_0^{++})^\dagger + (M_0^{--} + \mathscr{M}_0^{--})(M_0^{--} + \mathscr{M}_0^{--})^\dagger + \\
& + (M_0^{+-} + \mathscr{M}_0^{+-})(M_0^{+-} + \mathscr{M}_0^{+-})^\dagger + (M_0^{-+} + \mathscr{M}_0^{-+})(M_0^{-+} + \mathscr{M}_0^{-+})^\dagger = \\
& = (ReM_0^{++} + iImM_0^{++} + Re\mathscr{M}_0^{++} + iIm\mathscr{M}_0^{++})(ReM_0^{++} - iImM_0^{++} + Re\mathscr{M}_0^{++} - iIm\mathscr{M}_0^{++}) + \\
& + (ReM_0^{--} + iImM_0^{--} + Re\mathscr{M}_0^{--} + iIm\mathscr{M}_0^{--})(ReM_0^{--} - iImM_0^{--} + Re\mathscr{M}_0^{--} - iIm\mathscr{M}_0^{--}) + \\
& + (ReM_0^{+-} + iImM_0^{+-} + Re\mathscr{M}_0^{+-} + iIm\mathscr{M}_0^{+-})(ReM_0^{+-} - iImM_0^{+-} + Re\mathscr{M}_0^{+-} - iIm\mathscr{M}_0^{+-}) + \\
& + (ReM_0^{-+} + iImM_0^{-+} + Re\mathscr{M}_0^{-+} + iIm\mathscr{M}_0^{-+})(ReM_0^{-+} - iImM_0^{-+} + Re\mathscr{M}_0^{-+} - iIm\mathscr{M}_0^{-+}) = \\
& = ReM_0^{++} ReM_0^{++} \cancel{-iReM_0^{++} ImM_0^{++}} + ReM_0^{++} Re\mathscr{M}_0^{++} \cancel{-iReM_0^{++} Im\mathscr{M}_0^{++}} + \\
& \cancel{+iImM_0^{++} ReM_0^{++}} + ImM_0^{++} ImM_0^{++} + \cancel{iImM_0^{++} Re\mathscr{M}_0^{++}} + ImM_0^{++} Im\mathscr{M}_0^{++} + \\
& + Re\mathscr{M}_0^{++} ReM_0^{++} \cancel{-iRe\mathscr{M}_0^{++} ImM_0^{++}} + Re\mathscr{M}_0^{++} Re\mathscr{M}_0^{++} \cancel{-iRe\mathscr{M}_0^{++} Im\mathscr{M}_0^{++}} + \\
& \cancel{+iIm\mathscr{M}_0^{++} ReM_0^{++}} + Im\mathscr{M}_0^{++} ImM_0^{++} + \cancel{iIm\mathscr{M}_0^{++} Re\mathscr{M}_0^{++}} + Im\mathscr{M}_0^{++} Im\mathscr{M}_0^{++} + \\
& + ReM_0^{--} ReM_0^{--} \cancel{-iReM_0^{--} ImM_0^{--}} + ReM_0^{--} Re\mathscr{M}_0^{--} \cancel{-iReM_0^{--} Im\mathscr{M}_0^{--}} + \\
& \cancel{+iImM_0^{--} ReM_0^{--}} + ImM_0^{--} ImM_0^{--} + \cancel{iImM_0^{--} Re\mathscr{M}_0^{--}} + ImM_0^{--} Im\mathscr{M}_0^{--} + \\
& + Re\mathscr{M}_0^{--} ReM_0^{--} \cancel{-iRe\mathscr{M}_0^{--} ImM_0^{--}} + Re\mathscr{M}_0^{--} Re\mathscr{M}_0^{--} \cancel{-iRe\mathscr{M}_0^{--} Im\mathscr{M}_0^{--}} + \\
& \cancel{+iIm\mathscr{M}_0^{--} ReM_0^{--}} + Im\mathscr{M}_0^{--} ImM_0^{--} + \cancel{iIm\mathscr{M}_0^{--} Re\mathscr{M}_0^{--}} + Im\mathscr{M}_0^{--} Im\mathscr{M}_0^{--} + \\
& + ReM_0^{+-} ReM_0^{+-} \cancel{-iReM_0^{+-} ImM_0^{+-}} + ReM_0^{+-} Re\mathscr{M}_0^{+-} \cancel{-iReM_0^{+-} Im\mathscr{M}_0^{+-}} + \\
& \cancel{+iImM_0^{+-} ReM_0^{+-}} + ImM_0^{+-} ImM_0^{+-} + \cancel{iImM_0^{+-} Re\mathscr{M}_0^{+-}} + ImM_0^{+-} Im\mathscr{M}_0^{+-} + \\
& + Re\mathscr{M}_0^{+-} ReM_0^{+-} \cancel{-iRe\mathscr{M}_0^{+-} ImM_0^{+-}} + Re\mathscr{M}_0^{+-} Re\mathscr{M}_0^{+-} \cancel{-iRe\mathscr{M}_0^{+-} Im\mathscr{M}_0^{+-}} + \\
& \cancel{+iIm\mathscr{M}_0^{+-} ReM_0^{+-}} + Im\mathscr{M}_0^{+-} ImM_0^{+-} + \cancel{iIm\mathscr{M}_0^{+-} Re\mathscr{M}_0^{+-}} + Im\mathscr{M}_0^{+-} Im\mathscr{M}_0^{+-} + \\
& + ReM_0^{-+} ReM_0^{-+} \cancel{-iReM_0^{-+} ImM_0^{-+}} + ReM_0^{-+} Re\mathscr{M}_0^{-+} \cancel{-iReM_0^{-+} Im\mathscr{M}_0^{-+}} + \\
& \cancel{+iImM_0^{-+} ReM_0^{-+}} + ImM_0^{-+} ImM_0^{-+} + \cancel{iImM_0^{-+} Re\mathscr{M}_0^{-+}} + ImM_0^{-+} Im\mathscr{M}_0^{-+} + \\
& + Re\mathscr{M}_0^{-+} ReM_0^{-+} \cancel{-iRe\mathscr{M}_0^{-+} ImM_0^{-+}} + Re\mathscr{M}_0^{-+} Re\mathscr{M}_0^{-+} \cancel{-iRe\mathscr{M}_0^{-+} Im\mathscr{M}_0^{-+}} + \\
& \cancel{+iIm\mathscr{M}_0^{-+} ReM_0^{-+}} + Im\mathscr{M}_0^{-+} ImM_0^{-+} + \cancel{iIm\mathscr{M}_0^{-+} Re\mathscr{M}_0^{-+}} + Im\mathscr{M}_0^{-+} Im\mathscr{M}_0^{-+} = \\
& = (ReM_0^{++})^2 + 2ReM_0^{++} Re\mathscr{M}_0^{++} + (ImM_0^{++})^2 + 2ImM_0^{++} Im\mathscr{M}_0^{++} + (Im\mathscr{M}_0^{++})^2 + (Re\mathscr{M}_0^{++})^2 + \\
& + (ReM_0^{--})^2 + 2ReM_0^{--} Re\mathscr{M}_0^{--} + (ImM_0^{--})^2 + 2ImM_0^{--} Im\mathscr{M}_0^{--} + (Im\mathscr{M}_0^{--})^2 + (Re\mathscr{M}_0^{--})^2 + \\
& + (ReM_0^{+-})^2 + 2ReM_0^{+-} Re\mathscr{M}_0^{+-} + (ImM_0^{+-})^2 + 2ImM_0^{+-} Im\mathscr{M}_0^{+-} + (Im\mathscr{M}_0^{+-})^2 + (Re\mathscr{M}_0^{+-})^2 + \\
& + (ReM_0^{-+})^2 + 2ReM_0^{-+} Re\mathscr{M}_0^{-+} + (ImM_0^{-+})^2 + 2ImM_0^{-+} Im\mathscr{M}_0^{-+} + (Im\mathscr{M}_0^{-+})^2 + (Re\mathscr{M}_0^{-+})^2
\end{aligned}
\tag{91}
$$

The terms in the grid satisfy the relationships

$$
\mathscr{M}_1^{++} = \mathscr{M}_{-1}^{++} \; \mathscr{M}_0^{++} = -\mathscr{M}_0^{--} \; \mathscr{M}_1^{-+} = -\mathscr{M}_{-1}^{-+} \; \mathscr{M}_1^{+-} = -\mathscr{M}_{-1}^{+-} \; \mathscr{M}_1^{--} = \mathscr{M}_{-1}^{--} \tag{92}
$$

And from Tabs 8 and 9 we have that for $J^P = \frac{3}{2}^+$:

$$
M_1^{++} = -M_{-1}^{--} \; M_1^{+-} = M_{-1}^{-+} \; M_0^{+-} = -M_0^{-+} \; M_0^{++} = M_0^{--} \; M_{-1}^{++} = -M_1^{--} \; M_{-1}^{+-} = M_1^{-+} \tag{93}
$$

And we have:

$$H_{0,0} = (F'_{3/2}AQ\frac{3cos\theta^*_K - 1}{2}cos\frac{\theta^*_K}{2})^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + (Im\mathscr{M}_0^{++})^2 + (Re\mathscr{M}_0^{++})^2 +$$
$$+ (F'_{3/2}AQ\frac{3cos\theta^*_K - 1}{2}cos\frac{\theta^*_K}{2})^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + (Im\mathscr{M}_0^{--})^2 + (Re\mathscr{M}_0^{--})^2 +$$
$$+ (F'_{3/2}AQ(-\frac{3cos\theta^*_K + 1}{2}sin\frac{\theta^*_K}{2}))^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + (Im\mathscr{M}_0^{+-})^2 + (Re\mathscr{M}_0^{+-})^2 +$$
$$+ (F'_{3/2}AQ(-\frac{3cos\theta^*_K + 1}{2}sin\frac{\theta^*_K}{2}))^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + (Im\mathscr{M}_0^{-+})^2 + (Re\mathscr{M}_0^{-+})^2 +$$
$$- 2F'_{3/2}AQ\frac{3cos\theta^*_K - 1}{2}(sin\frac{\theta^*_K}{2})(Re\mathscr{M}_0^{+-}(M_r^2 - W^2) + Im\mathscr{M}_0^{+-}\Gamma_r M_r) +$$
$$- 2F'_{3/2}AQ\frac{3cos\theta^*_K - 1}{2}(sin\frac{\theta^*_K}{2})(Re\mathscr{M}_0^{-+}(M_r^2 - W^2) + Im\mathscr{M}_0^{-+}\Gamma_r M_r) \quad (94)$$

So, in conclusion we have:

$$H_{0,0} = 2(F'_{3/2}AQ\frac{3cos\theta^*_K - 1}{2}cos\frac{\theta^*_K}{2})^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + 2(F'_{3/2}AQ(\frac{3cos\theta^*_K + 1}{2}sin\frac{\theta^*_K}{2}))^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) +$$
$$+ (Im\mathscr{M}_0^{++})^2 + (Re\mathscr{M}_0^{++})^2 + (Im\mathscr{M}_0^{--})^2 + (Re\mathscr{M}_0^{--})^2 + (Im\mathscr{M}_0^{+-})^2 + (Re\mathscr{M}_0^{+-})^2 + (Im\mathscr{M}_0^{-+})^2 + (Re\mathscr{M}_0^{-+})^2 +$$
$$- 2F'_{3/2}AQ\frac{3cos\theta^*_K - 1}{2}(sin\frac{\theta^*_K}{2})(Re\mathscr{M}_0^{+-}(M_r^2 - W^2) + Im\mathscr{M}_0^{+-}\Gamma_r M_r) +$$
$$- 2F'_{3/2}AQ\frac{3cos\theta^*_K - 1}{2}(sin\frac{\theta^*_K}{2})(Re\mathscr{M}_0^{-+}(M_r^2 - W^2) + Im\mathscr{M}_0^{-+}\Gamma_r M_r) \quad (95)$$

Or

$$H_{0,0} = 2(F'_{3/2}AQ\frac{3cos\theta^*_K - 1}{2}cos\frac{\theta^*_K}{2})^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + 2(F'_{3/2}AQ(\frac{3cos\theta^*_K + 1}{2}sin\frac{\theta^*_K}{2}))^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) +$$
$$+ \mid \mathscr{M}_0^{++} \mid^2 + \mid \mathscr{M}_0^{--} \mid^2 + \mid \mathscr{M}_0^{+-} \mid^2 + \mid \mathscr{M}_0^{-+} \mid^2 +$$
$$- 2F'_{3/2}AQ\frac{3cos\theta^*_K - 1}{2}(sin\frac{\theta^*_K}{2})(Re\mathscr{M}_0^{+-}(M_r^2 - W^2) + Im\mathscr{M}_0^{+-}\Gamma_r M_r - Re\mathscr{M}_0^{-+}(M_r^2 - W^2) - Im\mathscr{M}_0^{-+}\Gamma_r M_r) \quad (96)$$

2.2.2 $H_{1,1}$

$$H_{1,1} = (M_1^{++} + \mathscr{M}_1^{++})(M_1^{++} + \mathscr{M}_1^{++})^\dagger + (M_1^{--} + \mathscr{M}_1^{--})(M_1^{--} + \mathscr{M}_1^{--})^\dagger +$$
$$+ (M_1^{+-} + \mathscr{M}_1^{+-})(M_1^{+-} + \mathscr{M}_1^{+-})^\dagger + (M_1^{-+} + \mathscr{M}_1^{-+})(M_1^{-+} + \mathscr{M}_1^{-+})^\dagger =$$
$$= (ReM_1^{++})^2 + 2ReM_1^{++}Re\mathscr{M}_1^{++} + (ImM_1^{++})^2 + 2ImM_1^{++}Im\mathscr{M}_1^{++} + (Im\mathscr{M}_1^{++})^2 + (Re\mathscr{M}_1^{++})^2 +$$
$$+ (ReM_1^{--})^2 + 2ReM_1^{--}Re\mathscr{M}_1^{--} + (ImM_1^{--})^2 + 2ImM_1^{--}Im\mathscr{M}_1^{--} + (Im\mathscr{M}_1^{--})^2 + (Re\mathscr{M}_1^{--})^2 +$$
$$+ (ReM_1^{+-})^2 + 2ReM_1^{+-}Re\mathscr{M}_1^{+-} + (ImM_1^{+-})^2 + 2ImM_1^{+-}Im\mathscr{M}_1^{+-} + (Im\mathscr{M}_1^{+-})^2 + (Re\mathscr{M}_1^{+-})^2 +$$
$$+ (ReM_1^{-+})^2 + 2ReM_1^{-+}Re\mathscr{M}_1^{-+} + (ImM_1^{-+})^2 + 2ImM_1^{-+}Im\mathscr{M}_1^{-+} + (Im\mathscr{M}_1^{-+})^2 + (Re\mathscr{M}_1^{-+})^2 \quad (97)$$

The terms in the grid satisfy the relationships

$$\mathscr{M}_1^{++} = \mathscr{M}_{-1}^{++} \;\; \mathscr{M}_0^{++} = -\mathscr{M}_0^{--} \;\; \mathscr{M}_1^{-+} = -\mathscr{M}_{-1}^{-+} \;\; \mathscr{M}_1^{+-} = -\mathscr{M}_{-1}^{+-} \;\; \mathscr{M}_1^{--} = \mathscr{M}_{-1}^{--} \quad (98)$$

And from Tabs 8 and 9 we have that for $J^P = \frac{3}{2}^+$:

$$M_1^{++} = -M_{-1}^{--} \;\; M_1^{+-} = M_{-1}^{-+} \;\; M_0^{+-} = -M_0^{-+} \;\; M_0^{++} = M_0^{--} \;\; M_{-1}^{++} = -M_1^{--} \;\; M_{-1}^{+-} = M_1^{-+} \quad (99)$$

In conclusion we have:

$$\begin{aligned}H_{1,1} = &(F'_{3/2}A(-\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2}))^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+(Im\mathscr{M}_1^{++})^2+(Re\mathscr{M}_1^{++})^2+\\
&+(F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2}))^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+(Im\mathscr{M}_1^{--})^2+(Re\mathscr{M}_1^{--})^2+\\
&+(F'_{3/2}A(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2}))^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+(Im\mathscr{M}_1^{+-})^2+(Re\mathscr{M}_1^{+-})^2+\\
&+(F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2}))^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+(Im\mathscr{M}_1^{-+})^2+(Re\mathscr{M}_1^{-+})^2+\\
&+2F'_{3/2}A(-\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)Re\mathscr{M}_1^{++}+2F'_{3/2}A(-\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})(\Gamma_r M_r)Im\mathscr{M}_1^{++}+\\
&+2F'_{3/2}A(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)Re\mathscr{M}_1^{+-}+2F'_{3/2}A(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})(\Gamma_r M_r)Im\mathscr{M}_1^{+-}+\\
&+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)Re\mathscr{M}_1^{-+}+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2})\Gamma_r M_r Im\mathscr{M}_1^{-+}+\\
&+2F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)Re\mathscr{M}_1^{--}+2F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2})\Gamma_r M_r Im\mathscr{M}_1^{--}\end{aligned} \quad (100)$$

Or

$$\begin{aligned}H_{1,1} = &(F'_{3/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)[(\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})^2+\frac{1}{Q^4}(\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2})^2+(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})^2+\\
&+\frac{1}{Q^4}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2})^2]+\mid\mathscr{M}_1^{++}\mid^2+\mid\mathscr{M}_1^{--}\mid^2+\mid\mathscr{M}_1^{+-}\mid^2+\mid\mathscr{M}_1^{-+}\mid^2+\\
&+2F'_{3/2}A(-\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)Re\mathscr{M}_1^{++}+2F'_{3/2}A(-\frac{3cos\theta^*_K+1}{2}sin\frac{\theta^*_K}{2})(\Gamma_r M_r)Im\mathscr{M}_1^{++}+\\
&+2F'_{3/2}A(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)Re\mathscr{M}_1^{+-}+2F'_{3/2}A(\frac{3cos\theta^*_K-1}{2}cos\frac{\theta^*_K}{2})(\Gamma_r M_r)Im\mathscr{M}_1^{+-}+\\
&+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2})(M_r^2-W^2)Re\mathscr{M}_1^{-+}+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta^*_K}{2}cos\frac{\theta^*_K}{2})\Gamma_r M_r Im\mathscr{M}_1^{-+}+\\
&+2F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2})(M_r^2-W^2)Re\mathscr{M}_1^{--}+2F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta^*_K}{2}sin\frac{\theta^*_K}{2})\Gamma_r M_r Im\mathscr{M}_1^{--}\end{aligned} \quad (101)$$

2.2.3 $H_{-1,-1}$

$$\begin{aligned}H_{-1,-1} = &(M_{-1}^{++}+\mathscr{M}_{-1}^{++})(M_{-1}^{++}+\mathscr{M}_{-1}^{++})^\dagger+(M_{-1}^{--}+\mathscr{M}_{-1}^{--})(M_{-1}^{--}+\mathscr{M}_{-1}^{--})^\dagger+\\
&+(M_{-1}^{+-}+\mathscr{M}_{-1}^{+-})(M_{-1}^{+-}+\mathscr{M}_{-1}^{+-})^\dagger+(M_{-1}^{-+}+\mathscr{M}_{-1}^{-+})(M_{-1}^{-+}+\mathscr{M}_{-1}^{-+})^\dagger=\\
&=(ReM_{-1}^{++})^2+2ReM_{-1}^{++}Re\mathscr{M}_{-1}^{++}+(ImM_{-1}^{++})^2+2ImM_{-1}^{++}Im\mathscr{M}_{-1}^{++}+(Im\mathscr{M}_{-1}^{++})^2+(Re\mathscr{M}_{-1}^{++})^2+\\
&+(ReM_{-1}^{--})^2+2ReM_{-1}^{--}Re\mathscr{M}_{-1}^{--}+(ImM_{-1}^{--})^2+2ImM_{-1}^{--}Im\mathscr{M}_{-1}^{--}+(Im\mathscr{M}_{-1}^{--})^2+(Re\mathscr{M}_{-1}^{--})^2+\\
&+(ReM_{-1}^{+-})^2+2ReM_{-1}^{+-}Re\mathscr{M}_{-1}^{+-}+(ImM_{-1}^{+-})^2+2ImM_{-1}^{+-}Im\mathscr{M}_{-1}^{+-}+(Im\mathscr{M}_{-1}^{+-})^2+(Re\mathscr{M}_{-1}^{+-})^2+\\
&+(ReM_{-1}^{-+})^2+2ReM_{-1}^{-+}Re\mathscr{M}_{-1}^{-+}+(ImM_{-1}^{-+})^2+2ImM_{-1}^{-+}Im\mathscr{M}_{-1}^{-+}+(Im\mathscr{M}_{-1}^{-+})^2+(Re\mathscr{M}_{-1}^{-+})^2\end{aligned} \quad (102)$$

The terms in the grid satisfy the relationships

$$\mathscr{M}_1^{++}=\mathscr{M}_{-1}^{++}\ \ \mathscr{M}_0^{++}=-\mathscr{M}_0^{--}\ \ \mathscr{M}_1^{-+}=-\mathscr{M}_{-1}^{-+}\ \ \mathscr{M}_1^{+-}=-\mathscr{M}_{-1}^{+-}\ \ \mathscr{M}_1^{--}=\mathscr{M}_{-1}^{--} \quad (103)$$

And from Tabs 8 and 9 we have that for $J^P=\frac{3}{2}^+$:

$$M_1^{++}=-M_{-1}^{--}\ \ M_1^{+-}=M_{-1}^{-+}\ \ M_0^{+-}=-M_0^{-+}\ \ M_0^{++}=M_0^{--}\ \ M_{-1}^{++}=-M_1^{--}\ \ M_{-1}^{+-}=M_1^{-+} \quad (104)$$

In conclusion we have:

$$H_{-1,-1} = (F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2}))^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+(Im\mathscr{M}_{-1}^{++})^2+(Re\mathscr{M}_{-1}^{++})^2+$$
$$+(F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2}))^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+(Im\mathscr{M}_{-1}^{--})^2+(Re\mathscr{M}_{-1}^{--})^2+$$
$$+(F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2}))^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+(Im\mathscr{M}_{-1}^{+-})^2+(Re\mathscr{M}_{-1}^{+-})^2+$$
$$+(F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2}))^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+(Im\mathscr{M}_{-1}^{-+})^2+(Re\mathscr{M}_{-1}^{-+})^2+$$
$$+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{++}+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{++}$$
$$+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{+-}+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{+-}+$$
$$+2F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+2F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+}+$$
$$+2F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{--}+2F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--} \tag{105}$$

Or

$$H_{-1,-1} = (F'_{3/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)[\frac{1}{Q^4}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})^2+(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})^2+$$
$$+\frac{1}{Q^4}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})^2+(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})^2]+\mid\mathscr{M}_{-1}^{++}\mid^2+\mid\mathscr{M}_{-1}^{--}\mid^2+\mid\mathscr{M}_{-1}^{+-}\mid^2+$$
$$+\mid\mathscr{M}_{-1}^{-+}\mid^2+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{++}+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{++}+$$
$$+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{+-}+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{+-}+$$
$$+2F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+2F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+}+$$
$$+2F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{--}+2F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--} \tag{106}$$

2.2.4 $H_{1,-1}$

We want to calculate the term $H_{1,-1}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N\lambda_Y} M_\lambda^{'\lambda_N\lambda_Y}(M_\lambda^{'\lambda_N\lambda_Y})^\dagger \tag{107}$$

and using

$$M_\lambda^{'\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y}+\mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{108}$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Table 7, for $J^P=\frac{3}{2}^+$. Specifically, the real parts are reported in Table 8, and the imaginary parts are reported in Table 9.
We have for $H_{1,-1}$ for $J^P=\frac{3}{2}^+$:

$$
\begin{aligned}
H_{1,-1} &= (M_1^{++}+\mathscr{M}_1^{++})(M_{-1}^{++}+\mathscr{M}_{-1}^{++})^\dagger + (M_1^{--}+\mathscr{M}_1^{--})(M_{-1}^{--}+\mathscr{M}_{-1}^{--})^\dagger +\\
&+ (M_1^{+-}+\mathscr{M}_1^{+-})(M_{-1}^{+-}+\mathscr{M}_{-1}^{+-})^\dagger + (M_1^{-+}+\mathscr{M}_1^{-+})(M_{-1}^{-+}+\mathscr{M}_{-1}^{-+})^\dagger =\\
&= (ReM_1^{++}+iImM_1^{++}+Re\mathscr{M}_1^{++}+iIm\mathscr{M}_1^{++})(ReM_{-1}^{++}-iImM_{-1}^{++}+Re\mathscr{M}_{-1}^{++}-iIm\mathscr{M}_{-1}^{++})+\\
&+ (ReM_1^{--}+iImM_1^{--}+Re\mathscr{M}_1^{--}+iIm\mathscr{M}_1^{--})(ReM_{-1}^{--}-iImM_{-1}^{--}+Re\mathscr{M}_{-1}^{--}-iIm\mathscr{M}_{-1}^{--})+\\
&+ (ReM_1^{+-}+iImM_1^{+-}+Re\mathscr{M}_1^{+-}+iIm\mathscr{M}_1^{+-})(ReM_{-1}^{+-}-iImM_{-1}^{+-}+Re\mathscr{M}_{-1}^{+-}-iIm\mathscr{M}_{-1}^{+-})+\\
&+ (ReM_1^{-+}+iImM_1^{-+}+Re\mathscr{M}_1^{-+}+iIm\mathscr{M}_1^{-+})(ReM_{-1}^{-+}-iImM_{-1}^{-+}+Re\mathscr{M}_{-1}^{-+}-iIm\mathscr{M}_{-1}^{-+}) =\\
&= ReM_1^{++}ReM_{-1}^{++} - iReM_1^{++}ImM_{-1}^{++} + ReM_1^{++}Re\mathscr{M}_{-1}^{++} - iReM_1^{++}Im\mathscr{M}_{-1}^{++}+\\
&+iImM_1^{++}ReM_{-1}^{++} + ImM_1^{++}ImM_{-1}^{++} + iImM_1^{++}Re\mathscr{M}_{-1}^{++} + ImM_1^{++}Im\mathscr{M}_{-1}^{++}+\\
&+Re\mathscr{M}_1^{++}ReM_{-1}^{++} - iRe\mathscr{M}_1^{++}ImM_{-1}^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++}+\\
&+iIm\mathscr{M}_1^{++}ReM_{-1}^{++} + Im\mathscr{M}_1^{++}ImM_{-1}^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++}+\\
&+ ReM_1^{--}ReM_{-1}^{--} - iReM_1^{--}ImM_{-1}^{--} + ReM_1^{--}Re\mathscr{M}_{-1}^{--} - iReM_1^{--}Im\mathscr{M}_{-1}^{--}+\\
&+iImM_1^{--}ReM_{-1}^{--} + ImM_1^{--}ImM_{-1}^{--} + iImM_1^{--}Re\mathscr{M}_{-1}^{--} + ImM_1^{--}Im\mathscr{M}_{-1}^{--}+\\
&+Re\mathscr{M}_1^{--}ReM_{-1}^{--} - iRe\mathscr{M}_1^{--}ImM_{-1}^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--}+\\
&+iIm\mathscr{M}_1^{--}ReM_{-1}^{--} + Im\mathscr{M}_1^{--}ImM_{-1}^{--} + iIm\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--}+\\
&+ ReM_1^{+-}ReM_{-1}^{+-} - iReM_1^{+-}ImM_{-1}^{+-} + ReM_1^{+-}Re\mathscr{M}_{-1}^{+-} - iReM_1^{+-}Im\mathscr{M}_{-1}^{+-}+\\
&+iImM_1^{+-}ReM_{-1}^{+-} + ImM_1^{+-}ImM_{-1}^{+-} + iImM_1^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_1^{+-}Im\mathscr{M}_{-1}^{+-}+\\
&+Re\mathscr{M}_1^{+-}ReM_{-1}^{+-} - iRe\mathscr{M}_1^{+-}ImM_{-1}^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-}+\\
&+iIm\mathscr{M}_1^{+-}ReM_{-1}^{+-} + Im\mathscr{M}_1^{+-}ImM_{-1}^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-}+\\
&+ ReM_1^{-+}ReM_{-1}^{-+} - iReM_1^{-+}ImM_{-1}^{-+} + ReM_1^{-+}Re\mathscr{M}_{-1}^{-+} - iReM_1^{-+}Im\mathscr{M}_{-1}^{-+}+\\
&+iImM_1^{-+}ReM_{-1}^{-+} + ImM_1^{-+}ImM_{-1}^{-+} + iImM_1^{-+}Re\mathscr{M}_{-1}^{-+} + ImM_1^{-+}Im\mathscr{M}_{-1}^{-+}+\\
&+Re\mathscr{M}_1^{-+}ReM_{-1}^{-+} - iRe\mathscr{M}_1^{-+}ImM_{-1}^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+}+\\
&+iIm\mathscr{M}_1^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_1^{-+}ImM_{-1}^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+}
\end{aligned} \tag{109}
$$

2.2.5 $H_{-1,1}$

We want to calculate the term $H_{-1,1}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N\lambda_Y} M_\lambda^{'\lambda_N\lambda_Y}(M_\lambda^{'\lambda_N\lambda_Y})^\dagger \tag{110}$$

and using

$$M_\lambda^{'\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y} + \mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{111}$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Table 7, for $J^P = \frac{3}{2}^+$. Specifically, the real parts are reported in Table 8, and the imaginary parts are reported in Table 9.
We have for $H_{-1,1}$ for $J^P = \frac{3}{2}^+$:

$$\begin{aligned}
H_{-1,1} &= (M_{-1}^{++} + \mathscr{M}_{-1}^{++})(M_{1}^{++} + \mathscr{M}_{1}^{++})^{\dagger} + (M_{-1}^{--} + \mathscr{M}_{-1}^{--})(M_{1}^{--} + \mathscr{M}_{1}^{--})^{\dagger} + \\
&+ (M_{-1}^{+-} + \mathscr{M}_{-1}^{+-})(M_{1}^{+-} + \mathscr{M}_{1}^{+-})^{\dagger} + (M_{-1}^{-+} + \mathscr{M}_{-1}^{-+})(M_{1}^{-+} + \mathscr{M}_{1}^{-+})^{\dagger} = \\
&= (ReM_{-1}^{++} + iImM_{-1}^{++} + Re\mathscr{M}_{-1}^{++} + iIm\mathscr{M}_{-1}^{++})(ReM_{1}^{++} - iImM_{1}^{++} + Re\mathscr{M}_{1}^{++} - iIm\mathscr{M}_{1}^{++}) + \\
&+ (ReM_{-1}^{--} + iImM_{-1}^{--} + Re\mathscr{M}_{-1}^{--} + iIm\mathscr{M}_{-1}^{--})(ReM_{1}^{--} - iImM_{1}^{--} + Re\mathscr{M}_{1}^{--} - iIm\mathscr{M}_{1}^{--}) + \\
&+ (ReM_{-1}^{+-} + iImM_{-1}^{+-} + Re\mathscr{M}_{-1}^{+-} + iIm\mathscr{M}_{-1}^{+-})(ReM_{1}^{+-} - iImM_{1}^{+-} + Re\mathscr{M}_{1}^{+-} - iIm\mathscr{M}_{1}^{+-}) + \\
&+ (ReM_{-1}^{-+} + iImM_{-1}^{-+} + Re\mathscr{M}_{-1}^{-+} + iIm\mathscr{M}_{-1}^{-+})(ReM_{1}^{-+} - iImM_{1}^{-+} + Re\mathscr{M}_{1}^{-+} - iIm\mathscr{M}_{1}^{-+}) = \\
&= ReM_{-1}^{++}ReM_{1}^{++} - iReM_{-1}^{++}ImM_{1}^{++} + ReM_{-1}^{++}Re\mathscr{M}_{1}^{++} - iReM_{-1}^{++}Im\mathscr{M}_{1}^{++} + \\
&+ iImM_{-1}^{++}ReM_{1}^{++} + ImM_{-1}^{++}ImM_{1}^{++} + iImM_{-1}^{++}Re\mathscr{M}_{1}^{++} + ImM_{-1}^{++}Im\mathscr{M}_{1}^{++} + \\
&+ Re\mathscr{M}_{-1}^{++}ReM_{1}^{++} - iRe\mathscr{M}_{-1}^{++}ImM_{1}^{++} + Re\mathscr{M}_{-1}^{++}Re\mathscr{M}_{1}^{++} - iRe\mathscr{M}_{-1}^{++}Im\mathscr{M}_{1}^{++} + \\
&+ iIm\mathscr{M}_{-1}^{++}ReM_{1}^{++} + Im\mathscr{M}_{-1}^{++}ImM_{1}^{++} + iIm\mathscr{M}_{-1}^{++}Re\mathscr{M}_{1}^{++} + Im\mathscr{M}_{-1}^{++}Im\mathscr{M}_{1}^{++} + \\
&+ ReM_{-1}^{--}ReM_{1}^{--} - iReM_{-1}^{--}ImM_{1}^{--} + ReM_{-1}^{--}Re\mathscr{M}_{1}^{--} - iReM_{-1}^{--}Im\mathscr{M}_{1}^{--} + \\
&+ iImM_{-1}^{--}ReM_{1}^{--} + ImM_{-1}^{--}ImM_{1}^{--} + iImM_{-1}^{--}Re\mathscr{M}_{1}^{--} + ImM_{-1}^{--}Im\mathscr{M}_{1}^{--} + \\
&+ Re\mathscr{M}_{-1}^{--}ReM_{1}^{--} - iRe\mathscr{M}_{-1}^{--}ImM_{1}^{--} + Re\mathscr{M}_{-1}^{--}Re\mathscr{M}_{1}^{--} - iRe\mathscr{M}_{-1}^{--}Im\mathscr{M}_{1}^{--} + \\
&+ iIm\mathscr{M}_{-1}^{--}ReM_{1}^{--} + Im\mathscr{M}_{-1}^{--}ImM_{1}^{--} + iIm\mathscr{M}_{-1}^{--}Re\mathscr{M}_{1}^{--} + Im\mathscr{M}_{-1}^{--}Im\mathscr{M}_{1}^{--} + \\
&+ ReM_{-1}^{+-}ReM_{1}^{+-} - iReM_{-1}^{+-}ImM_{1}^{+-} + ReM_{-1}^{+-}Re\mathscr{M}_{1}^{+-} - iReM_{-1}^{+-}Im\mathscr{M}_{1}^{+-} + \\
&+ iImM_{-1}^{+-}ReM_{1}^{+-} + ImM_{-1}^{+-}ImM_{1}^{+-} + iImM_{-1}^{+-}Re\mathscr{M}_{1}^{+-} + ImM_{-1}^{+-}Im\mathscr{M}_{1}^{+-} + \\
&+ Re\mathscr{M}_{-1}^{+-}ReM_{1}^{+-} - iRe\mathscr{M}_{-1}^{+-}ImM_{1}^{+-} + Re\mathscr{M}_{-1}^{+-}Re\mathscr{M}_{1}^{+-} - iRe\mathscr{M}_{-1}^{+-}Im\mathscr{M}_{1}^{+-} + \\
&+ iIm\mathscr{M}_{-1}^{+-}ReM_{1}^{+-} + Im\mathscr{M}_{-1}^{+-}ImM_{1}^{+-} + iIm\mathscr{M}_{-1}^{+-}Re\mathscr{M}_{1}^{+-} + Im\mathscr{M}_{-1}^{+-}Im\mathscr{M}_{1}^{+-} + \\
&+ ReM_{-1}^{-+}ReM_{1}^{-+} - iReM_{-1}^{-+}ImM_{1}^{-+} + ReM_{-1}^{-+}Re\mathscr{M}_{1}^{-+} - iReM_{-1}^{-+}Im\mathscr{M}_{1}^{-+} + \\
&+ iImM_{-1}^{-+}ReM_{1}^{-+} + ImM_{-1}^{-+}ImM_{1}^{-+} + iImM_{-1}^{-+}Re\mathscr{M}_{1}^{-+} + ImM_{-1}^{-+}Im\mathscr{M}_{1}^{-+} + \\
&+ Re\mathscr{M}_{-1}^{-+}ReM_{1}^{-+} - iRe\mathscr{M}_{-1}^{-+}ImM_{1}^{-+} + Re\mathscr{M}_{-1}^{-+}Re\mathscr{M}_{1}^{-+} - iRe\mathscr{M}_{-1}^{-+}Im\mathscr{M}_{1}^{-+} + \\
&+ iIm\mathscr{M}_{-1}^{-+}ReM_{1}^{-+} + Im\mathscr{M}_{-1}^{-+}ImM_{1}^{-+} + iIm\mathscr{M}_{-1}^{-+}Re\mathscr{M}_{1}^{-+} + Im\mathscr{M}_{-1}^{-+}Im\mathscr{M}_{1}^{-+}
\end{aligned} \tag{112}$$

2.2.6 $H_{0,1}$

We want to calculate the term $H_{0,1}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N \lambda_Y} M_{\lambda}^{'\lambda_N \lambda_Y} (M_{\lambda}^{'\lambda_N \lambda_Y})^{\dagger} \tag{113}$$

and using

$$M_{\lambda}^{'\lambda_N \lambda_Y} = M_{\lambda}^{\lambda_N \lambda_Y} + \mathscr{M}_{\lambda}^{\lambda_N \lambda_Y} \tag{114}$$

where the $\mathscr{M}_{\lambda}^{\lambda_N \lambda_Y}$ are the values from the grids, and the $M_{\lambda}^{\lambda_N \lambda_Y}$ are the values reported in Table 7, for $J^P = \frac{3}{2}^+$. Specifically, the real parts are reported in Table 8, and the imaginary parts are reported in Table 9. We have for $H_{0,1}$ for $J^P = \frac{3}{2}^+$:

$$
\begin{aligned}
H_{0,1} &= (M_0^{++} + \mathscr{M}_0^{++})(M_1^{++} + \mathscr{M}_1^{++})^\dagger + (M_0^{--} + \mathscr{M}_0^{--})(M_1^{--} + \mathscr{M}_1^{--})^\dagger + \\
&+ (M_0^{+-} + \mathscr{M}_0^{+-})(M_1^{+-} + \mathscr{M}_1^{+-})^\dagger + (M_0^{-+} + \mathscr{M}_0^{-+})(M_1^{-+} + \mathscr{M}_1^{-+})^\dagger = \\
&= (ReM_0^{++} + iImM_0^{++} + Re\mathscr{M}_0^{++} + iIm\mathscr{M}_0^{++})(ReM_1^{++} - iImM_1^{++} + Re\mathscr{M}_1^{++} - iIm\mathscr{M}_1^{++}) + \\
&+ (ReM_0^{--} + iImM_0^{--} + Re\mathscr{M}_0^{--} + iIm\mathscr{M}_0^{--})(ReM_1^{--} - iImM_1^{--} + Re\mathscr{M}_1^{--} - iIm\mathscr{M}_1^{--}) + \\
&+ (ReM_0^{+-} + iImM_0^{+-} + Re\mathscr{M}_0^{+-} + iIm\mathscr{M}_0^{+-})(ReM_1^{+-} - iImM_1^{+-} + Re\mathscr{M}_1^{+-} - iIm\mathscr{M}_1^{+-}) + \\
&+ (ReM_0^{-+} + iImM_0^{-+} + Re\mathscr{M}_0^{-+} + iIm\mathscr{M}_0^{-+})(ReM_1^{-+} - iImM_1^{-+} + Re\mathscr{M}_1^{-+} - iIm\mathscr{M}_1^{-+}) = \\
&= ReM_0^{++}ReM_1^{++} - iReM_0^{++}ImM_1^{++} + ReM_0^{++}Re\mathscr{M}_1^{++} - iReM_0^{++}Im\mathscr{M}_1^{++} + \\
&+ iImM_0^{++}ReM_1^{++} + ImM_0^{++}ImM_1^{++} + iImM_0^{++}Re\mathscr{M}_1^{++} + ImM_0^{++}Im\mathscr{M}_1^{++} + \\
&+ Re\mathscr{M}_0^{++}ReM_1^{++} - iRe\mathscr{M}_0^{++}ImM_1^{++} + Re\mathscr{M}_0^{++}Re\mathscr{M}_1^{++} - iRe\mathscr{M}_0^{++}Im\mathscr{M}_1^{++} + \\
&+ iIm\mathscr{M}_0^{++}ReM_1^{++} + Im\mathscr{M}_0^{++}ImM_1^{++} + iIm\mathscr{M}_0^{++}Re\mathscr{M}_1^{++} + Im\mathscr{M}_0^{++}Im\mathscr{M}_1^{++} + \\
&+ ReM_0^{--}ReM_1^{--} - iReM_0^{--}ImM_1^{--} + ReM_0^{--}Re\mathscr{M}_1^{--} - iReM_0^{--}Im\mathscr{M}_1^{--} + \\
&+ iImM_0^{--}ReM_1^{--} + ImM_0^{--}ImM_1^{--} + iImM_0^{--}Re\mathscr{M}_1^{--} + ImM_0^{--}Im\mathscr{M}_1^{--} + \\
&+ Re\mathscr{M}_0^{--}ReM_1^{--} - iRe\mathscr{M}_0^{--}ImM_1^{--} + Re\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} - iRe\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + \\
&+ iIm\mathscr{M}_0^{--}ReM_1^{--} + Im\mathscr{M}_0^{--}ImM_1^{--} + iIm\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} + Im\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + \\
&+ ReM_0^{+-}ReM_1^{+-} - iReM_0^{+-}ImM_1^{+-} + ReM_0^{+-}Re\mathscr{M}_1^{+-} - iReM_0^{+-}Im\mathscr{M}_1^{+-} + \\
&+ iImM_0^{+-}ReM_1^{+-} + ImM_0^{+-}ImM_1^{+-} + iImM_0^{+-}Re\mathscr{M}_1^{+-} + ImM_0^{+-}Im\mathscr{M}_1^{+-} + \\
&+ Re\mathscr{M}_0^{+-}ReM_1^{+-} - iRe\mathscr{M}_0^{+-}ImM_1^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} - iRe\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + \\
&+ iIm\mathscr{M}_0^{+-}ReM_1^{+-} + Im\mathscr{M}_0^{+-}ImM_1^{+-} + iIm\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + \\
&+ ReM_0^{-+}ReM_1^{-+} - iReM_0^{-+}ImM_1^{-+} + ReM_0^{-+}Re\mathscr{M}_1^{-+} - iReM_0^{-+}Im\mathscr{M}_1^{-+} + \\
&+ iImM_0^{-+}ReM_1^{-+} + ImM_0^{-+}ImM_1^{-+} + iImM_0^{-+}Re\mathscr{M}_1^{-+} + ImM_0^{-+}Im\mathscr{M}_1^{-+} + \\
&+ Re\mathscr{M}_0^{-+}ReM_1^{-+} - iRe\mathscr{M}_0^{-+}ImM_1^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} - iRe\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+} + \\
&+ iIm\mathscr{M}_0^{-+}ReM_1^{-+} + Im\mathscr{M}_0^{-+}ImM_1^{-+} + iIm\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+}
\end{aligned} \tag{115}
$$

2.2.7 $H_{1,0}$

We want to calculate the term $H_{1,0}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N \lambda_Y} M_\lambda^{'\lambda_N\lambda_Y} (M_\lambda^{'\lambda_N\lambda_Y})^\dagger \tag{116}$$

and using

$$M_\lambda^{'\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y} + \mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{117}$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Table 7 for $J^P = \frac{3}{2}^+$.
We have for $H_{1,0}$ for $J^P = \frac{3}{2}^+$:

$$
\begin{aligned}
H_{1,0} &= (M_1^{++} + \mathscr{M}_1^{++})(M_0^{++} + \mathscr{M}_0^{++})^\dagger + (M_1^{--} + \mathscr{M}_1^{--})(M_0^{--} + \mathscr{M}_0^{--})^\dagger + \\
&+ (M_1^{+-} + \mathscr{M}_1^{+-})(M_0^{+-} + \mathscr{M}_0^{+-})^\dagger + (M_1^{-+} + \mathscr{M}_1^{-+})(M_0^{-+} + \mathscr{M}_0^{-+})^\dagger = \\
&= (ReM_1^{++} + iImM_1^{++} + Re\mathscr{M}_1^{++} + iIm\mathscr{M}_1^{++})(ReM_0^{++} - iImM_0^{++} + Re\mathscr{M}_0^{++} - iIm\mathscr{M}_0^{++}) + \\
&+ (ReM_1^{--} + iImM_1^{--} + Re\mathscr{M}_1^{--} + iIm\mathscr{M}_1^{--})(ReM_0^{--} - iImM_0^{--} + Re\mathscr{M}_0^{--} - iIm\mathscr{M}_0^{--}) + \\
&+ (ReM_1^{+-} + iImM_1^{+-} + Re\mathscr{M}_1^{+-} + iIm\mathscr{M}_1^{+-})(ReM_0^{+-} - iImM_0^{+-} + Re\mathscr{M}_0^{+-} - iIm\mathscr{M}_0^{+-}) + \\
&+ (ReM_1^{-+} + iImM_1^{-+} + Re\mathscr{M}_1^{-+} + iIm\mathscr{M}_1^{-+})(ReM_0^{-+} - iImM_0^{-+} + Re\mathscr{M}_0^{-+} - iIm\mathscr{M}_0^{-+}) = \\
&= ReM_1^{++}ReM_0^{++} - iReM_1^{++}ImM_0^{++} + ReM_1^{++}Re\mathscr{M}_0^{++} - iReM_1^{++}Im\mathscr{M}_0^{++} + \\
&+ iImM_1^{++}ReM_0^{++} + ImM_1^{++}ImM_0^{++} + iImM_1^{++}Re\mathscr{M}_0^{++} + ImM_1^{++}Im\mathscr{M}_0^{++} + \\
&+ Re\mathscr{M}_1^{++}ReM_0^{++} - iRe\mathscr{M}_1^{++}ImM_0^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_0^{++} + \\
&+ iIm\mathscr{M}_1^{++}ReM_0^{++} + Im\mathscr{M}_1^{++}ImM_0^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_0^{++} + \\
&+ ReM_1^{--}ReM_0^{--} - iReM_1^{--}ImM_0^{--} + ReM_1^{--}Re\mathscr{M}_0^{--} - iReM_1^{--}Im\mathscr{M}_0^{--} + \\
&+ iImM_1^{--}ReM_0^{--} + ImM_1^{--}ImM_0^{--} + iImM_1^{--}Re\mathscr{M}_0^{--} + ImM_1^{--}Im\mathscr{M}_0^{--} + \\
&+ Re\mathscr{M}_1^{--}ReM_0^{--} - iRe\mathscr{M}_1^{--}ImM_0^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + \\
&+ iIm\mathscr{M}_1^{--}ReM_0^{--} + Im\mathscr{M}_1^{--}ImM_0^{--} + iIm\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + \\
&+ ReM_1^{+-}ReM_0^{+-} - iReM_1^{+-}ImM_0^{+-} + ReM_1^{+-}Re\mathscr{M}_0^{+-} - iReM_1^{+-}Im\mathscr{M}_0^{+-} + \\
&+ iImM_1^{+-}ReM_0^{+-} + ImM_1^{+-}ImM_0^{+-} + iImM_1^{+-}Re\mathscr{M}_0^{+-} + ImM_1^{+-}Im\mathscr{M}_0^{+-} + \\
&+ Re\mathscr{M}_1^{+-}ReM_0^{+-} - iRe\mathscr{M}_1^{+-}ImM_0^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + \\
&+ iIm\mathscr{M}_1^{+-}ReM_0^{+-} + Im\mathscr{M}_1^{+-}ImM_0^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + \\
&+ ReM_1^{-+}ReM_0^{-+} - iReM_1^{-+}ImM_0^{-+} + ReM_1^{-+}Re\mathscr{M}_0^{-+} - iReM_1^{-+}Im\mathscr{M}_0^{-+} + \\
&+ iImM_1^{-+}ReM_0^{-+} + ImM_1^{-+}ImM_0^{-+} + iImM_1^{-+}Re\mathscr{M}_0^{-+} + ImM_1^{-+}Im\mathscr{M}_0^{-+} + \\
&+ Re\mathscr{M}_1^{-+}ReM_0^{-+} - iRe\mathscr{M}_1^{-+}ImM_0^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+} + \\
&+ iIm\mathscr{M}_1^{-+}ReM_0^{-+} + Im\mathscr{M}_1^{-+}ImM_0^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+}
\end{aligned} \tag{118}
$$

2.2.8 $H_{0,-1}$

We want to calculate the term $H_{0,-1}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N\lambda_Y} M_\lambda'^{\lambda_N\lambda_Y} (M_\lambda'^{\lambda_N\lambda_Y})^\dagger \tag{119}$$

and using

$$M_\lambda'^{\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y} + \mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{120}$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Table 7, for $J^P = \frac{3}{2}^+$. We have for $H_{0,-1}$ for $J^P = \frac{3}{2}^+$:

$$
\begin{aligned}
H_{0,-1} &= (M_0^{++} + \mathscr{M}_0^{++})(M_{-1}^{++} + \mathscr{M}_{-1}^{++})^\dagger + (M_0^{--} + \mathscr{M}_0^{--})(M_{-1}^{--} + \mathscr{M}_{-1}^{--})^\dagger + \\
&+ (M_0^{+-} + \mathscr{M}_0^{+-})(M_{-1}^{+-} + \mathscr{M}_{-1}^{+-})^\dagger + (M_0^{-+} + \mathscr{M}_0^{-+})(M_{-1}^{-+} + \mathscr{M}_{-1}^{-+})^\dagger = \\
&= (ReM_0^{++} + iImM_0^{++} + Re\mathscr{M}_0^{++} + iIm\mathscr{M}_0^{++})(ReM_{-1}^{++} - iImM_{-1}^{++} + Re\mathscr{M}_{-1}^{++} - iIm\mathscr{M}_{-1}^{++}) + \\
&+ (ReM_0^{--} + iImM_0^{--} + Re\mathscr{M}_0^{--} + iIm\mathscr{M}_0^{--})(ReM_{-1}^{--} - iImM_{-1}^{--} + Re\mathscr{M}_{-1}^{--} - iIm\mathscr{M}_{-1}^{--}) + \\
&+ (ReM_0^{+-} + iImM_0^{+-} + Re\mathscr{M}_0^{+-} + iIm\mathscr{M}_0^{+-})(ReM_{-1}^{+-} - iImM_{-1}^{+-} + Re\mathscr{M}_{-1}^{+-} - iIm\mathscr{M}_{-1}^{+-}) + \\
&+ (ReM_0^{-+} + iImM_0^{-+} + Re\mathscr{M}_0^{-+} + iIm\mathscr{M}_0^{-+})(ReM_{-1}^{-+} - iImM_{-1}^{-+} + Re\mathscr{M}_{-1}^{-+} - iIm\mathscr{M}_{-1}^{-+}) = \\
&= ReM_0^{++}ReM_{-1}^{++} - iReM_0^{++}ImM_{-1}^{++} + ReM_0^{++}Re\mathscr{M}_{-1}^{++} - iReM_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ iImM_0^{++}ReM_{-1}^{++} + ImM_0^{++}ImM_{-1}^{++} + iImM_0^{++}Re\mathscr{M}_{-1}^{++} + ImM_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ Re\mathscr{M}_0^{++}ReM_{-1}^{++} - iRe\mathscr{M}_0^{++}ImM_{-1}^{++} + Re\mathscr{M}_0^{++}Re\mathscr{M}_{-1}^{++} - iRe\mathscr{M}_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ iIm\mathscr{M}_0^{++}ReM_{-1}^{++} + Im\mathscr{M}_0^{++}ImM_{-1}^{++} + iIm\mathscr{M}_0^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_0^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ ReM_0^{--}ReM_{-1}^{--} - iReM_0^{--}ImM_{-1}^{--} + ReM_0^{--}Re\mathscr{M}_{-1}^{--} - iReM_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ iImM_0^{--}ReM_{-1}^{--} + ImM_0^{--}ImM_{-1}^{--} + iImM_0^{--}Re\mathscr{M}_{-1}^{--} + ImM_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ Re\mathscr{M}_0^{--}ReM_{-1}^{--} - iRe\mathscr{M}_0^{--}ImM_{-1}^{--} + Re\mathscr{M}_0^{--}Re\mathscr{M}_{-1}^{--} - iRe\mathscr{M}_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ iIm\mathscr{M}_0^{--}ReM_{-1}^{--} + Im\mathscr{M}_0^{--}ImM_{-1}^{--} + iIm\mathscr{M}_0^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_0^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ ReM_0^{+-}ReM_{-1}^{+-} - iReM_0^{+-}ImM_{-1}^{+-} + ReM_0^{+-}Re\mathscr{M}_{-1}^{+-} - iReM_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ iImM_0^{+-}ReM_{-1}^{+-} + ImM_0^{+-}ImM_{-1}^{+-} + iImM_0^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ Re\mathscr{M}_0^{+-}ReM_{-1}^{+-} - iRe\mathscr{M}_0^{+-}ImM_{-1}^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_{-1}^{+-} - iRe\mathscr{M}_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ iIm\mathscr{M}_0^{+-}ReM_{-1}^{+-} + Im\mathscr{M}_0^{+-}ImM_{-1}^{+-} + iIm\mathscr{M}_0^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ ReM_0^{-+}ReM_{-1}^{-+} - iReM_0^{-+}ImM_{-1}^{-+} + ReM_0^{-+}Re\mathscr{M}_{-1}^{-+} - iReM_0^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ iImM_0^{-+}ReM_{-1}^{-+} + ImM_0^{-+}ImM_{-1}^{-+} + iImM_0^{-+}Re\mathscr{M}_{-1}^{-+} + ImM_0^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ Re\mathscr{M}_0^{-+}ReM_{-1}^{-+} - iRe\mathscr{M}_0^{-+}ImM_{-1}^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_{-1}^{-+} - iRe\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ iIm\mathscr{M}_0^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_0^{-+}ImM_{-1}^{-+} + iIm\mathscr{M}_0^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+}
\end{aligned}
\tag{121}
$$

2.2.9 $H_{-1,0}$

We want to calculate the term $H_{-1,0}$.
Considering the definition

$$H_{\lambda\lambda'} = \sum_{\lambda_N \lambda_Y} M_\lambda^{'\lambda_N\lambda_Y} (M_\lambda^{'\lambda_N\lambda_Y})^\dagger \tag{122}$$

and using

$$M_\lambda^{'\lambda_N\lambda_Y} = M_\lambda^{\lambda_N\lambda_Y} + \mathscr{M}_\lambda^{\lambda_N\lambda_Y} \tag{123}$$

where the $\mathscr{M}_\lambda^{\lambda_N\lambda_Y}$ are the values from the grids, and the $M_\lambda^{\lambda_N\lambda_Y}$ are the values reported in Table 7 for $J^P = \frac{3}{2}^+$.
We have for $H_{-1,0}$ for $J^P = \frac{3}{2}^+$:

$$
\begin{aligned}
H_{-1,0} &= (M_{-1}^{++} + \mathscr{M}_{-1}^{++})(M_0^{++} + \mathscr{M}_0^{++})^\dagger + (M_{-1}^{--} + \mathscr{M}_{-1}^{--})(M_0^{--} + \mathscr{M}_0^{--})^\dagger + \\
&+ (M_{-1}^{+-} + \mathscr{M}_{-1}^{+-})(M_0^{+-} + \mathscr{M}_0^{+-})^\dagger + (M_{-1}^{-+} + \mathscr{M}_{-1}^{-+})(M_0^{-+} + \mathscr{M}_0^{-+})^\dagger = \\
&= (ReM_{-1}^{++} + iImM_{-1}^{++} + Re\mathscr{M}_{-1}^{++} + iIm\mathscr{M}_{-1}^{++})(ReM_0^{++} - iImM_0^{++} + Re\mathscr{M}_0^{++} - iIm\mathscr{M}_0^{++}) + \\
&+ (ReM_{-1}^{--} + iImM_{-1}^{--} + Re\mathscr{M}_{-1}^{--} + iIm\mathscr{M}_{-1}^{--})(ReM_0^{--} - iImM_0^{--} + Re\mathscr{M}_0^{--} - iIm\mathscr{M}_0^{--}) + \\
&+ (ReM_{-1}^{+-} + iImM_{-1}^{+-} + Re\mathscr{M}_{-1}^{+-} + iIm\mathscr{M}_{-1}^{+-})(ReM_0^{+-} - iImM_0^{+-} + Re\mathscr{M}_0^{+-} - iIm\mathscr{M}_0^{+-}) + \\
&+ (ReM_{-1}^{-+} + iImM_{-1}^{-+} + Re\mathscr{M}_{-1}^{-+} + iIm\mathscr{M}_{-1}^{-+})(ReM_0^{-+} - iImM_0^{-+} + Re\mathscr{M}_0^{-+} - iIm\mathscr{M}_0^{-+}) = \\
&= ReM_{-1}^{++}ReM_0^{++} - iReM_{-1}^{++}ImM_0^{++} + ReM_{-1}^{++}Re\mathscr{M}_0^{++} - iReM_{-1}^{++}Im\mathscr{M}_0^{++} + \\
&+ iImM_{-1}^{++}ReM_0^{++} + ImM_{-1}^{++}ImM_0^{++} + iImM_{-1}^{++}Re\mathscr{M}_0^{++} + ImM_{-1}^{++}Im\mathscr{M}_0^{++} + \\
&+ Re\mathscr{M}_{-1}^{++}ReM_0^{++} - iRe\mathscr{M}_{-1}^{++}ImM_0^{++} + Re\mathscr{M}_{-1}^{++}Re\mathscr{M}_0^{++} - iRe\mathscr{M}_{-1}^{++}Im\mathscr{M}_0^{++} + \\
&+ iIm\mathscr{M}_{-1}^{++}ReM_0^{++} + Im\mathscr{M}_{-1}^{++}ImM_0^{++} + iIm\mathscr{M}_{-1}^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_{-1}^{++}Im\mathscr{M}_0^{++} + \\
&+ ReM_{-1}^{--}ReM_0^{--} - iReM_{-1}^{--}ImM_0^{--} + ReM_{-1}^{--}Re\mathscr{M}_0^{--} - iReM_{-1}^{--}Im\mathscr{M}_0^{--} + \\
&+ iImM_{-1}^{--}ReM_0^{--} + ImM_{-1}^{--}ImM_0^{--} + iImM_{-1}^{--}Re\mathscr{M}_0^{--} + ImM_{-1}^{--}Im\mathscr{M}_0^{--} + \\
&+ Re\mathscr{M}_{-1}^{--}ReM_0^{--} - iRe\mathscr{M}_{-1}^{--}ImM_0^{--} + Re\mathscr{M}_{-1}^{--}Re\mathscr{M}_0^{--} - iRe\mathscr{M}_{-1}^{--}Im\mathscr{M}_0^{--} + \\
&+ iIm\mathscr{M}_{-1}^{--}ReM_0^{--} + Im\mathscr{M}_{-1}^{--}ImM_0^{--} + iIm\mathscr{M}_{-1}^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_{-1}^{--}Im\mathscr{M}_0^{--} + \\
&+ ReM_{-1}^{+-}ReM_0^{+-} - iReM_{-1}^{+-}ImM_0^{+-} + ReM_{-1}^{+-}Re\mathscr{M}_0^{+-} - iReM_{-1}^{+-}Im\mathscr{M}_0^{+-} + \\
&+ iImM_{-1}^{+-}ReM_0^{+-} + ImM_{-1}^{+-}ImM_0^{+-} + iImM_{-1}^{+-}Re\mathscr{M}_0^{+-} + ImM_{-1}^{+-}Im\mathscr{M}_0^{+-} + \\
&+ Re\mathscr{M}_{-1}^{+-}ReM_0^{+-} - iRe\mathscr{M}_{-1}^{+-}ImM_0^{+-} + Re\mathscr{M}_{-1}^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_{-1}^{+-}Im\mathscr{M}_0^{+-} + \\
&+ iIm\mathscr{M}_{-1}^{+-}ReM_0^{+-} + Im\mathscr{M}_{-1}^{+-}ImM_0^{+-} + iIm\mathscr{M}_{-1}^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_{-1}^{+-}Im\mathscr{M}_0^{+-} + \\
&+ ReM_{-1}^{-+}ReM_0^{-+} - iReM_{-1}^{-+}ImM_0^{-+} + ReM_{-1}^{-+}Re\mathscr{M}_0^{-+} - iReM_{-1}^{-+}Im\mathscr{M}_0^{-+} + \\
&+ iImM_{-1}^{-+}ReM_0^{-+} + ImM_{-1}^{-+}ImM_0^{-+} + iImM_{-1}^{-+}Re\mathscr{M}_0^{-+} + ImM_{-1}^{-+}Im\mathscr{M}_0^{-+} + \\
&+ Re\mathscr{M}_{-1}^{-+}ReM_0^{-+} - iRe\mathscr{M}_{-1}^{-+}ImM_0^{-+} + Re\mathscr{M}_{-1}^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_{-1}^{-+}Im\mathscr{M}_0^{-+} + \\
&+ iIm\mathscr{M}_{-1}^{-+}ReM_0^{-+} + Im\mathscr{M}_{-1}^{-+}ImM_0^{-+} + iIm\mathscr{M}_{-1}^{-+}Re\mathscr{M}_0^{-+} + Im\mathscr{M}_{-1}^{-+}Im\mathscr{M}_0^{-+}
\end{aligned}
\tag{124}
$$

3 Calculation of differential cross sections

3.1 χ factor

To calculate χ, defined as:

$$\chi = \frac{1}{16 W m_P} \frac{|\mathbf{p}_K^*|}{K_H} \tag{125}$$

where W is the invariant mass of $K^+\Lambda$ system and m_P is 0.938 GeV/c^2, we need $|\mathbf{p}_K^*|$ and K_H.
As already obtained in Eq. 24, we obtain the first one using the formula for the two-body decay in the center of mass frame:

$$|\mathbf{p}_K^*| = |\mathbf{p}_\Lambda^*| = \frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W} \tag{126}$$

where m_K is 0.4937 GeV/c^2 and m_Λ is 1.116 GeV/c^2.
K_H is defined as

$$K_H = \omega_{LAB} - \frac{Q^2}{2m_P} \tag{127}$$

where Q^2 has been already defined in Eq. 17 as

$$Q^2 = -q^2 = -(\omega_{LAB}^2 - \mathbf{k}_{LAB}^2) = |\mathbf{k}_{LAB}|^2 - \omega_{LAB}^2 \tag{128}$$

In the laboratory frame holds the relationship 18:

$$\begin{aligned} W^2 &= (m_P + \omega_{LAB})^2 - (\mathbf{k}_{LAB})^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - |\mathbf{k}_{LAB}|^2 \\ &\Rightarrow |\mathbf{k}_{LAB}|^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - W^2 \end{aligned} \tag{129}$$

So, substituting 129 in 128, we have:

$$\begin{aligned} Q^2 &= |\mathbf{k}_{LAB}|^2 - \omega_{LAB}^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - W^2 - \omega_{LAB}^2 \\ &= m_P^2 + 2m_P\omega_{LAB} - W^2 \end{aligned} \tag{130}$$

and we can use this result in 127 obtaining

$$K_H = \omega_{LAB} - \frac{Q^2}{2m_P} = \omega_{LAB} - \frac{m_P^2 + 2m_P\omega_{LAB} - W^2}{2m_P} = \frac{W^2 - m_P^2}{2m_P} \tag{131}$$

In conclusion χ factor can be calculated as

$$\begin{aligned} \chi &= \frac{1}{16Wm_P}\frac{|\mathbf{p}_K^*|}{K_H} = \frac{1}{16Wm_P}\frac{\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}}{\frac{W^2 - m_P^2}{2m_P}} \\ &= \frac{1}{16Wm_P}\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\frac{2m_P}{W^2 - m_P^2} \end{aligned} \tag{132}$$

3.2 $J^P = \frac{1}{2}^+$

3.2.1 $\frac{d\sigma_L}{d\Omega}$

We use the definition

$$\frac{d\sigma_L}{d\Omega_K^*} = 2\chi\frac{1}{(4\pi)^2}(H_{0,0}) \tag{133}$$

employing χ factor from Eq. 132 and $H_{0,0}$ from Eq. 49 we have:

$$\begin{aligned} \frac{d\sigma_L}{d\Omega_K^*} &= 2\chi\frac{1}{(4\pi)^2}(H_{0,0}) = \frac{2}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\frac{2m_P}{W^2 - m_P^2}\cdot \\ &\big[2(F'_{1/2}AQ)^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + 2(Im\mathscr{M}_0^{++})^2 + 2(Re\mathscr{M}_0^{++})^2 + (Im\mathscr{M}_0^{+-})^2 + \\ &+ (Re\mathscr{M}_0^{+-})^2 + (Im\mathscr{M}_0^{-+})^2 + (Re\mathscr{M}_0^{-+})^2 + 2Re\mathscr{M}_0^{+-}F'_{1/2}AQ(sin\frac{\theta_K^*}{2})(M_r^2 - W^2) + \\ &+ 2Im\mathscr{M}_0^{+-}F'_{1/2}AQ(sin\frac{\theta_K^*}{2})(\Gamma_r M_r) + 2Re\mathscr{M}_0^{-+}F'_{1/2}AQ(-sin\frac{\theta_K^*}{2})(M_r^2 - W^2) + 2Im\mathscr{M}_0^{-+}F'_{1/2}AQ(-sin\frac{\theta_K^*}{2})(\Gamma_r M_r)\big] \end{aligned} \tag{134}$$

Or using χ factor from Eq. 132 and $H_{0,0}$ from Eq. 50:

$$\begin{aligned} &\frac{d\sigma_L}{d\Omega_K^*} = 2\chi\frac{1}{(4\pi)^2}(H_{0,0}) = \\ &= \frac{2}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\frac{2m_P}{W^2 - m_P^2}\cdot \\ &\big[2(F'_{1/2}AQ)^2((M_r^2 - W^2)^2 + (\Gamma_r M_r)^2) + 2\mid\mathscr{M}_0^{++}\mid^2 + \mid\mathscr{M}_0^{+-}\mid^2 + \mid\mathscr{M}_0^{-+}\mid^2 + \\ &+ 2F'_{1/2}AQ(sin\frac{\theta_K^*}{2})((M_r^2 - W^2)Re\mathscr{M}_0^{+-} + Im\mathscr{M}_0^{+-}(\Gamma_r M_r) - Re\mathscr{M}_0^{-+}(M_r^2 - W^2) - Im\mathscr{M}_0^{-+}(\Gamma_r M_r))\big] \end{aligned} \tag{135}$$

3.2.2 $\frac{d\sigma_T}{d\Omega}$

The terms in the grid satisfy the relationships

$$\mathscr{M}_1^{++} = \mathscr{M}_{-1}^{++} \;\; \mathscr{M}_0^{++} = -\mathscr{M}_0^{--} \;\; \mathscr{M}_1^{-+} = -\mathscr{M}_{-1}^{-+} \;\; \mathscr{M}_1^{+-} = -\mathscr{M}_{-1}^{+-} \;\; \mathscr{M}_1^{--} = \mathscr{M}_{-1}^{--} \tag{136}$$

Using χ factor from Eq. 132, $H_{1,1}$ from Eq. 54 and $H_{-1,-1}$ Eq. 58:

$$\begin{aligned}
\frac{d\sigma_T}{d\Omega_K^*} &= \chi\frac{1}{(4\pi)^2}(H_{1,1}+H_{-1,-1}) = \\
&= \frac{\chi}{(4\pi)^2}\cdot\big[(F'_{1/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+\mid\mathscr{M}_1^{++}\mid^2+\mid\mathscr{M}_1^{--}\mid^2+\mid\mathscr{M}_1^{+-}\mid^2+\mid\mathscr{M}_1^{-+}\mid^2+\\
&+2F'_{1/2}A(-sin\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_1^{++}+2F'_{1/2}A(-sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{++}+\\
&+2F'_{1/2}A(cos\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_1^{+-}+2F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{+-}+\\
&+(F'_{1/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+\mid\mathscr{M}_{-1}^{++}\mid^2+\mid\mathscr{M}_{-1}^{--}\mid^2+\mid\mathscr{M}_{-1}^{+-}\mid^2+\mid\mathscr{M}_{-1}^{-+}\mid^2+\\
&+2F'_{1/2}A(cos\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+2F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+}+\\
&+2F'_{1/2}A(sin\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_{-1}^{--}+2F'_{1/2}A(sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--}\big] = \\
&= \frac{\chi}{(4\pi)^2}\cdot\big[2(F'_{1/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+\mid\mathscr{M}_1^{++}\mid^2+\mid\mathscr{M}_1^{--}\mid^2+\mid\mathscr{M}_1^{+-}\mid^2+\mid\mathscr{M}_1^{-+}\mid^2+\\
&+2F'_{1/2}A(-sin\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_1^{++}+2F'_{1/2}A(-sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{++}+\\
&+2F'_{1/2}A(cos\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_1^{+-}+2F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{+-}+\\
&+\mid\mathscr{M}_{-1}^{++}\mid^2+\mid\mathscr{M}_{-1}^{--}\mid^2+\mid\mathscr{M}_{-1}^{+-}\mid^2+\mid\mathscr{M}_{-1}^{-+}\mid^2+\\
&+2F'_{1/2}A(cos\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+2F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+}+\\
&+2F'_{1/2}A(sin\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_{-1}^{--}+2F'_{1/2}A(sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--}\big] = \\
&= \frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot\\
&2\big[(F'_{1/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+\mid\mathscr{M}_1^{++}\mid^2+\mid\mathscr{M}_1^{--}\mid^2+\mid\mathscr{M}_1^{+-}\mid^2+\mid\mathscr{M}_1^{-+}\mid^2+\\
&-F'_{1/2}A(sin\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_1^{++}-F'_{1/2}A(sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{++}+F'_{1/2}A(cos\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_1^{+-}+\\
&+F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_1^{+-}+F'_{1/2}A(cos\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+F'_{1/2}A(cos\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+}+\\
&+F'_{1/2}A(sin\theta_K^*/2)(M_r^2-W^2)Re\mathscr{M}_{-1}^{--}+F'_{1/2}A(sin\theta_K^*/2)(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--}\big]
\end{aligned} \tag{137}$$

In conclusion

$$\begin{aligned}
\frac{d\sigma_T}{d\Omega_K^*} &= \chi\frac{1}{(4\pi)^2}(H_{1,1}+H_{-1,-1}) = \\
&= \frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot\\
&2\big[(F'_{1/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+\mid\mathscr{M}_1^{++}\mid^2+\mid\mathscr{M}_1^{--}\mid^2+\mid\mathscr{M}_1^{+-}\mid^2+\mid\mathscr{M}_1^{-+}\mid^2+\\
&+F'_{1/2}A(sin\theta_K^*/2)((M_r^2-W^2)Re\mathscr{M}_{-1}^{--}+(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--}-(M_r^2-W^2)Re\mathscr{M}_1^{++}-(\Gamma_r M_r)Im\mathscr{M}_1^{++})\\
&+F'_{1/2}A(cos\theta_K^*/2)((M_r^2-W^2)Re\mathscr{M}_1^{+-}+(\Gamma_r M_r)Im\mathscr{M}_1^{+-}+(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+})\big]
\end{aligned} \tag{138}$$

3.2.3 $\frac{d\sigma_{LT}}{d\Omega}$

$$
\begin{aligned}
&\frac{d\sigma_{LT}}{d\Omega_K^*} = -\chi\frac{1}{(4\pi)^2}(H_{1,-1}+H_{-1,1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&\big[(ReM_1^{++}Re\mathscr{M}_{-1}^{++} - iReM_1^{++}Im\mathscr{M}_{-1}^{++} + iImM_1^{++}Re\mathscr{M}_{-1}^{++} + ImM_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+Re\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+Re\mathscr{M}_1^{--}ReM_{-1}^{--} - iRe\mathscr{M}_1^{--}ImM_{-1}^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+iIm\mathscr{M}_1^{--}ReM_{-1}^{--} + Im\mathscr{M}_1^{--}ImM_{-1}^{--} + iIm\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ReM_1^{+-}Re\mathscr{M}_{-1}^{+-} - iReM_1^{+-}Im\mathscr{M}_{-1}^{+-} + iImM_1^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+Re\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+Re\mathscr{M}_1^{-+}ReM_{-1}^{-+} - iRe\mathscr{M}_1^{-+}ImM_{-1}^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+iIm\mathscr{M}_1^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_1^{-+}ImM_{-1}^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+}) + \\
&+(+Re\mathscr{M}_{-1}^{++}ReM_1^{++} - iRe\mathscr{M}_{-1}^{++}ImM_1^{++} + Re\mathscr{M}_{-1}^{++}Re\mathscr{M}_1^{++} - iRe\mathscr{M}_{-1}^{++}Im\mathscr{M}_1^{++} + \\
&+iIm\mathscr{M}_{-1}^{++}ReM_1^{++} + Im\mathscr{M}_{-1}^{++}ImM_1^{++} + iIm\mathscr{M}_{-1}^{++}Re\mathscr{M}_1^{++} + Im\mathscr{M}_{-1}^{++}Im\mathscr{M}_1^{++} + \\
&+ReM_{-1}^{--}Re\mathscr{M}_1^{--} - iReM_{-1}^{--}Im\mathscr{M}_1^{--} + iImM_{-1}^{--}Re\mathscr{M}_1^{--} + ImM_{-1}^{--}Im\mathscr{M}_1^{--} + \\
&+Re\mathscr{M}_{-1}^{--}Re\mathscr{M}_1^{--} - iRe\mathscr{M}_{-1}^{--}Im\mathscr{M}_1^{--} + iIm\mathscr{M}_{-1}^{--}Re\mathscr{M}_1^{--} + Im\mathscr{M}_{-1}^{--}Im\mathscr{M}_1^{--} + \\
&+Re\mathscr{M}_{-1}^{+-}ReM_1^{+-} - iRe\mathscr{M}_{-1}^{+-}ImM_1^{+-} + Re\mathscr{M}_{-1}^{+-}Re\mathscr{M}_1^{+-} - iRe\mathscr{M}_{-1}^{+-}Im\mathscr{M}_1^{+-} + \\
&+iIm\mathscr{M}_{-1}^{+-}ReM_1^{+-} + Im\mathscr{M}_{-1}^{+-}ImM_1^{+-} + iIm\mathscr{M}_{-1}^{+-}Re\mathscr{M}_1^{+-} + Im\mathscr{M}_{-1}^{+-}Im\mathscr{M}_1^{+-} + \\
&+ReM_{-1}^{-+}Re\mathscr{M}_1^{-+} - iReM_{-1}^{-+}Im\mathscr{M}_1^{-+} + iImM_{-1}^{-+}Re\mathscr{M}_1^{-+} + ImM_{-1}^{-+}Im\mathscr{M}_1^{-+} + \\
&+Re\mathscr{M}_{-1}^{-+}Re\mathscr{M}_1^{-+} - iRe\mathscr{M}_{-1}^{-+}Im\mathscr{M}_1^{-+} + iIm\mathscr{M}_{-1}^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_{-1}^{-+}Im\mathscr{M}_1^{-+} \\
&)\big]
\end{aligned}
\tag{139}
$$

The terms in the grid satisfy the relationships

$$
\mathscr{M}_1^{++} = \mathscr{M}_{-1}^{++} \;\; \mathscr{M}_0^{++} = -\mathscr{M}_0^{--} \;\; \mathscr{M}_1^{-+} = -\mathscr{M}_{-1}^{-+} \;\; \mathscr{M}_1^{+-} = -\mathscr{M}_{-1}^{+-} \;\; \mathscr{M}_1^{--} = \mathscr{M}_{-1}^{--}
\tag{140}
$$

And from Table 3 we have that

$$
M_1^{++} = -M_{-1}^{--} \;\; M_1^{+-} = M_{-1}^{-+} \;\; M_0^{+-} = -M_0^{-+} \;\; M_0^{++} = M_0^{--}
\tag{141}
$$

And we can simplify considering that there are opposite terms:

$$
\begin{aligned}
&\frac{d\sigma_{LT}}{d\Omega_K^*} = -\chi\frac{1}{(4\pi)^2}(H_{1,-1}+H_{-1,1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&2\big(ReM_1^{++}Re\mathscr{M}_{-1}^{++} + ImM_1^{++}Im\mathscr{M}_{-1}^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ Re\mathscr{M}_1^{--}ReM_{-1}^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_1^{--}ImM_{-1}^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ ReM_1^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_1^{+-}Im\mathscr{M}_{-1}^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ Re\mathscr{M}_1^{-+}ReM_{-1}^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_1^{-+}ImM_{-1}^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+}\big) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&2\big(ReM_1^{++}Re\mathscr{M}_{-1}^{++} + ImM_1^{++}Im\mathscr{M}_{-1}^{++} + |\mathscr{M}_1^{++}|^2 + |\mathscr{M}_{-1}^{--}|^2 + Re\mathscr{M}_1^{--}ReM_{-1}^{--} + Im\mathscr{M}_1^{--}ImM_{-1}^{--} + \\
&+ ReM_1^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_1^{+-}Im\mathscr{M}_{-1}^{+-} - |\mathscr{M}_1^{+-}|^2 - |\mathscr{M}_1^{-+}|^2 + Re\mathscr{M}_1^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_1^{-+}ImM_{-1}^{-+}\big)
\end{aligned}
\tag{142}
$$

Finally we substitute values from Tabs 4 and 5:

$$
\begin{aligned}
&\frac{d\sigma_{LT}}{d\Omega_K^*} = -\chi \frac{1}{(4\pi)^2}(H_{1,-1} + H_{-1,1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\frac{2m_P}{W^2 - m_P^2}\cdot \\
&2\big(F'_{1/2}A(-sin\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_{-1}^{++} - F'_{1/2}A(sin\theta_K^*/2)\Gamma_r M_r Im\mathscr{M}_{-1}^{++} + |\mathscr{M}_1^{++}|^2 + |\mathscr{M}_{-1}^{--}|^2 + \\
&+ Re\mathscr{M}_1^{--}F'_{1/2}A(sin\theta_K^*/2)(M_r^2 - W^2) + Im\mathscr{M}_1^{--}F'_{1/2}A(sin\theta_K^*/2)\Gamma_r M_r + \\
&+ F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_{-1}^{+-} + F'_{1/2}A(cos\theta_K^*/2)\Gamma_r M_r Im\mathscr{M}_{-1}^{+-} - |\mathscr{M}_1^{+-}|^2 - |\mathscr{M}_1^{-+}|^2 + \\
&+ Re\mathscr{M}_1^{-+}F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2) + Im\mathscr{M}_1^{-+}F'_{1/2}A(cos\theta_K^*/2)\Gamma_r M_r\big)
\end{aligned}
\tag{143}
$$

In conclusion:

$$
\begin{aligned}
&\frac{d\sigma_{LT}}{d\Omega_K^*} = -\chi \frac{1}{(4\pi)^2}(H_{1,-1} + H_{-1,1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\frac{2m_P}{W^2 - m_P^2}\cdot 2\big(|\mathscr{M}_1^{++}|^2 + |\mathscr{M}_{-1}^{--}|^2 - |\mathscr{M}_1^{+-}|^2 - |\mathscr{M}_1^{-+}|^2 + \\
&+ F'_{1/2}A(sin\theta_K^*/2)(Re\mathscr{M}_1^{--}(M_r^2 - W^2) + Im\mathscr{M}_1^{--}\Gamma_r M_r - (M_r^2 - W^2)Re\mathscr{M}_{-1}^{++} - \Gamma_r M_r Im\mathscr{M}_{-1}^{++}) \\
&+ F'_{1/2}A(cos\theta_K^*/2)((M_r^2 - W^2)Re\mathscr{M}_{-1}^{+-} + \Gamma_r M_r Im\mathscr{M}_{-1}^{+-} + Re\mathscr{M}_1^{-+}(M_r^2 - W^2) + Im\mathscr{M}_1^{-+}\Gamma_r M_r)\big)
\end{aligned}
\tag{144}
$$

3.2.4 $\frac{d\sigma_{TT}}{d\Omega}$

$$
\begin{gathered}
\frac{d\sigma_{TT}}{d\Omega_K^*} = -\chi\frac{1}{(4\pi)^2}(H_{0,1}+H_{1,0}-H_{-1,0}-H_{0,-1}) = \\
= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
\big[ReM_0^{++}ReM_1^{++} - iReM_0^{++}ImM_1^{++} + ReM_0^{++}Re\mathscr{M}_1^{++} - iReM_0^{++}Im\mathscr{M}_1^{++} + iImM_0^{++}ReM_1^{++}+ \\
+ImM_0^{++}ImM_1^{++} + iImM_0^{++}Re\mathscr{M}_1^{++} + ImM_0^{++}Im\mathscr{M}_1^{++} + Re\mathscr{M}_0^{++}ReM_1^{++} - iRe\mathscr{M}_0^{++}ImM_1^{++}+ \\
+Re\mathscr{M}_0^{++}Re\mathscr{M}_1^{++} - iRe\mathscr{M}_0^{++}Im\mathscr{M}_1^{++} + iIm\mathscr{M}_0^{++}ReM_1^{++} + Im\mathscr{M}_0^{++}ImM_1^{++} + iIm\mathscr{M}_0^{++}Re\mathscr{M}_1^{++}+ \\
+Im\mathscr{M}_0^{++}Im\mathscr{M}_1^{++} + ReM_0^{--}Re\mathscr{M}_1^{--} - iReM_0^{--}Im\mathscr{M}_1^{--} + iImM_0^{--}Re\mathscr{M}_1^{--} + ImM_0^{--}Im\mathscr{M}_1^{--}+ \\
+Re\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} - iRe\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + iIm\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} + Im\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + ReM_0^{+-}ReM_1^{+-}+ \\
-iReM_0^{+-}ImM_1^{+-} + ReM_0^{+-}Re\mathscr{M}_1^{+-} - iReM_0^{+-}Im\mathscr{M}_1^{+-} + iImM_0^{+-}ReM_1^{+-} + ImM_0^{+-}ImM_1^{+-}+ \\
+iImM_0^{+-}Re\mathscr{M}_1^{+-} + ImM_0^{+-}Im\mathscr{M}_1^{+-} + Re\mathscr{M}_0^{+-}ReM_1^{+-} - iRe\mathscr{M}_0^{+-}ImM_1^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-}+ \\
-iRe\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + iIm\mathscr{M}_0^{+-}ReM_1^{+-} + Im\mathscr{M}_0^{+-}ImM_1^{+-} + iIm\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-}+ \\
+ReM_0^{-+}Re\mathscr{M}_1^{-+} - iReM_0^{-+}Im\mathscr{M}_1^{-+} + iImM_0^{-+}Re\mathscr{M}_1^{-+} + ImM_0^{-+}Im\mathscr{M}_1^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+}+ \\
-iRe\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+} + iIm\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+} + ReM_1^{++}ReM_0^{++} - iReM_1^{++}ImM_0^{++}+ \\
+ReM_1^{++}Re\mathscr{M}_0^{++} - iReM_1^{++}Im\mathscr{M}_0^{++} + iImM_1^{++}ReM_0^{++} + ImM_1^{++}ImM_0^{++} + iImM_1^{++}Re\mathscr{M}_0^{++}+ \\
+ImM_1^{++}Im\mathscr{M}_0^{++} + Re\mathscr{M}_1^{++}ReM_0^{++} - iRe\mathscr{M}_1^{++}ImM_0^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_0^{++}+ \\
+iIm\mathscr{M}_1^{++}ReM_0^{++} + Im\mathscr{M}_1^{++}ImM_0^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_0^{++} + Re\mathscr{M}_1^{--}ReM_0^{--}+ \\
-iRe\mathscr{M}_1^{--}ImM_0^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + iIm\mathscr{M}_1^{--}ReM_0^{--} + Im\mathscr{M}_1^{--}ImM_0^{--}+ \\
+iIm\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + ReM_1^{+-}ReM_0^{+-} - iReM_1^{+-}ImM_0^{+-} + ReM_1^{+-}Re\mathscr{M}_0^{+-}+ \\
-iReM_1^{+-}Im\mathscr{M}_0^{+-} + iImM_1^{+-}ReM_0^{+-} + ImM_1^{+-}ImM_0^{+-} + iImM_1^{+-}Re\mathscr{M}_0^{+-} + ImM_1^{+-}Im\mathscr{M}_0^{+-}+ \\
+Re\mathscr{M}_1^{+-}ReM_0^{+-} - iRe\mathscr{M}_1^{+-}ImM_0^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + iIm\mathscr{M}_1^{+-}ReM_0^{+-}+ \\
+Im\mathscr{M}_1^{+-}ImM_0^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + Re\mathscr{M}_1^{-+}ReM_0^{-+} - iRe\mathscr{M}_1^{-+}ImM_0^{-+}+ \\
+Re\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+} + iIm\mathscr{M}_1^{-+}ReM_0^{-+} + Im\mathscr{M}_1^{-+}ImM_0^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+}+ \\
+Im\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+} - \big(Re\mathscr{M}_{-1}^{++}ReM_0^{++} - iRe\mathscr{M}_{-1}^{++}ImM_0^{++} + Re\mathscr{M}_{-1}^{++}Re\mathscr{M}_0^{++} - iRe\mathscr{M}_{-1}^{++}Im\mathscr{M}_0^{++}+ \\
+iIm\mathscr{M}_{-1}^{++}ReM_0^{++} + Im\mathscr{M}_{-1}^{++}ImM_0^{++} + iIm\mathscr{M}_{-1}^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_{-1}^{++}Im\mathscr{M}_0^{++} + ReM_{-1}^{--}ReM_0^{--} \\
-iReM_{-1}^{--}ImM_0^{--} + ReM_{-1}^{--}Re\mathscr{M}_0^{--} - iReM_{-1}^{--}Im\mathscr{M}_0^{--} + iImM_{-1}^{--}ReM_0^{--} + ImM_{-1}^{--}ImM_0^{--}+ \\
+iImM_{-1}^{--}Re\mathscr{M}_0^{--} + ImM_{-1}^{--}Im\mathscr{M}_0^{--} + Re\mathscr{M}_{-1}^{--}ReM_0^{--} - iRe\mathscr{M}_{-1}^{--}ImM_0^{--} + Re\mathscr{M}_{-1}^{--}Re\mathscr{M}_0^{--}+ \\
-iRe\mathscr{M}_{-1}^{--}Im\mathscr{M}_0^{--} + iIm\mathscr{M}_{-1}^{--}ReM_0^{--} + Im\mathscr{M}_{-1}^{--}ImM_0^{--} + iIm\mathscr{M}_{-1}^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_{-1}^{--}Im\mathscr{M}_0^{--}+ \\
+Re\mathscr{M}_{-1}^{+-}ReM_0^{+-} - iRe\mathscr{M}_{-1}^{+-}ImM_0^{+-} + Re\mathscr{M}_{-1}^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_{-1}^{+-}Im\mathscr{M}_0^{+-} + iIm\mathscr{M}_{-1}^{+-}ReM_0^{+-}+ \\
+Im\mathscr{M}_{-1}^{+-}ImM_0^{+-} + iIm\mathscr{M}_{-1}^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_{-1}^{+-}Im\mathscr{M}_0^{+-} + ReM_{-1}^{-+}ReM_0^{-+} - iReM_{-1}^{-+}ImM_0^{-+}+ \\
+ReM_{-1}^{-+}Re\mathscr{M}_0^{-+} - iReM_{-1}^{-+}Im\mathscr{M}_0^{-+} + iImM_{-1}^{-+}ReM_0^{-+} + ImM_{-1}^{-+}ImM_0^{-+} + iImM_{-1}^{-+}Re\mathscr{M}_0^{-+}+ \\
+ImM_{-1}^{-+}Im\mathscr{M}_0^{-+} + Re\mathscr{M}_{-1}^{-+}ReM_0^{-+} - iRe\mathscr{M}_{-1}^{-+}ImM_0^{-+} + Re\mathscr{M}_{-1}^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_{-1}^{-+}Im\mathscr{M}_0^{-+}+ \\
+iIm\mathscr{M}_{-1}^{-+}ReM_0^{-+} + Im\mathscr{M}_{-1}^{-+}ImM_0^{-+} + iIm\mathscr{M}_{-1}^{-+}Re\mathscr{M}_0^{-+} + Im\mathscr{M}_{-1}^{-+}Im\mathscr{M}_0^{-+}\big) - \big(ReM_1^{++}ReM_0^{++}+ \\
-iReM_1^{++}ImM_0^{++} + ReM_1^{++}Re\mathscr{M}_0^{++} - iReM_1^{++}Im\mathscr{M}_0^{++} + iImM_1^{++}ReM_0^{++} + ImM_1^{++}ImM_0^{++}+ \\
+iImM_1^{++}Re\mathscr{M}_0^{++} + ImM_1^{++}Im\mathscr{M}_0^{++} + Re\mathscr{M}_1^{++}ReM_0^{++} - iRe\mathscr{M}_1^{++}ImM_0^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_0^{++}+ \\
-iRe\mathscr{M}_1^{++}Im\mathscr{M}_0^{++} + iIm\mathscr{M}_1^{++}ReM_0^{++} + Im\mathscr{M}_1^{++}ImM_0^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_0^{++}+ \\
+Re\mathscr{M}_1^{--}ReM_0^{--} - iRe\mathscr{M}_1^{--}ImM_0^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + iIm\mathscr{M}_1^{--}ReM_0^{--}+ \\
+Im\mathscr{M}_1^{--}ImM_0^{--} + iIm\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + ReM_1^{+-}ReM_0^{+-} - iReM_1^{+-}ImM_0^{+-}+ \\
+ReM_1^{+-}Re\mathscr{M}_0^{+-} - iReM_1^{+-}Im\mathscr{M}_0^{+-} + iImM_1^{+-}ReM_0^{+-} + ImM_1^{+-}ImM_0^{+-} + iImM_1^{+-}Re\mathscr{M}_0^{+-}+ \\
+ImM_1^{+-}Im\mathscr{M}_0^{+-} + Re\mathscr{M}_1^{+-}ReM_0^{+-} - iRe\mathscr{M}_1^{+-}ImM_0^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-}+ \\
+iIm\mathscr{M}_1^{+-}ReM_0^{+-} + Im\mathscr{M}_1^{+-}ImM_0^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + Re\mathscr{M}_1^{-+}ReM_0^{-+}+ \\
-iRe\mathscr{M}_1^{-+}ImM_0^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+} + iIm\mathscr{M}_1^{-+}ReM_0^{-+} + Im\mathscr{M}_1^{-+}ImM_0^{-+}+ \\
+iIm\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+}\big)\big]
\end{gathered} \tag{145}
$$

The terms in the grid satisfy the relationships

$$
\mathscr{M}_1^{++} = \mathscr{M}_{-1}^{++} \quad \mathscr{M}_0^{++} = -\mathscr{M}_0^{--} \quad \mathscr{M}_1^{-+} = -\mathscr{M}_{-1}^{-+} \quad \mathscr{M}_1^{+-} = -\mathscr{M}_{-1}^{+-} \quad \mathscr{M}_1^{--} = \mathscr{M}_{-1}^{--} \tag{146}
$$

And from Table 3 we have that

$$M_1^{++} = -M_{-1}^{--} \; M_1^{+-} = M_{-1}^{-+} \; M_0^{+-} = -M_0^{-+} \; M_0^{++} = M_0^{--} \tag{147}$$

And we can simplify considering that there are opposite terms, obtaining:

$$\begin{aligned}
&\frac{d\sigma_{TT}}{d\Omega_K^*} = -\chi\frac{1}{(4\pi)^2}(H_{0,1} + H_{1,0} - H_{-1,0} - H_{0,-1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\frac{2m_P}{W^2 - m_P^2}\cdot \\
&\big[4(ReM_0^{++}ReM_1^{++} + ImM_0^{++}ImM_1^{++} + ReM_0^{+-}ReM_1^{+-} + ImM_0^{+-}ImM_1^{+-} + \\
&+ Im\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} + \\
&ReM_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_1^{-+}ImM_0^{-+} + ReM_0^{+-}Re\mathscr{M}_1^{+-} + ImM_0^{+-}Im\mathscr{M}_1^{+-}) + \\
&+ 2(Re\mathscr{M}_0^{+-}ReM_1^{+-} + Im\mathscr{M}_0^{+-}ImM_1^{+-} - Re\mathscr{M}_0^{-+}ReM_{-1}^{-+} - Im\mathscr{M}_0^{-+}ImM_{-1}^{-+}))\big]
\end{aligned} \tag{148}$$

We can substitute values from Tabs 4 and 5:

$$\begin{aligned}
&\frac{d\sigma_{TT}}{d\Omega_K^*} = -\chi\frac{1}{(4\pi)^2}(H_{0,1} + H_{1,0} - H_{-1,0} - H_{0,-1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\frac{2m_P}{W^2 - m_P^2}\cdot \\
&\big[4(F'_{1/2}AQ(cos\theta_K^*/2)F'_{1/2}A(-sin\theta_K^*/2)(M_r^2 - W^2)^2 + F'_{1/2}AQ(cos\theta_K^*/2)F'_{1/2}A(-sin\theta_K^*/2)\Gamma_r^2M_r^2 + \\
&+ F'_{1/2}AQ(sin\theta_K^*/2)F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2)^2 + F'_{1/2}AQ(sin\theta_K^*/2)F'_{1/2}A(cos\theta_K^*/2)(\Gamma_rM_r)^2 + \\
&+ Im\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} + \\
&- F'_{1/2}AQ(sin\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_1^{-+} + Im\mathscr{M}_1^{-+}F'_{1/2}AQ(-sin\theta_K^*/2)\Gamma_rM_r + \\
&+ F'_{1/2}AQ(sin\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_1^{+-} + F'_{1/2}AQ(sin\theta_K^*/2)\Gamma_rM_rIm\mathscr{M}_1^{+-}) + \\
&+ 2(Re\mathscr{M}_0^{+-}F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2) + Im\mathscr{M}_0^{+-}F'_{1/2}A(cos\theta_K^*/2)\Gamma_rM_r + \\
&- Re\mathscr{M}_0^{-+}F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2) - Im\mathscr{M}_0^{-+}F'_{1/2}A(cos\theta_K^*/2)\Gamma_rM_r)\big]
\end{aligned} \tag{149}$$

Simplifying opposite terms we obtain:

$$\begin{aligned}
&\frac{d\sigma_{TT}}{d\Omega_K^*} = -\chi\frac{1}{(4\pi)^2}(H_{0,1} + H_{1,0} - H_{-1,0} - H_{0,-1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\frac{2m_P}{W^2 - m_P^2}\cdot \\
&\big[4(Im\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} + \\
&- F'_{1/2}AQ(sin\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_1^{-+} + Im\mathscr{M}_1^{-+}F'_{1/2}AQ(-sin\theta_K^*/2)\Gamma_rM_r + \\
&+ F'_{1/2}AQ(sin\theta_K^*/2)(M_r^2 - W^2)Re\mathscr{M}_1^{+-} + F'_{1/2}AQ(sin\theta_K^*/2)\Gamma_rM_rIm\mathscr{M}_1^{+-}) + \\
&+ 2(Re\mathscr{M}_0^{+-}F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2) + Im\mathscr{M}_0^{+-}F'_{1/2}A(cos\theta_K^*/2)\Gamma_rM_r + \\
&- Re\mathscr{M}_0^{-+}F'_{1/2}A(cos\theta_K^*/2)(M_r^2 - W^2) - Im\mathscr{M}_0^{-+}F'_{1/2}A(cos\theta_K^*/2)\Gamma_rM_r)\big]
\end{aligned} \tag{150}$$

In conclusion:

$$\begin{aligned}
&\frac{d\sigma_{TT}}{d\Omega_K^*} = -\chi\frac{1}{(4\pi)^2}(H_{0,1} + H_{1,0} - H_{-1,0} - H_{0,-1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\frac{2m_P}{W^2 - m_P^2}\cdot \\
&\big[4(Im\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} + \\
&+ F'_{1/2}AQ(sin\theta_K^*/2)(-(M_r^2 - W^2)Re\mathscr{M}_1^{-+} - Im\mathscr{M}_1^{-+}\Gamma_rM_r + (M_r^2 - W^2)Re\mathscr{M}_1^{+-} + \Gamma_rM_rIm\mathscr{M}_1^{+-})) + \\
&+ 2F'_{1/2}A(cos\theta_K^*/2)(Re\mathscr{M}_0^{+-}(M_r^2 - W^2) + Im\mathscr{M}_0^{+-}\Gamma_rM_r - Re\mathscr{M}_0^{-+}(M_r^2 - W^2) - Im\mathscr{M}_0^{-+}\Gamma_rM_r)\big]
\end{aligned} \tag{151}$$

3.3 $J^P = \frac{3}{2}^+$

3.3.1 $\frac{d\sigma_L}{d\Omega}$

We use the definition

$$\frac{d\sigma_L}{d\Omega_K^*} = 2\chi\frac{1}{(4\pi)^2}(H_{0,0}) \tag{152}$$

employing χ factor from Eq. 132 and $H_{0,0}$ from Eq. 95 we have:

$$\begin{aligned}
\frac{d\sigma_L}{d\Omega_K^*} &= 2\chi\frac{1}{(4\pi)^2}(H_{0,0}) = \\
&= \frac{2}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&\big[2(F'_{3/2}AQ\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})^2((M_r^2-W^2)^2+(\Gamma_rM_r)^2)+2(F'_{3/2}AQ(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2}))^2((M_r^2-W^2)^2+(\Gamma_rM_r)^2)+ \\
&+(Im\mathscr{M}_0^{++})^2+(Re\mathscr{M}_0^{++})^2+(Im\mathscr{M}_0^{--})^2+(Re\mathscr{M}_0^{--})^2+(Im\mathscr{M}_0^{+-})^2+(Re\mathscr{M}_0^{+-})^2+(Im\mathscr{M}_0^{-+})^2+(Re\mathscr{M}_0^{-+})^2+ \\
&-2F'_{3/2}AQ\frac{3cos\theta_K^*-1}{2}(sin\frac{\theta_K^*}{2})(Re\mathscr{M}_0^{+-}(M_r^2-W^2)+Im\mathscr{M}_0^{+-}\Gamma_rM_r)+ \\
&-2F'_{3/2}AQ\frac{3cos\theta_K^*-1}{2}(sin\frac{\theta_K^*}{2})(Re\mathscr{M}_0^{-+}(M_r^2-W^2)+Im\mathscr{M}_0^{-+}\Gamma_rM_r)\big]
\end{aligned} \tag{153}$$

employing χ factor from Eq. 132 and $H_{0,0}$ from Eq. 96 we have:

$$\begin{aligned}
\frac{d\sigma_L}{d\Omega_K^*} &= 2\chi\frac{1}{(4\pi)^2}(H_{0,0}) = \\
&= \frac{2}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&\big[2(F'_{3/2}AQ\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})^2((M_r^2-W^2)^2+(\Gamma_rM_r)^2)+2(F'_{3/2}AQ(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2}))^2((M_r^2-W^2)^2+(\Gamma_rM_r)^2)+ \\
&+\mid\mathscr{M}_0^{++}\mid^2+\mid\mathscr{M}_0^{--}\mid^2+\mid\mathscr{M}_0^{+-}\mid^2+\mid\mathscr{M}_0^{-+}\mid^2+ \\
&-2F'_{3/2}AQ\frac{3cos\theta_K^*-1}{2}(sin\frac{\theta_K^*}{2})(Re\mathscr{M}_0^{+-}(M_r^2-W^2)+Im\mathscr{M}_0^{+-}\Gamma_rM_r-Re\mathscr{M}_0^{-+}(M_r^2-W^2)-Im\mathscr{M}_0^{-+}\Gamma_rM_r)\big]
\end{aligned} \tag{154}$$

3.3.2 $\frac{d\sigma_T}{d\Omega}$

The terms in the grid satisfy the relationships

$$\mathscr{M}_1^{++}=\mathscr{M}_{-1}^{++}\;\; \mathscr{M}_0^{++}=-\mathscr{M}_0^{--}\;\; \mathscr{M}_1^{-+}=-\mathscr{M}_{-1}^{-+}\;\; \mathscr{M}_1^{+-}=-\mathscr{M}_{-1}^{+-}\;\; \mathscr{M}_1^{--}=\mathscr{M}_{-1}^{--} \tag{155}$$

Using χ factor from Eq. 132, $H_{1,1}$ from Eq. 101 and $H_{-1,-1}$ Eq. 106:

$$
\begin{aligned}
&\frac{d\sigma_T}{d\Omega_K^*} = \chi\frac{1}{(4\pi)^2}(H_{1,1}+H_{-1,-1}) = \\
&= \frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&[(F'_{3/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)[(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})^2+\frac{1}{Q^4}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})^2+(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})^2+ \\
&+\frac{1}{Q^4}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})^2]+\mid\mathscr{M}_1^{++}\mid^2+\mid\mathscr{M}_1^{--}\mid^2+\mid\mathscr{M}_1^{+-}\mid^2+\mid\mathscr{M}_1^{-+}\mid^2+ \\
&+2F'_{3/2}A(-\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_1^{++}+2F'_{3/2}A(-\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_1^{++}+ \\
&+2F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_1^{+-}+2F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_1^{+-}+ \\
&+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_1^{-+}+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})\Gamma_r M_r Im\mathscr{M}_1^{-+}+ \\
&+2F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_1^{--}+2F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})\Gamma_r M_r Im\mathscr{M}_1^{--}+ \\
&+(F'_{3/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)[\frac{1}{Q^4}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})^2+(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})^2+ \\
&+\frac{1}{Q^4}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})^2+(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})^2]+\mid\mathscr{M}_{-1}^{++}\mid^2+\mid\mathscr{M}_{-1}^{--}\mid^2+\mid\mathscr{M}_{-1}^{+-}\mid^2+ \\
&+\mid\mathscr{M}_{-1}^{-+}\mid^2+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{++}+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{++}+ \\
&+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{+-}+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{+-}+ \\
&+2F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+2F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+}+ \\
&+2F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{--}+2F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--}] = \\
&= \frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&[2(F'_{3/2}A)^2((M_r^2-W^2)^2+(\Gamma_r M_r)^2)[(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})^2+\frac{1}{Q^4}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})^2+(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})^2+ \\
&+\frac{1}{Q^4}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})^2]+2\mid\mathscr{M}_1^{++}\mid^2+2\mid\mathscr{M}_1^{--}\mid^2+2\mid\mathscr{M}_1^{+-}\mid^2+2\mid\mathscr{M}_1^{-+}\mid^2+ \\
&+2F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})[(M_r^2-W^2)Re\mathscr{M}_{-1}^{--}+(\Gamma_r M_r)Im\mathscr{M}_{-1}^{--}-(M_r^2-W^2)Re\mathscr{M}_1^{++}-(\Gamma_r M_r)Im\mathscr{M}_1^{++}]+ \\
&+2F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})[(M_r^2-W^2)Re\mathscr{M}_1^{+-}+(\Gamma_r M_r)Im\mathscr{M}_1^{+-}+(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+(\Gamma_r M_r)Im\mathscr{M}_{-1}^{-+}]+ \\
&+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})[(M_r^2-W^2)Re\mathscr{M}_1^{-+}+\Gamma_r M_r Im\mathscr{M}_1^{-+}+(M_r^2-W^2)Re\mathscr{M}_{-1}^{+-}+(\Gamma_r M_r)Im\mathscr{M}_{-1}^{+-}]+ \\
&+2F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})[(M_r^2-W^2)Re\mathscr{M}_{-1}^{++}+(\Gamma_r M_r)Im\mathscr{M}_{-1}^{++}-(M_r^2-W^2)Re\mathscr{M}_1^{--}-\Gamma_r M_r Im\mathscr{M}_1^{--}]]
\end{aligned}
\tag{156}
$$

3.3.3 $\frac{d\sigma_{LT}}{d\Omega}$

$$
\begin{aligned}
\frac{d\sigma_{LT}}{d\Omega_K^*} &= -\chi\frac{1}{(4\pi)^2}(H_{1,-1}+H_{-1,1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&\big[\big(ReM_1^{++}ReM_{-1}^{++} - iReM_1^{++}ImM_{-1}^{++} + ReM_1^{++}Re\mathscr{M}_{-1}^{++} - iReM_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+iImM_1^{++}ReM_{-1}^{++} + ImM_1^{++}ImM_{-1}^{++} + iImM_1^{++}Re\mathscr{M}_{-1}^{++} + ImM_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+Re\mathscr{M}_1^{++}ReM_{-1}^{++} - iRe\mathscr{M}_1^{++}ImM_{-1}^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+iIm\mathscr{M}_1^{++}ReM_{-1}^{++} + Im\mathscr{M}_1^{++}ImM_{-1}^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ReM_1^{--}ReM_{-1}^{--} - iReM_1^{--}ImM_{-1}^{--} + ReM_1^{--}Re\mathscr{M}_{-1}^{--} - iReM_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+iImM_1^{--}ReM_{-1}^{--} + ImM_1^{--}ImM_{-1}^{--} + iImM_1^{--}Re\mathscr{M}_{-1}^{--} + ImM_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+Re\mathscr{M}_1^{--}ReM_{-1}^{--} - iRe\mathscr{M}_1^{--}ImM_{-1}^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+iIm\mathscr{M}_1^{--}ReM_{-1}^{--} + Im\mathscr{M}_1^{--}ImM_{-1}^{--} + iIm\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ReM_1^{+-}ReM_{-1}^{+-} - iReM_1^{+-}ImM_{-1}^{+-} + ReM_1^{+-}Re\mathscr{M}_{-1}^{+-} - iReM_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+iImM_1^{+-}ReM_{-1}^{+-} + ImM_1^{+-}ImM_{-1}^{+-} + iImM_1^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+Re\mathscr{M}_1^{+-}ReM_{-1}^{+-} - iRe\mathscr{M}_1^{+-}ImM_{-1}^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+iIm\mathscr{M}_1^{+-}ReM_{-1}^{+-} + Im\mathscr{M}_1^{+-}ImM_{-1}^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ReM_1^{-+}ReM_{-1}^{-+} - iReM_1^{-+}ImM_{-1}^{-+} + ReM_1^{-+}Re\mathscr{M}_{-1}^{-+} - iReM_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+iImM_1^{-+}ReM_{-1}^{-+} + ImM_1^{-+}ImM_{-1}^{-+} + iImM_1^{-+}Re\mathscr{M}_{-1}^{-+} + ImM_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+Re\mathscr{M}_1^{-+}ReM_{-1}^{-+} - iRe\mathscr{M}_1^{-+}ImM_{-1}^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+iIm\mathscr{M}_1^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_1^{-+}ImM_{-1}^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+}\big) \\
&+\big(ReM_{-1}^{++}ReM_1^{++} - iReM_{-1}^{++}ImM_1^{++} + ReM_{-1}^{++}Re\mathscr{M}_1^{++} - iReM_{-1}^{++}Im\mathscr{M}_1^{++} + \\
&+iImM_{-1}^{++}ReM_1^{++} + ImM_{-1}^{++}ImM_1^{++} + iImM_{-1}^{++}Re\mathscr{M}_1^{++} + ImM_{-1}^{++}Im\mathscr{M}_1^{++} + \\
&+Re\mathscr{M}_{-1}^{++}ReM_1^{++} - iRe\mathscr{M}_{-1}^{++}ImM_1^{++} + Re\mathscr{M}_{-1}^{++}Re\mathscr{M}_1^{++} - iRe\mathscr{M}_{-1}^{++}Im\mathscr{M}_1^{++} + \\
&+iIm\mathscr{M}_{-1}^{++}ReM_1^{++} + Im\mathscr{M}_{-1}^{++}ImM_1^{++} + iIm\mathscr{M}_{-1}^{++}Re\mathscr{M}_1^{++} + Im\mathscr{M}_{-1}^{++}Im\mathscr{M}_1^{++} + \\
&+ReM_{-1}^{--}ReM_1^{--} - iReM_{-1}^{--}ImM_1^{--} + ReM_{-1}^{--}Re\mathscr{M}_1^{--} - iReM_{-1}^{--}Im\mathscr{M}_1^{--} + \\
&+iImM_{-1}^{--}ReM_1^{--} + ImM_{-1}^{--}ImM_1^{--} + iImM_{-1}^{--}Re\mathscr{M}_1^{--} + ImM_{-1}^{--}Im\mathscr{M}_1^{--} + \\
&+Re\mathscr{M}_{-1}^{--}ReM_1^{--} - iRe\mathscr{M}_{-1}^{--}ImM_1^{--} + Re\mathscr{M}_{-1}^{--}Re\mathscr{M}_1^{--} - iRe\mathscr{M}_{-1}^{--}Im\mathscr{M}_1^{--} + \\
&+iIm\mathscr{M}_{-1}^{--}ReM_1^{--} + Im\mathscr{M}_{-1}^{--}ImM_1^{--} + iIm\mathscr{M}_{-1}^{--}Re\mathscr{M}_1^{--} + Im\mathscr{M}_{-1}^{--}Im\mathscr{M}_1^{--} + \\
&+ReM_{-1}^{+-}ReM_1^{+-} - iReM_{-1}^{+-}ImM_1^{+-} + ReM_{-1}^{+-}Re\mathscr{M}_1^{+-} - iReM_{-1}^{+-}Im\mathscr{M}_1^{+-} + \\
&+iImM_{-1}^{+-}ReM_1^{+-} + ImM_{-1}^{+-}ImM_1^{+-} + iImM_{-1}^{+-}Re\mathscr{M}_1^{+-} + ImM_{-1}^{+-}Im\mathscr{M}_1^{+-} + \\
&+Re\mathscr{M}_{-1}^{+-}ReM_1^{+-} - iRe\mathscr{M}_{-1}^{+-}ImM_1^{+-} + Re\mathscr{M}_{-1}^{+-}Re\mathscr{M}_1^{+-} - iRe\mathscr{M}_{-1}^{+-}Im\mathscr{M}_1^{+-} + \\
&+iIm\mathscr{M}_{-1}^{+-}ReM_1^{+-} + Im\mathscr{M}_{-1}^{+-}ImM_1^{+-} + iIm\mathscr{M}_{-1}^{+-}Re\mathscr{M}_1^{+-} + Im\mathscr{M}_{-1}^{+-}Im\mathscr{M}_1^{+-} + \\
&+ReM_{-1}^{-+}ReM_1^{-+} - iReM_{-1}^{-+}ImM_1^{-+} + ReM_{-1}^{-+}Re\mathscr{M}_1^{-+} - iReM_{-1}^{-+}Im\mathscr{M}_1^{-+} + \\
&+iImM_{-1}^{-+}ReM_1^{-+} + ImM_{-1}^{-+}ImM_1^{-+} + iImM_{-1}^{-+}Re\mathscr{M}_1^{-+} + ImM_{-1}^{-+}Im\mathscr{M}_1^{-+} + \\
&+Re\mathscr{M}_{-1}^{-+}ReM_1^{-+} - iRe\mathscr{M}_{-1}^{-+}ImM_1^{-+} + Re\mathscr{M}_{-1}^{-+}Re\mathscr{M}_1^{-+} - iRe\mathscr{M}_{-1}^{-+}Im\mathscr{M}_1^{-+} + \\
&+iIm\mathscr{M}_{-1}^{-+}ReM_1^{-+} + Im\mathscr{M}_{-1}^{-+}ImM_1^{-+} + iIm\mathscr{M}_{-1}^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_{-1}^{-+}Im\mathscr{M}_1^{-+} \\
&\big)\big]
\end{aligned}
\tag{157}
$$

The terms in the grid satisfy the relationships

$$
\mathscr{M}_1^{++} = \mathscr{M}_{-1}^{++} \;\; \mathscr{M}_0^{++} = -\mathscr{M}_0^{--} \;\; \mathscr{M}_1^{-+} = -\mathscr{M}_{-1}^{-+} \;\; \mathscr{M}_1^{+-} = -\mathscr{M}_{-1}^{+-} \;\; \mathscr{M}_1^{--} = \mathscr{M}_{-1}^{--}
\tag{158}
$$

And from Tabs 8 and 9 we have that

$$M_1^{++} = -M_{-1}^{--} \; M_1^{+-} = M_{-1}^{-+} \; M_0^{+-} = -M_0^{-+} \; M_0^{++} = M_0^{--} \; M_{-1}^{++} = -M_1^{--} \; M_{-1}^{+-} = M_1^{-+} \tag{159}$$

And we can simplify considering that there are opposite terms:

$$\begin{aligned}
\frac{d\sigma_{LT}}{d\Omega_K^*} &= -\chi\frac{1}{(4\pi)^2}(H_{1,-1}+H_{-1,1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&\big(ReM_1^{++}ReM_{-1}^{++} + ReM_1^{++}Re\mathscr{M}_{-1}^{++} + ImM_1^{++}ImM_{-1}^{++} + ImM_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ Re\mathscr{M}_1^{++}ReM_{-1}^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_1^{++}ImM_{-1}^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ ReM_1^{--}ReM_{-1}^{--} + ReM_1^{--}Re\mathscr{M}_{-1}^{--} + ImM_1^{--}ImM_{-1}^{--} + ImM_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ Re\mathscr{M}_1^{--}ReM_{-1}^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_1^{--}ImM_{-1}^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ ReM_1^{+-}ReM_{-1}^{+-} + ReM_1^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_1^{+-}ImM_{-1}^{+-} + ImM_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ Re\mathscr{M}_1^{+-}ReM_{-1}^{+-} + Re\mathscr{M}_1^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_1^{+-}ImM_{-1}^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ ReM_1^{-+}ReM_{-1}^{-+} + ReM_1^{-+}Re\mathscr{M}_{-1}^{-+} + ImM_1^{-+}ImM_{-1}^{-+} + ImM_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ Re\mathscr{M}_1^{-+}ReM_{-1}^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_1^{-+}ImM_{-1}^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_{-1}^{-+}\big) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&\big(2ReM_1^{++}ReM_{-1}^{++} + ReM_1^{++}Re\mathscr{M}_{-1}^{++} + 2ImM_1^{++}ImM_{-1}^{++} + ImM_1^{++}Im\mathscr{M}_{-1}^{++} + \\
&+ Re\mathscr{M}_1^{++}ReM_{-1}^{++} + |\mathscr{M}_1^{++}|^2 + Im\mathscr{M}_1^{++}ImM_{-1}^{++} + ReM_1^{--}Re\mathscr{M}_{-1}^{--} + ImM_1^{--}Im\mathscr{M}_{-1}^{--} + \\
&+ Re\mathscr{M}_1^{--}ReM_{-1}^{--} + |\mathscr{M}_1^{--}|^2 + Im\mathscr{M}_1^{--}ImM_{-1}^{--} + \\
&+ 2ReM_1^{+-}ReM_{-1}^{+-} + ReM_1^{+-}Re\mathscr{M}_{-1}^{+-} + 2ImM_1^{+-}ImM_{-1}^{+-} + ImM_1^{+-}Im\mathscr{M}_{-1}^{+-} + \\
&+ Re\mathscr{M}_1^{+-}ReM_{-1}^{+-} + |\mathscr{M}_1^{+-}|^2 + Im\mathscr{M}_1^{+-}ImM_{-1}^{+-} + ReM_1^{-+}Re\mathscr{M}_{-1}^{-+} + ImM_1^{-+}Im\mathscr{M}_{-1}^{-+} + \\
&+ Re\mathscr{M}_1^{-+}ReM_{-1}^{-+} + |\mathscr{M}_1^{-+}|^2 + Im\mathscr{M}_1^{-+}ImM_{-1}^{-+}\big)
\end{aligned} \tag{160}$$

Finally we substitute values from Tabs 8 and 9 and we obtain:

$$
\begin{aligned}
\frac{d\sigma_{LT}}{d\Omega_K^*} &= -\chi \frac{1}{(4\pi)^2}(H_{1,-1}+H_{-1,1}) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&\Big(2(F'_{3/2})^2(-\frac{3cos\theta_K^*+1}{2}sin^2\frac{\theta_K^*}{2})\frac{A^2}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2})(M_r^2-W^2)^2+F'_{3/2}A(-\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{++}+ \\
&+2(F'_{3/2})^2(-\frac{3cos\theta_K^*+1}{2}sin^2\frac{\theta_K^*}{2})\frac{A^2}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2})(\Gamma_rM_r)^2+F'_{3/2}A(-\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})\Gamma_rM_rIm\mathscr{M}_{-1}^{++}+ \\
&+Re\mathscr{M}_1^{++}F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)+|\mathscr{M}_1^{++}|^2+Im\mathscr{M}_1^{++}F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})\Gamma_rM_r+ \\
&+F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{--}+F'_{3/2}\frac{A}{Q^2}(-\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})\Gamma_rM_rIm\mathscr{M}_{-1}^{--}+ \\
&+Re\mathscr{M}_1^{--}F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)+|\mathscr{M}_1^{--}|^2+Im\mathscr{M}_1^{--}F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})\Gamma_rM_r+ \\
&+2F'^2_{3/2}(\frac{3cos\theta_K^*-1}{2}cos^2\frac{\theta_K^*}{2})\frac{A^2}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2})(M_r^2-W^2)^2+F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{+-}+ \\
&+2F'^2_{3/2}(\frac{3cos\theta_K^*-1}{2}cos^2\frac{\theta_K^*}{2})\frac{A^2}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2})(\Gamma_rM_r)^2+F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})\Gamma_rM_rIm\mathscr{M}_{-1}^{+-}+ \\
&+Re\mathscr{M}_1^{+-}F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)+|\mathscr{M}_1^{+-}|^2+Im\mathscr{M}_1^{+-}F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})\Gamma_rM_r+ \\
&F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})\Gamma_rM_rIm\mathscr{M}_{-1}^{-+}+ \\
&+Re\mathscr{M}_1^{-+}F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(M_r^2-W^2)+|\mathscr{M}_1^{-+}|^2+Im\mathscr{M}_1^{-+}F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})\Gamma_rM_r\Big) = \\
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot \\
&\Big(2(F'_{3/2})^2(-\frac{3cos\theta_K^*+1}{2}sin^2\frac{\theta_K^*}{2})\frac{A^2}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2})[(M_r^2-W^2)^2+(\Gamma_rM_r)^2]+|\mathscr{M}_1^{--}|^2+|\mathscr{M}_1^{-+}|^2+ \\
&+F'_{3/2}A(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})[Re\mathscr{M}_1^{--}(M_r^2-W^2)+Im\mathscr{M}_1^{--}\Gamma_rM_r-(M_r^2-W^2)Re\mathscr{M}_{-1}^{++}-\Gamma_rM_rIm\mathscr{M}_{-1}^{++}]+ \\
&+F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})[Re\mathscr{M}_1^{++}(M_r^2-W^2)+Im\mathscr{M}_1^{++}\Gamma_rM_r-(M_r^2-W^2)Re\mathscr{M}_{-1}^{--}-\Gamma_rM_rIm\mathscr{M}_{-1}^{--}]+ \\
&+2F'^2_{3/2}(\frac{3cos\theta_K^*-1}{2}cos^2\frac{\theta_K^*}{2})\frac{A^2}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2})[(M_r^2-W^2)^2+(\Gamma_rM_r)^2]+|\mathscr{M}_1^{++}|^2+|\mathscr{M}_1^{+-}|^2+ \\
&+F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})[(M_r^2-W^2)Re\mathscr{M}_{-1}^{+-}+\Gamma_rM_rIm\mathscr{M}_{-1}^{+-}+Re\mathscr{M}_1^{-+}(M_r^2-W^2)+Im\mathscr{M}_1^{-+}\Gamma_rM_r]+ \\
&+F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})[Re\mathscr{M}_1^{+-}(M_r^2-W^2)+Im\mathscr{M}_1^{+-}\Gamma_rM_r+(M_r^2-W^2)Re\mathscr{M}_{-1}^{-+}+\Gamma_rM_rIm\mathscr{M}_{-1}^{-+}]\Big)
\end{aligned}
\tag{161}
$$

3.3.4 $\frac{d\sigma_{TT}}{d\Omega}$

$$
\frac{d\sigma_{TT}}{d\Omega^*_K} = -\chi\frac{1}{(4\pi)^2}(H_{0,1}+H_{1,0}-H_{-1,0}-H_{0,-1}) =
$$

$$
\begin{aligned}
&= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot\\
&\Big[\Big(ReM_0^{++}ReM_1^{++} - iReM_0^{++}ImM_1^{++} + ReM_0^{++}Re\mathscr{M}_1^{++} - iReM_0^{++}Im\mathscr{M}_1^{++} + iImM_0^{++}ReM_1^{++} + ImM_0^{++}ImM_1^{++}+\\
&+iImM_0^{++}Re\mathscr{M}_1^{++} + ImM_0^{++}Im\mathscr{M}_1^{++} + Re\mathscr{M}_0^{++}ReM_1^{++} - iRe\mathscr{M}_0^{++}ImM_1^{++} + Re\mathscr{M}_0^{++}Re\mathscr{M}_1^{++} - iRe\mathscr{M}_0^{++}Im\mathscr{M}_1^{++}+\\
&+iIm\mathscr{M}_0^{++}ReM_1^{++} + Im\mathscr{M}_0^{++}ImM_1^{++} + iIm\mathscr{M}_0^{++}Re\mathscr{M}_1^{++} + Im\mathscr{M}_0^{++}Im\mathscr{M}_1^{++} + ReM_0^{--}ReM_1^{--} - iReM_0^{--}ImM_1^{--}\\
&+ReM_0^{--}Re\mathscr{M}_1^{--} - iReM_0^{--}Im\mathscr{M}_1^{--} + iImM_0^{--}ReM_1^{--} + ImM_0^{--}ImM_1^{--} + iImM_0^{--}Re\mathscr{M}_1^{--} + ImM_0^{--}Im\mathscr{M}_1^{--}+\\
&+Re\mathscr{M}_0^{--}ReM_1^{--} - iRe\mathscr{M}_0^{--}ImM_1^{--} + Re\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} - iRe\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + iIm\mathscr{M}_0^{--}ReM_1^{--} + Im\mathscr{M}_0^{--}ImM_1^{--}+\\
&+iIm\mathscr{M}_0^{--}Re\mathscr{M}_1^{--} + Im\mathscr{M}_0^{--}Im\mathscr{M}_1^{--} + ReM_0^{+-}ReM_1^{+-} - iReM_0^{+-}ImM_1^{+-} + ReM_0^{+-}Re\mathscr{M}_1^{+-} - iReM_0^{+-}Im\mathscr{M}_1^{+-}+\\
&+iImM_0^{+-}ReM_1^{+-} + ImM_0^{+-}ImM_1^{+-} + iImM_0^{+-}Re\mathscr{M}_1^{+-} + ImM_0^{+-}Im\mathscr{M}_1^{+-} + Re\mathscr{M}_0^{+-}ReM_1^{+-} - iRe\mathscr{M}_0^{+-}ImM_1^{+-}+\\
&+Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} - iRe\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-} + iIm\mathscr{M}_0^{+-}ReM_1^{+-} + Im\mathscr{M}_0^{+-}ImM_1^{+-} + iIm\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-}+\\
&+ReM_0^{-+}ReM_1^{-+} - iReM_0^{-+}ImM_1^{-+} + ReM_0^{-+}Re\mathscr{M}_1^{-+} - iReM_0^{-+}Im\mathscr{M}_1^{-+} + iImM_0^{-+}ReM_1^{-+} + ImM_0^{-+}ImM_1^{-+}+\\
&+iImM_0^{-+}Re\mathscr{M}_1^{-+} + ImM_0^{-+}Im\mathscr{M}_1^{-+} + Re\mathscr{M}_0^{-+}ReM_1^{-+} - iRe\mathscr{M}_0^{-+}ImM_1^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} - iRe\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+}+\\
&+iIm\mathscr{M}_0^{-+}ReM_1^{-+} + Im\mathscr{M}_0^{-+}ImM_1^{-+} + iIm\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+}\Big)+\\
&\Big(ReM_1^{++}ReM_0^{++} - iReM_1^{++}ImM_0^{++} + ReM_1^{++}Re\mathscr{M}_0^{++} - iReM_1^{++}Im\mathscr{M}_0^{++} + iImM_1^{++}ReM_0^{++} + ImM_1^{++}ImM_0^{++}+\\
&+iImM_1^{++}Re\mathscr{M}_0^{++} + ImM_1^{++}Im\mathscr{M}_0^{++} + Re\mathscr{M}_1^{++}ReM_0^{++} - iRe\mathscr{M}_1^{++}ImM_0^{++} + Re\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} - iRe\mathscr{M}_1^{++}Im\mathscr{M}_0^{++}+\\
&+iIm\mathscr{M}_1^{++}ReM_0^{++} + Im\mathscr{M}_1^{++}ImM_0^{++} + iIm\mathscr{M}_1^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_1^{++}Im\mathscr{M}_0^{++} + ReM_1^{--}ReM_0^{--} - iReM_1^{--}ImM_0^{--}+\\
&+ReM_1^{--}Re\mathscr{M}_0^{--} - iReM_1^{--}Im\mathscr{M}_0^{--} + iImM_1^{--}ReM_0^{--} + ImM_1^{--}ImM_0^{--} + iImM_1^{--}Re\mathscr{M}_0^{--} + ImM_1^{--}Im\mathscr{M}_0^{--}+\\
&+Re\mathscr{M}_1^{--}ReM_0^{--} - iRe\mathscr{M}_1^{--}ImM_0^{--} + Re\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} - iRe\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + iIm\mathscr{M}_1^{--}ReM_0^{--} + Im\mathscr{M}_1^{--}ImM_0^{--}+\\
&+iIm\mathscr{M}_1^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_1^{--}Im\mathscr{M}_0^{--} + ReM_1^{+-}ReM_0^{+-} - iReM_1^{+-}ImM_0^{+-} + ReM_1^{+-}Re\mathscr{M}_0^{+-} - iReM_1^{+-}Im\mathscr{M}_0^{+-}+\\
&+iImM_1^{+-}ReM_0^{+-} + ImM_1^{+-}ImM_0^{+-} + iImM_1^{+-}Re\mathscr{M}_0^{+-} + ImM_1^{+-}Im\mathscr{M}_0^{+-} + Re\mathscr{M}_1^{+-}ReM_0^{+-} - iRe\mathscr{M}_1^{+-}ImM_0^{+-}+\\
&+Re\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-} + iIm\mathscr{M}_1^{+-}ReM_0^{+-} + Im\mathscr{M}_1^{+-}ImM_0^{+-} + iIm\mathscr{M}_1^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_1^{+-}Im\mathscr{M}_0^{+-}+\\
&+ReM_1^{-+}ReM_0^{-+} - iReM_1^{-+}ImM_0^{-+} + ReM_1^{-+}Re\mathscr{M}_0^{-+} - iReM_1^{-+}Im\mathscr{M}_0^{-+} + iImM_1^{-+}ReM_0^{-+} + ImM_1^{-+}ImM_0^{-+}+\\
&+iImM_1^{-+}Re\mathscr{M}_0^{-+} + ImM_1^{-+}Im\mathscr{M}_0^{-+} + Re\mathscr{M}_1^{-+}ReM_0^{-+} - iRe\mathscr{M}_1^{-+}ImM_0^{-+} + Re\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+}+\\
&+iIm\mathscr{M}_1^{-+}ReM_0^{-+} + Im\mathscr{M}_1^{-+}ImM_0^{-+} + iIm\mathscr{M}_1^{-+}Re\mathscr{M}_0^{-+} + Im\mathscr{M}_1^{-+}Im\mathscr{M}_0^{-+}\Big)\\
&-\Big(ReM_{-1}^{++}ReM_0^{++} - iReM_{-1}^{++}ImM_0^{++} + ReM_{-1}^{++}Re\mathscr{M}_0^{++} - iReM_{-1}^{++}Im\mathscr{M}_0^{++} + iImM_{-1}^{++}ReM_0^{++} + ImM_{-1}^{++}ImM_0^{++}+\\
&+iImM_{-1}^{++}Re\mathscr{M}_0^{++} + ImM_{-1}^{++}Im\mathscr{M}_0^{++} + Re\mathscr{M}_{-1}^{++}ReM_0^{++} - iRe\mathscr{M}_{-1}^{++}ImM_0^{++} + Re\mathscr{M}_{-1}^{++}Re\mathscr{M}_0^{++} - iRe\mathscr{M}_{-1}^{++}Im\mathscr{M}_0^{++}+\\
&+iIm\mathscr{M}_{-1}^{++}ReM_0^{++} + Im\mathscr{M}_{-1}^{++}ImM_0^{++} + iIm\mathscr{M}_{-1}^{++}Re\mathscr{M}_0^{++} + Im\mathscr{M}_{-1}^{++}Im\mathscr{M}_0^{++} + ReM_{-1}^{--}ReM_0^{--} - iReM_{-1}^{--}ImM_0^{--}+\\
&+ReM_{-1}^{--}Re\mathscr{M}_0^{--} - iReM_{-1}^{--}Im\mathscr{M}_0^{--} + iImM_{-1}^{--}ReM_0^{--} + ImM_{-1}^{--}ImM_0^{--} + iImM_{-1}^{--}Re\mathscr{M}_0^{--} + ImM_{-1}^{--}Im\mathscr{M}_0^{--}+\\
&+Re\mathscr{M}_{-1}^{--}ReM_0^{--} - iRe\mathscr{M}_{-1}^{--}ImM_0^{--} + Re\mathscr{M}_{-1}^{--}Re\mathscr{M}_0^{--} - iRe\mathscr{M}_{-1}^{--}Im\mathscr{M}_0^{--} + iIm\mathscr{M}_{-1}^{--}ReM_0^{--} + Im\mathscr{M}_{-1}^{--}ImM_0^{--}+\\
&+iIm\mathscr{M}_{-1}^{--}Re\mathscr{M}_0^{--} + Im\mathscr{M}_{-1}^{--}Im\mathscr{M}_0^{--} + ReM_{-1}^{+-}ReM_0^{+-} - iReM_{-1}^{+-}ImM_0^{+-} + ReM_{-1}^{+-}Re\mathscr{M}_0^{+-} - iReM_{-1}^{+-}Im\mathscr{M}_0^{+-}+\\
&+iImM_{-1}^{+-}ReM_0^{+-} + ImM_{-1}^{+-}ImM_0^{+-} + iImM_{-1}^{+-}Re\mathscr{M}_0^{+-} + ImM_{-1}^{+-}Im\mathscr{M}_0^{+-} + Re\mathscr{M}_{-1}^{+-}ReM_0^{+-} - iRe\mathscr{M}_{-1}^{+-}ImM_0^{+-}+\\
&+Re\mathscr{M}_{-1}^{+-}Re\mathscr{M}_0^{+-} - iRe\mathscr{M}_{-1}^{+-}Im\mathscr{M}_0^{+-} + iIm\mathscr{M}_{-1}^{+-}ReM_0^{+-} + Im\mathscr{M}_{-1}^{+-}ImM_0^{+-} + iIm\mathscr{M}_{-1}^{+-}Re\mathscr{M}_0^{+-} + Im\mathscr{M}_{-1}^{+-}Im\mathscr{M}_0^{+-}+\\
&+ReM_{-1}^{-+}ReM_0^{-+} - iReM_{-1}^{-+}ImM_0^{-+} + ReM_{-1}^{-+}Re\mathscr{M}_0^{-+} - iReM_{-1}^{-+}Im\mathscr{M}_0^{-+} + iImM_{-1}^{-+}ReM_0^{-+} + ImM_{-1}^{-+}ImM_0^{-+}+\\
&+iImM_{-1}^{-+}Re\mathscr{M}_0^{-+} + ImM_{-1}^{-+}Im\mathscr{M}_0^{-+} + Re\mathscr{M}_{-1}^{-+}ReM_0^{-+} - iRe\mathscr{M}_{-1}^{-+}ImM_0^{-+} + Re\mathscr{M}_{-1}^{-+}Re\mathscr{M}_0^{-+} - iRe\mathscr{M}_{-1}^{-+}Im\mathscr{M}_0^{-+}+\\
&+iIm\mathscr{M}_{-1}^{-+}ReM_0^{-+} + Im\mathscr{M}_{-1}^{-+}ImM_0^{-+} + iIm\mathscr{M}_{-1}^{-+}Re\mathscr{M}_0^{-+} + Im\mathscr{M}_{-1}^{-+}Im\mathscr{M}_0^{-+}\Big)\\
&-\Big(ReM_0^{++}ReM_{-1}^{++} - iReM_0^{++}ImM_{-1}^{++} + ReM_0^{++}Re\mathscr{M}_{-1}^{++} - iReM_0^{++}Im\mathscr{M}_{-1}^{++} + iImM_0^{++}ReM_{-1}^{++} + ImM_0^{++}ImM_{-1}^{++}+\\
&+iImM_0^{++}Re\mathscr{M}_{-1}^{++} + ImM_0^{++}Im\mathscr{M}_{-1}^{++} + Re\mathscr{M}_0^{++}ReM_{-1}^{++} - iRe\mathscr{M}_0^{++}ImM_{-1}^{++} + Re\mathscr{M}_0^{++}Re\mathscr{M}_{-1}^{++} - iRe\mathscr{M}_0^{++}Im\mathscr{M}_{-1}^{++}+\\
&+iIm\mathscr{M}_0^{++}ReM_{-1}^{++} + Im\mathscr{M}_0^{++}ImM_{-1}^{++} + iIm\mathscr{M}_0^{++}Re\mathscr{M}_{-1}^{++} + Im\mathscr{M}_0^{++}Im\mathscr{M}_{-1}^{++} + ReM_0^{--}ReM_{-1}^{--} - iReM_0^{--}ImM_{-1}^{--}+\\
&+ReM_0^{--}Re\mathscr{M}_{-1}^{--} - iReM_0^{--}Im\mathscr{M}_{-1}^{--} + iImM_0^{--}ReM_{-1}^{--} + ImM_0^{--}ImM_{-1}^{--} + iImM_0^{--}Re\mathscr{M}_{-1}^{--} + ImM_0^{--}Im\mathscr{M}_{-1}^{--}+\\
&+Re\mathscr{M}_0^{--}ReM_{-1}^{--} - iRe\mathscr{M}_0^{--}ImM_{-1}^{--} + Re\mathscr{M}_0^{--}Re\mathscr{M}_{-1}^{--} - iRe\mathscr{M}_0^{--}Im\mathscr{M}_{-1}^{--} + iIm\mathscr{M}_0^{--}ReM_{-1}^{--} + Im\mathscr{M}_0^{--}ImM_{-1}^{--}+\\
&+iIm\mathscr{M}_0^{--}Re\mathscr{M}_{-1}^{--} + Im\mathscr{M}_0^{--}Im\mathscr{M}_{-1}^{--} + ReM_0^{+-}ReM_{-1}^{+-} - iReM_0^{+-}ImM_{-1}^{+-} + ReM_0^{+-}Re\mathscr{M}_{-1}^{+-} - iReM_0^{+-}Im\mathscr{M}_{-1}^{+-}+\\
&+iImM_0^{+-}ReM_{-1}^{+-} + ImM_0^{+-}ImM_{-1}^{+-} + iImM_0^{+-}Re\mathscr{M}_{-1}^{+-} + ImM_0^{+-}Im\mathscr{M}_{-1}^{+-} + Re\mathscr{M}_0^{+-}ReM_{-1}^{+-} - iRe\mathscr{M}_0^{+-}ImM_{-1}^{+-}+\\
&+Re\mathscr{M}_0^{+-}Re\mathscr{M}_{-1}^{+-} - iRe\mathscr{M}_0^{+-}Im\mathscr{M}_{-1}^{+-} + iIm\mathscr{M}_0^{+-}ReM_{-1}^{+-} + Im\mathscr{M}_0^{+-}ImM_{-1}^{+-} + iIm\mathscr{M}_0^{+-}Re\mathscr{M}_{-1}^{+-} + Im\mathscr{M}_0^{+-}Im\mathscr{M}_{-1}^{+-}+\\
&+ReM_0^{-+}ReM_{-1}^{-+} - iReM_0^{-+}ImM_{-1}^{-+} + ReM_0^{-+}Re\mathscr{M}_{-1}^{-+} - iReM_0^{-+}Im\mathscr{M}_{-1}^{-+} + iImM_0^{-+}ReM_{-1}^{-+} + ImM_0^{-+}ImM_{-1}^{-+}+\\
&+iImM_0^{-+}Re\mathscr{M}_{-1}^{-+} + ImM_0^{-+}Im\mathscr{M}_{-1}^{-+} + Re\mathscr{M}_0^{-+}ReM_{-1}^{-+} - iRe\mathscr{M}_0^{-+}ImM_{-1}^{-+} + Re\mathscr{M}_0^{-+}Re\mathscr{M}_{-1}^{-+} - iRe\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+}+\\
&+iIm\mathscr{M}_0^{-+}ReM_{-1}^{-+} + Im\mathscr{M}_0^{-+}ImM_{-1}^{-+} + iIm\mathscr{M}_0^{-+}Re\mathscr{M}_{-1}^{-+} + Im\mathscr{M}_0^{-+}Im\mathscr{M}_{-1}^{-+}\Big)\Big]
\end{aligned}
\tag{162}
$$

The terms in the grid satisfy the relationships

$$\mathscr{M}_1^{++}=\mathscr{M}_{-1}^{++}\ \mathscr{M}_0^{++}=-\mathscr{M}_0^{--}\ \mathscr{M}_1^{-+}=-\mathscr{M}_{-1}^{-+}\ \mathscr{M}_1^{+-}=-\mathscr{M}_{-1}^{+-}\ \mathscr{M}_1^{--}=\mathscr{M}_{-1}^{--} \tag{163}$$

And from Tabs 8 and 9 we have that

$$M_1^{++}=-M_{-1}^{--}\ M_1^{+-}=M_{-1}^{-+}\ M_0^{+-}=-M_0^{-+}\ M_0^{++}=M_0^{--}\ M_{-1}^{++}=-M_1^{--}\ M_{-1}^{+-}=M_1^{-+} \tag{164}$$

And we can simplify considering that there are opposite terms:

$$\begin{aligned}
&\frac{d\sigma_{TT}}{d\Omega_K^*}=-\chi\frac{1}{(4\pi)^2}(H_{0,1}+H_{1,0}-H_{-1,0}-H_{0,-1})=\\
&=-\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot\\
&[4\big(ReM_0^{++}ReM_1^{++}+ImM_0^{++}ImM_1^{++}+ReM_0^{--}ReM_1^{--}+ImM_0^{--}ImM_1^{--}+\\
&+ReM_0^{+-}ReM_1^{+-}+ImM_0^{+-}ImM_1^{+-}+ReM_0^{+-}Re\mathscr{M}_1^{+-}+ImM_0^{+-}Im\mathscr{M}_1^{+-}+\\
&+Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-}+Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-}+ReM_0^{-+}ReM_1^{-+}+ImM_0^{-+}ImM_1^{-+}+\\
&+ReM_0^{-+}Re\mathscr{M}_1^{-+}+ImM_0^{-+}Im\mathscr{M}_1^{-+}+Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+}+Im\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+}\big)+\\
&+2\big(Re\mathscr{M}_0^{+-}ReM_1^{+-}+Im\mathscr{M}_0^{+-}ImM_1^{+-}+Re\mathscr{M}_0^{-+}ReM_1^{-+}+Im\mathscr{M}_0^{-+}ImM_1^{-+}+\\
&+ReM_{-1}^{+-}Re\mathscr{M}_0^{+-}+ImM_{-1}^{+-}Im\mathscr{M}_0^{+-}+ReM_{-1}^{-+}Re\mathscr{M}_0^{-+}+ImM_{-1}^{-+}Im\mathscr{M}_0^{-+}\big)]
\end{aligned} \tag{165}$$

We can substitute values from Tabs 8 and 9 obtaining:

$$\begin{aligned}
&\frac{d\sigma_{TT}}{d\Omega_K^*}=-\chi\frac{1}{(4\pi)^2}(H_{0,1}+H_{1,0}-H_{-1,0}-H_{0,-1})=\\
&=-\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot\\
&[4\big(F_{3/2}^{'2}A^2Q(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})(-\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})((M_r^2-W^2)^2+(\Gamma_rM_r)^2)+\\
&+F_{3/2}^{'2}(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})\frac{A^2}{Q}(-\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})((M_r^2-W^2)^2+(\Gamma_rM_r)^2)+\\
&+F_{3/2}^{'2}A^2Q(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})((M_r^2-W^2)^2+(\Gamma_rM_r)^2)+\\
&+F'_{3/2}AQ(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_1^{+-}+F'_{3/2}AQ(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})\Gamma_rM_rIm\mathscr{M}_1^{+-}+Im\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+}+\\
&+Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-}+Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-}+F_{3/2}^{'2}(-\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})\frac{A^2}{Q}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})((M_r^2-W^2)^2+(\Gamma_rM_r)^2)+\\
&+F'_{3/2}AQ(-\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})(M_r^2-W^2)Re\mathscr{M}_1^{-+}+F'_{3/2}AQ(-\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})\Gamma_rM_rIm\mathscr{M}_1^{-+}+Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+}\big)+\\
&+2\big(F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})((M_r^2-W^2)Re\mathscr{M}_0^{+-}+Im\mathscr{M}_0^{+-}\Gamma_rM_r+Re\mathscr{M}_0^{-+}(M_r^2-W^2)+Im\mathscr{M}_0^{-+}\Gamma_rM_r)+\\
&+F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})((M_r^2-W^2)Re\mathscr{M}_0^{+-}+\Gamma_rM_rIm\mathscr{M}_0^{+-}+(M_r^2-W^2)Re\mathscr{M}_0^{-+}+\Gamma_rM_rIm\mathscr{M}_0^{-+}))]
\end{aligned} \tag{166}$$

And the final result

$$\frac{d\sigma_{TT}}{d\Omega_K^*} = -\chi\frac{1}{(4\pi)^2}(H_{0,1}+H_{1,0}-H_{-1,0}-H_{0,-1}) =$$

$$= -\frac{1}{(4\pi)^2}\frac{1}{16Wm_P}\frac{\sqrt{W^4+(m_K^2-m_\Lambda^2)^2-2W^2(m_K^2+m_\Lambda^2)}}{2W}\frac{2m_P}{W^2-m_P^2}\cdot$$

$$[4\big(F_{3/2}^{\prime 2}(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})\frac{A^2}{Q}(-\sqrt{3}\frac{1+cos\theta_K^*}{2}sin\frac{\theta_K^*}{2})((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+$$

$$+F'_{3/2}AQ(\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})[(M_r^2-W^2)Re\mathscr{M}_1^{+-}+\Gamma_r M_r Im\mathscr{M}_1^{+-}-(M_r^2-W^2)Re\mathscr{M}_1^{-+}-\Gamma_r M_r Im\mathscr{M}_1^{-+}]+$$

$$+F_{3/2}^{\prime 2}(-\frac{3cos\theta_K^*+1}{2}sin\frac{\theta_K^*}{2})\frac{A^2}{Q}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})((M_r^2-W^2)^2+(\Gamma_r M_r)^2)+$$

$$+Re\mathscr{M}_0^{+-}Re\mathscr{M}_1^{+-}+Im\mathscr{M}_0^{+-}Im\mathscr{M}_1^{+-}+Im\mathscr{M}_0^{-+}Im\mathscr{M}_1^{-+}+Re\mathscr{M}_0^{-+}Re\mathscr{M}_1^{-+}\big)+$$

$$+2\big(F'_{3/2}\frac{A}{Q^2}(\sqrt{3}\frac{1-cos\theta_K^*}{2}cos\frac{\theta_K^*}{2})((M_r^2-W^2)Re\mathscr{M}_0^{+-}+Im\mathscr{M}_0^{+-}\Gamma_r M_r+Re\mathscr{M}_0^{-+}(M_r^2-W^2)+Im\mathscr{M}_0^{-+}\Gamma_r M_r)+$$

$$+F'_{3/2}A(\frac{3cos\theta_K^*-1}{2}cos\frac{\theta_K^*}{2})((M_r^2-W^2)Re\mathscr{M}_0^{+-}+\Gamma_r M_r Im\mathscr{M}_0^{+-}+(M_r^2-W^2)Re\mathscr{M}_0^{-+}+\Gamma_r M_r Im\mathscr{M}_0^{-+})\big)] \quad (167)$$

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