GLUONIC NUCLEONS IN A VALON BAG MODEL

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Two isomers of the nucleon, consisting of three 1S quarks and a transverse electric gluon, are predicted by the bag model to exist at masses around 1600 MeV. Mixing of these states with the nucleons seems unacceptably large. With this motivation we reformulate the problem in terms of valons – dressed quark and gluon single-particle eigenstates of (part of) the QCD bag hamiltonian. Now composed of valons, the nucleon and gluonic nucleons are decoupled, thereby retaining previous desciptions of the nucleon. One isomer should be visible in πN elastic scattering and might be the $N^*(1710)$.

1. Introduction. The MIT bag model predicts massless quark S states with energies $\omega(1S) = 2.04/R$, $\omega(2S) = 5.40/R$ and gluon states $\omega(\mathrm{TE10}) = 2.75/R$ [1] in terms of the bag radius R. The purpose of this work is to understand the properties of gluonic nucleon [2] excitations. We concentrate on two spin 1/2, isospin 1/2 states N^1_{qg} and N^2_{qg} made up of three 1S quarks and a (TE10) gluon in distinct symmetry states.

A straightforward application of bag model techniques [3,4], however, leads to an apparently untenable situation. The gluonic nucleons are predicted to mix very strongly via the (quark) to (quark plus gluon) interaction, with the ordinary nucleon [3]. The ground state nucleon becomes roughly equal amplitudes of $(1S)^3$, N^1_{qg}, and N^2_{qg}. All the preexisting success of the bag model in describing the static properties of the nucleon is threatened.

We eliminate this coupling between nucleon and gluonic nucleon by using the idea of valons introduced by Hwa [5,6]. Valons are dressed quark or gluon single-particle eigenstates of (part of) the QCD hamiltonian in the bag. Denote the bare quark and gluon by q,g; and the dressed quark-like and gluon-like valons by Q, G. Then, generically speaking, the Yukawa part of the QCD hamiltonian

$$H_0 = \omega_q q^+ q + \omega_g g^+ g + \kappa q^+ (g^+ + g) q \tag{1}$$

is diagonalized by the transformation to the valon dressed single-particle states to

$$H_D = \omega_Q Q^+ Q + \omega_G G^+ G - (\kappa^2/\omega_q) : Q^+ Q Q^+ Q : + \ldots, \tag{2}$$

with $Q = q + (\kappa/\omega_g) gq$ and $G = g + (\kappa/\omega_g) q^+ q$ to first order in the coupling parameter κ. Bare quarks emit bare gluons but dressed Q valons do not emit dressed G valons. The Yukawa type interaction $\kappa q^+(g^+ + g)q$ which leads to self interactions, has been eliminated by expressing the hamiltonian in terms of dressed operators which include the self interactions.

Now view the nucleon as three 1S Q's. The valon energy $\omega_Q \simeq \omega_q - \kappa^2/\omega_g$ so to recover the baryon spectrum predicted by the original three-quark bag model, it is necessary to adjust the bag model parameters Z and B. Note that the spin dependent gluon exchange term, which splits the nucleon and delta as a second order perturbation on the energy levels of the 3q baryons, is replaced by a first order four-Q interaction $-(\kappa^2/\omega_g) : Q^+ Q Q^+ Q :$ which produces the same splitting on the energy levels of 3Q baryons. Thus the spectroscopic successes of the quark bag model are retained in the valon bag model.

The old scalar model of a scalar meson interacting with a static spinless nucleon is an explicitly solvable example of the structure visualized here [7–9]. To paraphrase that simple case, the valon state $|Q\rangle$ is built from the quark state $|q\rangle$ by a unitary transformation U

$$U = 1 - (\kappa/\omega_g) q^+ (g^+ - g) q\,, \tag{3}$$

which generates $|q\rangle$, $|qg\rangle$, $|qgg\rangle$... states coherently so that $H_1|Q\rangle = \omega_Q|Q\rangle$ where $|Q\rangle \to |q\rangle$ as $\kappa \to 0$. The same unitary transformation generates the state $|G\rangle$ from $|g\rangle$. The scalar model is solvable because it is just a translated simple harmonic oscillator. In QCD, even with the neglect of g^3 and g^4 terms in the hamiltonian, and the inclusion of only the lowest (1S) quark and (TE10) gluon bag modes, the spin–color structure precludes an exact solution. Nonetheless, we will reformulate the QCD bag hamiltonian in terms of the valons Q,G. Then we can discuss the baryon spectrum in terms of the decoupled states $|Q^3\rangle$ and $|Q^3G\rangle$. Except for explicitly containing some valence glue, the 3Q baryons will have the same properties – up to a point – as do the 3q baryons. The essential difference is that the theory contains no QQG vertices so gluonic corrections must be treated differently.

2. *Valon hamiltonian.* The quark–gluon bag hamiltonian including only 1S quark modes and (TE10) gluon modes and ignoring three- and four-gluon terms is [10]

$$H_o = H_q + H_g + H_{qqg}\,, \tag{4}$$

where

$$H_q = \omega_q \sum_m q_m^+ q_m\,, \quad H_g = \omega_g \sum_\alpha g_\alpha^+ g_\alpha$$

and

$$H_{qqg} = \kappa \sum_{mm'\alpha} \langle m|\Theta_\alpha|m'\rangle q_m^+ q_{m'} (g_\alpha^+ + g_\alpha)\,, \tag{5}$$

with $\omega_q = 2.04/R$, $\omega_g = 2.74/R$, $\kappa = -0.545/R$ for QCD coupling strength $g^2/4\pi = 1.2$. The quark annihilation operator q_m is labeled by a color, flavor, and spin index m; the gluon operator g_α is labeled by a color and spin index α. The operator $\Theta = \lambda\sigma/2$ is the product of the Gell-Mann color matrix $\lambda/2$ and the Pauli spin matrix σ.

The unitary transformation $U = e^{iS}$ with

$$S(q,g) = i\frac{\kappa}{\omega_g} \sum_{mm'\alpha} q_{m'}^+ q_m (g_\alpha^+ - g_\alpha)\langle m'|\Theta_\alpha|m\rangle \tag{6}$$

generates valon degrees of freedom which, to first order in κ, are

$$Q_M^+ = q_M^+ + \frac{\kappa}{\omega_g} \sum_{m,\alpha} \langle m|\Theta_\alpha|M\rangle q_m^+ (g_\alpha^+ - g_\alpha)\,, \tag{7}$$

$$G_\alpha^+ = g_\alpha^+ + \frac{\kappa}{\omega_g} \sum_{mm'} q_{m'}^+ \langle m'|\Theta_\alpha|m\rangle q_m\,. \tag{8}$$

The hamiltonian in terms of dressed valons is

$$H_D(Q,G) = e^{-iS(Q,G)} H_o(Q,G) e^{iS(Q,G)}$$

$$= H_o(Q,G) - i[S,H_o]_- + \tfrac{1}{2}[[S,H_o],S]_-\,. \tag{9}$$

So,

$$H_D(Q,G) = \omega_Q \sum_m Q_m^+ Q_m + \omega_G \sum_\alpha G_\alpha^+ G_\alpha$$

$$- \frac{\kappa^2}{\omega_G} \sum :Q_M^+ Q_{M'} Q_m^+ Q_{m'}: \langle M|\Theta_\alpha|M'\rangle\langle m|\Theta_\alpha|m'\rangle$$

$$- \frac{\kappa^2}{\omega_G} \sum Q_M^+ Q_{M'} G_\alpha^+ G_\beta \{\langle M|\Theta_\alpha|M''\rangle\langle M''|\Theta_\beta|M'\rangle$$

$$- \langle M|\Theta_\beta|M''\rangle\langle M''|\Theta_\alpha|M'\rangle\} + O(\kappa^3)\,. \tag{10}$$

There are no QQG vertices. The third term splits the N and Δ as usual.

$$\omega_G = \omega_g\,,$$

$$\omega_Q = \omega_q - \frac{\kappa^2}{\omega_g} \sum_\alpha \langle m|\Theta_\alpha|m\rangle^2 = \omega_q - 4\kappa^2/\omega_g$$

$$= (2.04 - 0.43)/R\,. \tag{11}$$

Three- and four-gluon terms in the QCD hamiltonian must be considered as a perturbation hamiltonian. The only one used here will be

$$H_{3g} = \delta \sum f^{[\alpha\beta\gamma]} \epsilon^{[\alpha\beta\gamma]} (g_\alpha + g_\alpha^+)(g_\beta + g_\beta^+)(g_\gamma + g_\gamma^+)\,, \tag{12}$$

where $\delta = 0.083/R$, $f^{[\alpha\beta\gamma]}$ are the antisymmetric structure constants of color SU(3) and $\epsilon^{[\alpha\beta\gamma]} = +1, 0, -1$ for spin SU(2). This term generates a QG–QG vertex

$$H_{QQGG} = -\frac{6\delta\kappa}{\omega_g} \sum f^{[\alpha\beta\gamma]} \epsilon^{[\alpha\beta\gamma]} Q_{m'}^+ Q_m \langle m'|\Theta_\alpha|m\rangle$$

$$\times (G_\beta + G_\beta^+)(G_\gamma + G_\gamma^+)\,. \tag{13}$$

Antiquarks can be included in the dressed hamiltonian by a simple generalization. Pair producing terms must, of course, be relegated to the perturbation hamiltonian.

3. Gluonic nucleons. We construct the two gluonic nucleon [2] states as color octets of 3q's coupled with a g to form color singlet spin 1/2 states.

$$|\mathrm{N}^1_{\mathrm{qg}}\rangle = \sum R_S \{I_{\mathrm{ms}} C^\alpha_{\mathrm{ma}} - I_{\mathrm{ma}} C^\alpha_{\mathrm{ms}}\} S^i_s \mathrm{g}^{k+}_\alpha /4$$
$$\times (3/2, i; 1, k \,|\, 1/2, \mu) \,|0\rangle \qquad (14)$$

and

$$|\mathrm{N}^2_{\mathrm{qg}}\rangle = \sum R_s (I_{\mathrm{ms}} C^\alpha_{\mathrm{ms}} S^i_{\mathrm{ma}} + I_{\mathrm{ms}} C^\alpha_{\mathrm{ma}} S^i_{\mathrm{ms}} + I_{\mathrm{ma}} C^\alpha_{\mathrm{ms}} S^i_{\mathrm{ms}}$$
$$- I_{\mathrm{ma}} C^\alpha_{\mathrm{ma}} S^i_{\mathrm{ma}})(\mathrm{g}^{k+}_\alpha/\sqrt{32})(1/2, i; 1, k \,|\, 1/2, \mu)\,|0\rangle\,, \qquad (15)$$

where I, C, S, R are flavor, color, spin, and radial three-quark functions of symmetric (s), mixed symmetric (ms) and mixed antisymmetric (ma) type defined by Close [11] for three 1S q's and a (TE10) g. Now replace q → Q and g → G to get valon states $|\mathrm{N}^1_{\mathrm{QG}}\rangle$ and $|\mathrm{N}^2_{\mathrm{QG}}\rangle$.

The mass matrix for $|\mathrm{N_q}\rangle$ and $|\mathrm{N}^{1,2}_{\mathrm{qg}}\rangle$ states is illustrated in figs. 1a–1b including $O(g^2)$ gluonic corrections and $O(f^2_{\pi\mathrm{qq}})$ pionic corrections using the cloudy bag model [12,13]. The offending matrix element [4] is

$$\langle \mathrm{N_q} | H_{\mathrm{qqg}} | \mathrm{N}^1_{\mathrm{qg}} \rangle = \langle \mathrm{N_q} | H_{\mathrm{qqg}} | \mathrm{N}^2_{\mathrm{qg}} \rangle = -2\kappa/R$$
$$= -264\ \mathrm{MeV} \quad \text{for} \quad R = 5.8\ \mathrm{GeV}^{-1}\,. \qquad (16)$$

In contrast the mass matrix for $|\mathrm{N_Q}\rangle$ and $|\mathrm{N}^{1,2}_{\mathrm{QG}}\rangle$ in figs. 1k–1q contains no mixing and has all diagrams with QQG vertices eliminated. The gluon Compton scattering amplitudes of figs. 1f, 1i, 1j are compacted into fig. 1o. Z graphs involving pair production are not eliminated but have not been included here (see ref. [2]). The mass matrix for $|\mathrm{N}^1_{\mathrm{qg}}\rangle, |\mathrm{N}^2_{\mathrm{qg}}\rangle$ is [‡1]

$$M_0(\mathrm{N_{qg}}) + \underset{(\mathrm{f})}{\begin{pmatrix} -165 & -33 \\ -33 & -66 \end{pmatrix}} + \underset{(\mathrm{g})}{\begin{pmatrix} 18 & 0 \\ 0 & -18 \end{pmatrix}}$$
$$+ \underset{(\mathrm{h})}{\begin{pmatrix} -15 & -5 \\ -5 & -11 \end{pmatrix}} + \underset{(\mathrm{ij})}{\begin{pmatrix} 43 & 31 \\ 31 & 68 \end{pmatrix}}\ \mathrm{MeV}$$

and for $|\mathrm{N}^1_{\mathrm{QG}}\rangle, |\mathrm{N}^2_{\mathrm{QG}}\rangle$ is

$$M_0(\mathrm{N_{QG}}) + \underset{(\mathrm{o})}{\begin{pmatrix} -165 & -33 \\ -33 & -66 \end{pmatrix}} + \underset{(\mathrm{p})}{\begin{pmatrix} 18 & 0 \\ 0 & -18 \end{pmatrix}}$$
$$+ \underset{(\mathrm{q})}{\begin{pmatrix} -15 & -5 \\ -5 & -11 \end{pmatrix}} + \underset{(\mathrm{o})}{\begin{pmatrix} 43 & 31 \\ 31 & 68 \end{pmatrix}}\ \mathrm{MeV}\,,$$

‡1 Beware the subtleties noted by Golowich et al. [3].

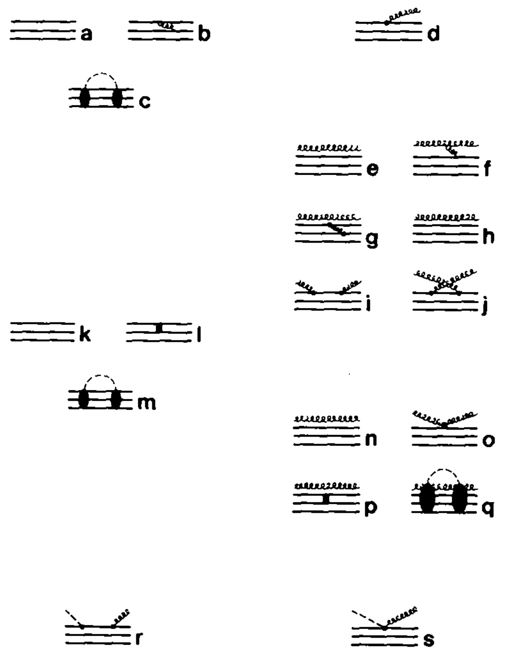

Fig. 1. Mass matrix for nucleon and gluonic nucleons. (a–c) nucleon mass terms including gluon and pion exchange terms; (d) coupling between N and $\mathrm{N_g}$ due to gluon emission q → qg; (e–j) gluonic nucleon mass including 3g coupling, gluon exchange, pion exchange, gluon Compton scattering; (k–m) nucleon mass terms in the valon model; (n–q) gluonic nucleon mass terms in the valon model; (r,s) amplitudes for pionic excitation $\pi\mathrm{N}-\mathrm{N_{qg}}$ or $\pi\mathrm{N}-\mathrm{N_{QG}}$.

for the processes in figs. 1n–1q, all evaluated at R = 5.8 GeV^{-1} [14]. The physical states which diagonalize this mass matrix are

$$|\mathrm{N}^{\mathrm{A}}_{\mathrm{QG}}\rangle = \cos\Theta\, |\mathrm{N}^1_{\mathrm{QG}}\rangle + \sin\Theta\, |\mathrm{N}^2_{\mathrm{QG}}\rangle\,,$$
$$|\mathrm{N}^{\mathrm{B}}_{\mathrm{QG}}\rangle = -\sin\Theta\, |\mathrm{N}^1_{\mathrm{QG}}\rangle + \cos\Theta\, |\mathrm{N}^2_{\mathrm{QG}}\rangle\,,$$

with $\cos\Theta = 0.99$, $\sin\Theta = 0.06$ and $E_{\mathrm{A}} = M_0(\mathrm{N_{QG}}) - 119$ MeV, $E_{\mathrm{B}} = M_0(\mathrm{N_{QG}}) - 27$ MeV.

In fig. 2 we have tried to establish a best value $M_0(\mathrm{N_{QG}})$ by following DeTar [12] and the standard bag model practice of minimizing $E(B, Z, R)$ with respect to R and eliminating center of mass motion to get $M(B, Z)$. B is fixed by requiring the 3Q nucleon mass of figs. 1k–1m to be the experimental value. We then calculate the ρ meson mass as a $\mathrm{Q\bar{Q}}$ state following DeTar but including interactions only of the type of figs. 1k–1m and pion radiative corrections $\rho \to \pi\omega \to \rho$ but not $\rho \to \pi\pi \to \rho$. For acceptable values of R_{N} we

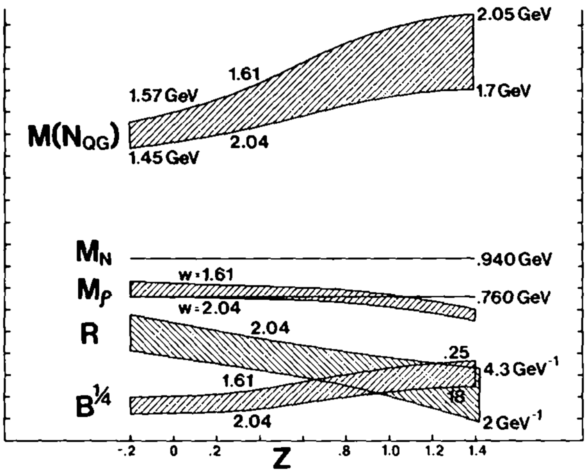

Fig. 2. Determination of bag parameters Z and B in the valon model with $\omega_Q = (2.04 - 0.43)/R$. M_N is fixed leaving $M(N_{QG})$, M_ρ, B, and R_N as functions of the zero point energy Z. Bands indicate spread as ω_Q changes from $2.04/R$ to $1.61/R$. Vertical scales are in the indicated units, with a suppressed zero.

never find a Z which produces M_ρ = 760 MeV. Our ρ masses are about 50 MeV higher, close to those of DeTar. The result is that Z is ill constrained and one can find a mass $M_0(N_{QG})$ from 1.65 to 1.80 GeV for $0 < Z < 0.7$ giving $M(N^A_{QG}) = 1.53 - 1.68$ GeV and $M(N^B_{QG}) = 1.62 - 1.77$ GeV. Our range for Z is consistent with other similarly ill constrained fits in the literature, where, for example, Z can be found to vary from -0.65 [12] and +1 [15] for models that subtract center-of-mass motion effects.

The width of these states for pion emission can be calculated in the cloudy bag model [13] which succesfully parametrizes the pionic couplings in the NΔ multiplet [13] and the N* transitions [16]. The second order amplitude of fig. 1r [2] for the (qg) theory must be replaced by the amplitude of fig. 1s for the (QG) theory. The numbers are the same. The state N^A_{QG} is almost completely decoupled from the πN channel and has a large width in the $\pi\Delta$ channel so would not be seen in πN elastic scattering. We get

$$\Gamma(N^A_{QG} \to \pi N) = 1 \text{ MeV}, \quad \Gamma(N^A_{QG} \to \pi\Delta) = 250 \text{ MeV},$$

$$\Gamma(N^B_{QG} \to \pi N) = 78 \text{ MeV}, \quad \Gamma(N^B_{QG} \to \pi\Delta) = 35 \text{ MeV}.$$

We might conjecture that the N^B_{QG} is the N*(1710) [2,16].

In summary, we emphasize that the valon bag model does not differ in any essential way from standard bag models. The spectrum and decay widths found in the valon theory are just those of the MIT and cloudy bag models. The flavor dynamics of the valon bag model is the same as that of the MIT bag model except that quark vertex functions are renormalized by gluon exchange radiative corrections. What is changed is the starting point. In the standard model bare constituent quarks and gluons make up zeroth order states whose group structures are then mixed by perturbative virtual interactions. Witness the mixing of the $(1s)^3$ 56 plet of q's with N^1_{qg}, and N^2_{qg} in the physical nucleon. In the valon picture the dressed constituents already include virtual interactions so the nucleon remains a $(1s)^3$ 56 plet of Q's. This choice of valon basis states is not changed by the qqg coupling.

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