



# Hyperspherical Harmonics and Their Physical Applications

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Just as spherical harmonics are an indispensable tool for representing single-particle wave-functions in 3D, hyperspherical harmonics can capture the high-dimensional geometry of interacting multi-particle ensembles in few-body systems. They have been extremely useful in nuclear physics, reactive scattering, and for evaluating difficult molecular electron repulsion integrals in quantum chemistry.

This book aims to change the theory of hyperspherical harmonics from an esoteric field, mastered by specialists, into an easily-used tool with a place in the working kit of all theoretical physicists, theoretical chemists and mathematicians.

The theory presented here is accessible without the knowledge of Lie-groups and representation theory, and can be understood with an ordinary knowledge of calculus.

With programs and exercises designed for teaching and practical use.

- Exercises are included at the end of each chapter
- The computer programs and electronic versions of the exercises and solutions are available at <http://harmonics.kvante.org>.

**Readership:** Researchers and students of theoretical physics, theoretical chemistry, and mathematics

Browse sample chapter at <http://bit.ly/hyperspherical>

## Contents

- Computer Programs and Exercises
- Harmonic Functions
- Generalized Angular Momentum
- Gegenbauer Polynomials
- Fourier Transforms in d Dimensions
- Fock's Treatment of Hydrogenlike Atoms and Its Generalization
- D-Dimensional Hydrogenlike Orbitals in Direct Space
- Generalized Sturmians
- Choosing Appropriate Hyperspherical Representations
- Molecular Integrals from Hyperspherical Harmonics
- Lagrangians for Particles and Fields
- Coordinate Transformations for N Bodies
- Some Illustrative Examples
- **Appendices:**
  - The D-Dimensional Harmonic Oscillator
  - Molecular Integrals for Slater-type Orbitals

## About the Authors



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### John Scales Avery

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