

The experimental asymmetry is defined in one ϕ -bin by the number of counts for each helicity :

$$PA = A_{\text{exp}} = \frac{N^+ - N^-}{N^+ + N^-} = \frac{N^+ - N^-}{N}$$

Define the shorthand

$$\partial_+ = \frac{\partial}{\partial N^+}$$

and similarly for N_- . Then

$$\partial_+ A_{\text{exp}} = \frac{2N^-}{N^2} \quad \partial_- A_{\text{exp}} = -\frac{2N^+}{N^2}$$

For uncorrelated helicity counts, the error matrix is diagonal

$$\begin{aligned} (\sigma_{A_{\text{exp}}})^2 &= (\partial_+ A_{\text{exp}} \quad \partial_- A_{\text{exp}}) \begin{pmatrix} \sigma_{N^+}^2 & 0 \\ 0 & \sigma_{N^-}^2 \end{pmatrix} \begin{pmatrix} \partial_+ A_{\text{exp}} \\ \partial_- A_{\text{exp}} \end{pmatrix} \\ &= (\partial_+ A_{\text{exp}})^2 \sigma_{N^+}^2 + (\partial_- A_{\text{exp}})^2 \sigma_{N^-}^2 \\ &= \frac{4N^{-2}}{N^4} \sigma_{N^+}^2 + \frac{4N^{+2}}{N^4} \sigma_{N^-}^2 \end{aligned}$$

In the case $N^- = 0$,

$$\sigma_{A_{\text{exp}}} = \frac{2}{N^+} \sigma_{N^-}$$

In the case that both helicity counts are non-zero

$$(\sigma_{A_{\text{exp}}})^2 = \frac{4N^{+2}N^{-2}}{N^4} \left[\left(\frac{\sigma_{N^+}}{N^+} \right)^2 + \left(\frac{\sigma_{N^-}}{N^-} \right)^2 \right]$$

Assume Poisson distribution of counts and errors

$$\begin{aligned} (\sigma_{A_{\text{exp}}})^2 &= \frac{4N^{+2}N^{-2}}{N^4} \left[\frac{1}{N^+} + \frac{1}{N^-} \right] \\ &= \frac{4N^+N^-}{N^3} \\ &= \frac{N^2 - (A_{\text{exp}}N)^2}{N^3} = \frac{1 - A_{\text{exp}}^2}{N} \\ \sigma_{A_{\text{exp}}} &= \sqrt{\frac{1 - A_{\text{exp}}^2}{N}} \end{aligned}$$

Finally, put back in polarization

$$\boxed{\sigma_A = \frac{1}{P} \sqrt{\frac{1 - (AP)^2}{N}}}$$

Check that the result is correct if A_{exp} is large. Take

$$\begin{aligned} N^- &= \epsilon N^+ \\ A_{\text{exp}} &= \frac{1 - \epsilon}{1 + \epsilon} \approx 1 - 2\epsilon \\ A_{\text{exp}}^2 &\approx 1 - 4\epsilon \\ (\sigma_{A_{\text{exp}}})^2 &\approx \frac{4\epsilon}{N^+} \\ \sigma_{A_{\text{exp}}} &\approx \frac{2}{N^+} \sqrt{\epsilon N^+} = \frac{2}{N^+} \sigma_{N^-} \end{aligned}$$