NOTE ON IMPLEMENTING KN AMPLITUDES IN CLAS PWA

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1. introduction

In this note I will (ultimately) describe the implementation of the meson-baryon interactions in CLAS PWA. There are several possibilities for doing the fits, which depend on the choice of kinematical variables. I will discuss two: the one currently used and emphasizes the upper vertex (meson-meson interaction) and the Dalitz plot analysis.

Here I will begin with the standard CLAS PWA and focus on implementation of the baryon amplitudes.

2. Kinematics

The reaction is

$$\gamma(p_A,\sigma) + N(p_B,\lambda) \to K^-(p_1) + K^+(p_2) + N(p_D,\lambda') \tag{1}$$

The variables in parenthesis refer to momenta and helicities, respectively. The GJ frame is defined as the rest frame of the $K\bar{K}$ pair, with the y-axis perpendicular to the production plane, the z-axis in the direction of the incoming photon, and the nucleons having negative x-momentum components [1] (see Fig. 1)

It is convenient to introduce a set of Lorentz invaraints defined in Fig. 2. Being invariant these can be computed from any set (*e.g.* lab-frame) 4-vectors and used to express all

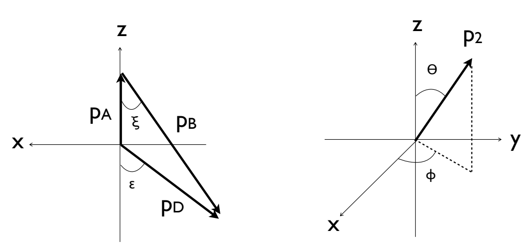

Figure 1. GJ kinematics

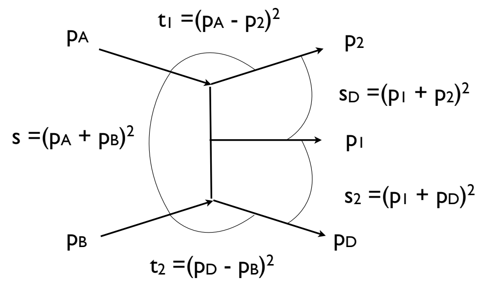

FIGURE 2. Lorentz invaraints

kinematical variables needed in computation of the amplitudes. Specifically, in the current PWA, the independent variables are: $s, t_2, s_D, \theta, \phi$ with $\sqrt{s}$ being the overall c.m. energy, t_2 the square of momentum transfer to the nucleon, $\sqrt{s_D}$ the $K\bar{K}$ invariant mass and finally θ, ϕ describing the direction of motion of the K^+ in the GJ frame. In terms of these variables, the other variables needed for evaluation of amplitudes are given by (with μ and M being the kaon and nucleon mass, respectively)

$$
\begin{aligned}
&E_A = \frac{s_D - t_2}{2\sqrt{s_D}},\ |p_A| = E_A \\
&E_D = \frac{s - s_D - M^2}{2\sqrt{s_D}},\ |p_D| = \sqrt{E_D^2 - M^2} \\
&E_B = E_D + \sqrt{s_D} - E_A,\ |p_B| = \sqrt{E_B^2 - M^2} \\
&\cos\epsilon = \frac{p_B^2 - p_D^2 - p_A^2}{2|p_D||p_A|} \\
&\cos\xi = \frac{|p_A| + |p_D|\cos\epsilon}{|p_B|} \\
&E_1 = \frac{\sqrt{s_D}}{2},\ |p_1| = \sqrt{E_1^2 - \mu^2} \\
&s_2 = \mu^2 + M^2 + 2E_D E_1 - 2|p_D||p_1|(\cos\epsilon\cos\theta + \sin\epsilon\sin\theta\cos\phi) \\
&t_1 = \mu^2 - E_A\sqrt{s_D} + 2|p_A||p_1|\cos\theta \qquad (2)
\end{aligned}
$$

2.1. **Amplitudes.** The amplitudes describing the "upper" vertex *i.e.* the amplitudes in the current PWA are expressed as a sum over partial waves (in the following I drop the explicit dependence on s assuming fixed photon energy).

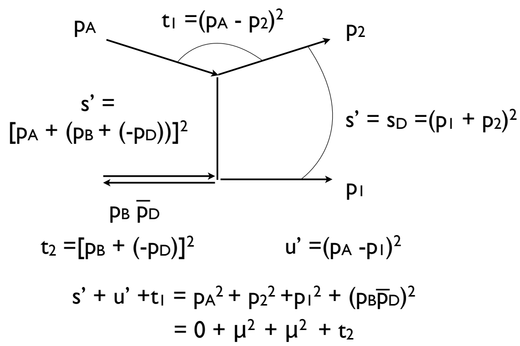

FIGURE 3. Kinematics relevant for Regge parametrization of the K-exchange

$$A = A_{\sigma\lambda\lambda'}(t_2, s_D, \theta, \phi) = \sum_{LM} a^{LM}_{\sigma\lambda\lambda'}(t_2, s_D) Y_{LM}(\theta, \phi) \tag{3}$$

At fixed t_2 and s_D the partial wave amplitudes a^{LM} are obtained by fitting the θ, ϕ dependence. The baryon K^-p amplitude is given by (schematically)

$$B = B_{\sigma\lambda\lambda'}(t_2, s_D, \theta, \phi) = \beta_\sigma(t_1) R(s' - u', t_1) A^{KN}_{\lambda\lambda'}(p_B, p_D, p_1) \tag{4}$$

where (*cf.* Fig. 3)

$$\frac{s' - u'}{2} = s_D + \frac{t_1 - t_2}{2} - \mu^2 \tag{5}$$

The details of the residue function β, Regge propagator, R and the KN amplitude A^{KN} will be provided soon. The bottom line however, is that once the explicit formulas for these functions are given, B can be expressed (binned) in exactly the same variables as A, *i.e.* momentum transfer, t_1 and the $K\bar{K}$ invariant mass s_D. Thus for a given set of helicity combinations, $H = \sigma\lambda\lambda'$ the total amplitude is then given by

$$T_{\sigma\lambda\lambda'}(t_2, s_D, \theta, \phi) = \sum_{LM} a^{LM}_{\sigma\lambda\lambda'}(t_2, s_D) Y_{LM}(\theta, \phi) + B_{\sigma\lambda\lambda'}(t_2, s_D, \theta, \phi) \tag{6}$$

(To be precise there will be two sets of B-amplitudes, one for K^-p and the other for K^+p)

2.2. **Parity invariance, reflectivity and rank.** In the rest frame of the $K\bar{K}$ system other particles are in the xz plane. It is thus convenient to combine parity with rotation by 180^0 around the y axis, *i.e.* use the reflection in the direction perpendicular to the reaction plane, since it leaves all momenta unchanged.

$$Y = e^{-i\pi J_y} P \tag{7}$$

For a particle with helicity λ moving in the xz plane it is easy to show that

$$Y|J,\lambda\rangle = \eta(-1)^{J-\lambda}|J-\lambda\rangle \tag{8}$$

with momentum unchanged and thus not shown explicitly. Parity invariance (for either A or B) amplitudes implies,

$$T_{\sigma\lambda\lambda'}(t_2, s_D, \theta, \phi) = -(-1)^{\lambda'-\lambda} T_{-\sigma-\lambda-\lambda'}(t_2, s_D, \theta, -\phi) \tag{9}$$

For the partial wave amplitudes a^{LM}

$$\begin{aligned} a^{LM}_{-\sigma-\lambda-\lambda'}(t_2, s_D) &= -(-1)^{\lambda-\lambda'} \int d\cos\theta d\phi Y^*_{LM}(\theta,\phi) A_{\sigma\lambda\lambda'}(t_2, s_D, \theta, -\phi) \\ &= -(-1)^{\lambda-\lambda'-M} \int d\cos\theta d\phi Y^*_{L-M}(\theta,-\phi) A_{\sigma\lambda\lambda'}(t_2, s_D, \theta, -\phi) \end{aligned} \tag{10}$$

it leads to

$$a^{L-M}_{-\sigma-\lambda-\lambda'}(t_2, s_D) = -(-1)^{\lambda-\lambda'+M} a^{LM}_{\sigma\lambda\lambda'}(t_2, s_D) \tag{11}$$

The reflectivity basis amplitudes are defined as linear combinations of the a's, such that they separate symmetric from anti-symmetric component of the amplitude under Y-reflection:

$$\begin{aligned} &\sum_{LM} a^{LM}_{\sigma\lambda\lambda'} Y_{LM} = \sum_{L,M>1} \left[a^{LM}_{\sigma\lambda\lambda'} Y_{LM} - (-1)^{\lambda'-\lambda+M} a^{LM}_{-\sigma-\lambda-\lambda'} Y_{L-M} \right] + \sum_{L,M=0} a^{L0}_{\sigma\lambda\lambda'} Y_{L0} \\ &= \sum_{L,M>1} \sqrt{\frac{2L+1}{4\pi}} \left[a^{LM}_{\sigma\lambda\lambda'} d^L_{M0}(\theta) e^{iM\phi} - (-1)^{\lambda'-\lambda} a^{LM}_{-\sigma-\lambda-\lambda'} d^L_{M0}(\theta) e^{-iM\phi} \right] \\ &+ \sum_{L,M=0} \sqrt{\frac{2L+1}{4\pi}} \frac{1}{2} \left(a^{L0}_{\sigma\lambda\lambda'} - (-1)^{\lambda'-\lambda} a^{L0}_{-\sigma-\lambda-\lambda'} \right) d^L_{00}(\theta) \\ &= \sum_{L,M\geq 0} \sqrt{\frac{2L+1}{4\pi}} \left[a^{LM+}_{\sigma\lambda\lambda'} d^L_{M0}(\theta) \cos(M\phi) + i a^{LM-}_{\sigma\lambda\lambda'} d^L_{M0}(\theta) \sin(M\phi) \right] \end{aligned} \tag{12}$$

where we defined

$$a^{L|M|\epsilon}_{\sigma\lambda\lambda'} \equiv a^{L|M|}_{\sigma\lambda\lambda'} - \epsilon(-1)^{\lambda'-\lambda} a^{L|M|}_{-\sigma-\lambda-\lambda'} \tag{13}$$

for $M > 0$ and

$$a^{L0+}_{\sigma\lambda\lambda'} \equiv \frac{1}{2} \left(a^{L0}_{\sigma\lambda\lambda'} - (-1)^{\lambda'-\lambda} a^{L0}_{-\sigma-\lambda-\lambda'} \right) \tag{14}$$

for $M = 0$.

The amplitudes $a_H^{L|M|\epsilon}$ ($H = \sigma\lambda\lambda'$) satisfy

$$a_H^{L|M|\epsilon} = -\epsilon(-1)^{\lambda-\lambda'} a_{-H}^{L|M|\epsilon} \tag{15}$$

In absence of the B amplitudes, the unpolarized cross section is proportional to

$$I^A = \sum_H |A_{\sigma\lambda\lambda'}|^2 = \sum_{H=\sigma\lambda\lambda'} \sum_{LL',MM'\geq 0} \sqrt{\frac{2L+1}{4\pi}}\sqrt{\frac{2L'+1}{4\pi}} d^L_{M0}(\theta) d^L_{M'0}(\theta)$$

$$\left[(a_H^{LM+})^*(a_H^{L'M'+})\cos(M\phi)\cos(M'\phi) + (a_H^{LM-})^*(a_H^{L'M'-})\sin(M\phi)\sin(M'\phi)\right] \tag{16}$$

Thus there is no interference between $\epsilon = +$ and $\epsilon = -$ amplitudes. Including the B amplitudes gives

$$T_H = \sum_{L,M\geq 0} \sqrt{\frac{2L+1}{4\pi}} \left[a^{LM+}_{\sigma\lambda\lambda'} d^L_{M0}(\theta)\cos(M\phi) + i a^{LM-}_{\sigma\lambda\lambda'} d^L_{M0}(\theta)\sin(M\phi)\right] + B_H \tag{17}$$

$$I = I^A + I^{A+B} + I^{A-B} + I^B \tag{18}$$

with

$$I^B = \sum_H (B_H)^* B_H \tag{19}$$

$$I^{A+B} = \sum_{L,M\geq 0} \sqrt{\frac{2L+1}{4\pi}} \left[(a_H^{LM+})^* B_H + (a_H^{LM+}) B_H^*\right] d^L_{M0}(\theta)\cos(M\phi) \tag{20}$$

$$I^{A-B} = \sum_{L,M\geq 0} \sqrt{\frac{2L+1}{4\pi}} \left[(a_H^{LM-})^* B_H + (a_H^{LM-}) B_H^*\right] d^L_{M0}(\theta)\sin(M\phi) \tag{21}$$

References

[1] G.Ascoli, L.M.Jones, B.Weinstein, and H.W.Wyld, Phys. Rev. D**8**, 3895 (1973).