Rate Calculations from MottG4 Simulation

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1 Purpose of Calculating Rates

The calculation of rates from simulation is what allows us direct comparison to data. Quite simply the rate is a calculation of how many events hit our detectors per second per micro-Amp from a given generator. The rate quantity is comparable between different generators and between simulations and data. The differential rate in our detector from one point in phase space $\vec{v}$ is:

$$d\mathcal{R}(\vec{v}) = \mathcal{L}(\vec{v})\sigma(\vec{v})\epsilon(\vec{v})dv \tag{1}$$

where $\mathcal{L}(\vec{v})$ is the luminosity, $\sigma(\vec{v})$ is the cross-section of the physics of interest and $\epsilon(\vec{v})$ is the acceptance function of our detectors (essentially the chance that an event near $\vec{v}$ will be detcted). While $\mathcal{L}(\vec{v})$ and $\sigma(\vec{v})$ are calculated exactly based on theory, $\epsilon(\vec{v})$ is a value obtained by simulation. Typically we only determine the shape of this function with respect to scattering angle since our simulated detectors care little about what happens in the target and have perfect energy resolution.

2 Single Scattering Calculation

For a single scattering event, our phase space vector becomes $\vec{v} = (x, y, z, E, \chi, \psi)$ where χ is the scattering angle and ψ is the azimuthal angle. The total rate in a detector is then

$$\mathcal{R} = \int_V d\mathcal{R}(\vec{v}) \tag{2}$$

where the integral is carried out over the entire phase space available V. The integrals over x, y are trivial while the dependence upon z and E are small enough to simply ignore. This yields:

$$\mathcal{R} = \frac{N_A\rho}{A}N_B d\int_{\psi_{min}}^{\psi_{max}}\int_{\chi_{min}}^{\chi_{max}}\sigma(\chi,\psi)\epsilon(\chi,\psi)\sin\chi d\chi d\psi \tag{3}$$

From this point there are two methods that I've attempted to approximate this integral numerically.

2.1 Method 1: Reimann Sum

We divide the integral into bins in χ and ψ

$$\mathcal{R} \approx \frac{N_A \rho}{A} N_B d \sum_i \sum_j \sigma_{ij} \epsilon_{ij} \sin \chi_i \Delta\chi \Delta\psi \tag{4}$$

This method gives good results which are in decent agreement with data as seen in Table 1.

d (nm)	$\mathcal{R}_{\text{sim}}$	$\mathcal{R}_{\text{data}}$
52	10.25 ± 0.67	9.93 ± 0.09
215	42.49 ± 1.46	46.50 ± 0.48
389	77.15 ± 2.08	82.58 ± 1.04
487	95.75 ± 2.40	97.74 ± 1.00
561	109.89 ± 2.62	128.66 ± 1.32
775	153.20 ± 3.28	178.30 ± 1.86
837	163.88 ± 3.45	209.30 ± 2.15
944	186.50 ± 3.77	246.00 ± 2.53

Table 1: Simulated rates for single scattering averaged from left and right detectors. Data averaged from `https://wiki.jlab.org/ciswiki/images/e/ef/Rates.pdf`

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2.2 Method 2: Monte Carlo Integration

In this method we try to solve the integral with a Monte Carlo Estimator method. Events are generated from a distribution, $g(\vec{v}) = C \sin \chi$. If we define, $f(\vec{v}) = \sigma(\chi, \psi)\epsilon(\chi, \psi) \sin \chi$ can use the following estimator:

$$\mathcal{R} = \frac{N_A \rho}{A} N_B d \int_{\psi_{min}}^{\psi_{max}} \int_{\chi_{min}}^{\chi_{max}} f(\vec{v}) d\chi d\psi \tag{5}$$

$$= \frac{N_A \rho}{A} N_B d \int_{\psi_{min}}^{\psi_{max}} \int_{\chi_{min}}^{\chi_{max}} \frac{f(\vec{v})}{g(\vec{v})} g(\vec{v}) d\chi d\psi \tag{6}$$

$$\approx \frac{1}{n} \frac{N_A \rho}{A} N_B d \sum_i^n \frac{f(\vec{v}_i)}{g(\vec{v}_i)} \tag{7}$$

$$\approx \frac{1}{n} \frac{N_A \rho}{A} N_B d \sum_i^n \frac{\sigma(\chi_i, \psi_i)\epsilon(\chi_i, \psi_i)}{C} \tag{8}$$

Where n is the number of events generated and, by definition:

$$\frac{1}{C} = \int_{\psi_{min}}^{\psi_{max}} d\psi \int_{\chi_{min}}^{\chi_{max}} \sin \chi d\chi \tag{9}$$

$$= \frac{\pi}{9} \left[\cos \frac{\pi}{36} - \cos \frac{\pi}{18} \right] \tag{10}$$

Using this method we see some issues as highlighted in Table 2

	Method 1		Method 2	
d (nm)	$\mathcal{R}_L$	$\mathcal{R}_R$	$\mathcal{R}_L$	$\mathcal{R}_R$
52	4.97	15.52	5.34	8.28
215	20.63	64.34	22.10	34.55
389	37.50	116.80	40.29	62.99
487	46.32	145.19	49.86	78.91
561	53.96	165.83	57.81	88.77
775	74.40	232.01	80.35	122.98
837	80.11	247.65	86.09	133.39
944	90.76	282.25	98.08	152.05

Table 2: Simulated rates for single scattering averaged for each detector from each method. I currently have no explanation for why Left roughly matches but right does not.

3 Double Scattering Rates

We look again at the differential form of the scattering rate. In this case we consider the rate by pieces. The rate from the initial scattering at position (x, y, z) and energy (prior to entering the target), E towards the second scattering position along direction (θ, ϕ) is given by:

$$d\mathcal{R}_1(\vec{v}) = \mathcal{L}(\vec{v})\sigma_1(\vec{v})dv \tag{11}$$

$$= \frac{N_A\rho}{A}\frac{N_B}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_E}\exp\left[\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}+\frac{E^2}{2\sigma_E^2}\right]\sigma_1(z,E,\theta,\phi)\sin\theta dxdydzdEd\theta d\phi \tag{12}$$

The rate that our detector sees then from the second scattering is then:

$$d\mathcal{R}(\vec{v}) = \frac{N_A\rho}{A}d\mathcal{R}_1(\vec{v})\sigma_2(z,E,\xi,\theta,\phi,\chi,\psi)\epsilon(\chi,\psi)\sin\chi d\xi d\chi d\psi \tag{13}$$

We define:

$$f(\vec{v}) = \exp\left[\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}+\frac{E^2}{2\sigma_E^2}\right]\sigma_1(\vec{v})\sigma_2(\vec{v})\epsilon(\chi,\psi)\sin\theta\sin\chi \tag{14}$$

and note that we throw from a distribution:

$$g(\vec{v}) = C\exp\left[\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}+\frac{E^2}{2\sigma_E^2}\right]\sin\theta\sin\chi \tag{15}$$

and then we can define our integral of interest in terms of these functions:

$$\mathcal{R} = \left(\frac{N_A\rho}{A}\right)^2\frac{N_B}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_E}\int_V\frac{f(\vec{v})}{g(\vec{v})}g(\vec{v})dv \tag{16}$$

Using the Reimann sum method is not available to us due to the high dimension of the integral and the difficulty of the integration limits. So we instead try to integrate numerically with a Monte

Carlo estimator:

$$\mathcal{R} = \frac{1}{n}\left(\frac{N_A\rho}{A}\right)^2 \frac{N_B}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_E}\sum_i^n \frac{f(\vec{v}_i)}{g(\vec{v}_i)} \tag{17}$$

$$= \frac{1}{C}\frac{1}{n}\left(\frac{N_A\rho}{A}\right)^2 \frac{N_B}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_E}\sum_i^n \sigma_1(\vec{v}_i)\sigma_2(\vec{v}_i)\epsilon(\chi_i,\psi_i) \tag{18}$$

The problems begin with the integral over V as evidenced by the determination of C.

$$1 = \int_V g(\vec{v})dv \tag{19}$$

$$= CI\int_{-\infty}^{\infty} e^{-x^2/2\sigma_x^2}dx\int_{-\infty}^{\infty} e^{-y^2/2\sigma_y^2}dy\int_{-\infty}^{\infty} e^{-E^2/2\sigma_E^2}dE\int_0^{2\pi}d\phi\int_{\psi_{min}}^{\psi_{max}}d\psi\int_{\chi_{min}}^{\chi_{max}}\sin\chi d\chi \tag{20}$$

Where

$$I = \int_0^d\int_0^\pi\left[\int_0^{\xi_{max}(\theta,z)}d\xi\right]\sin\theta d\theta dz \tag{21}$$

The variable ξ refers to distance from the first scattering point and $\chi_{max}(\theta,z)$ is the maximum distance the second scattering can be generated given the initial scattering position and angle. Since the simulation also has the caveat that all electrons generated must not have lost more than 500 keV in the target (these would not be counted in our physical asymmetry in any case), we put a distance limit for those particles travelling at $\theta \approx \pi/2$. Thus we define:

$$\chi_{max}(\theta,z) = \frac{d-z}{|\cos\theta|}\left[1 - H\left(\frac{d-z}{|\cos\theta|} - D\right)\right] \tag{22}$$

where $D = 157\,\mu\text{m}$. Since this function is symmetric about $\theta = \pi/2$ we have

$$I = 2\int_0^d\int_0^{\pi/2}\frac{d-z}{|\cos\theta|}\left[1 - H\left(\frac{d-z}{|\cos\theta|} - D\right)\right]\sin\theta d\theta dz \tag{23}$$

$$= 2\int_0^d(d-z)\left[\int_0^{\cos\theta=\alpha(z)}\frac{\sin\theta}{\cos\theta}d\theta\right]dz \tag{24}$$

where $\alpha(z) = (d-z)/D$ and $0 \le \alpha(z) < 1$ based on our target dimensions. We then see:

$$I = 2\int_0^d(d-z)\left[\int_\alpha^1\frac{du}{u}\right]dz \tag{25}$$

$$= -2\int_0^d(d-Z)\log\left(\frac{d-z}{157\,\mu\text{m}}\right)dz \tag{26}$$

$$= \frac{1}{2}d^2\left[1 - 2\log\left(\frac{d}{D}\right)\right] \tag{27}$$

All of our target thickness behavior is encoded in this and unfortunately this depends upon an arbitrary cutoff, D. This leads to non-physical results when I look at the two-scattering processes using my assumption of $D = 157\,\mu$m. I need additional input to help me determine where I'm going wrong with these calculations. My intuition tells me that there is probably some exponential behavior that governs the probability of the second scattering occuring at some length, ξ along it's path in the target. That is

$$P(\xi) \propto e^{-\xi/\lambda} \tag{28}$$

But I'm unsure how to generate an appropriate physical value for λ.