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High precision 5 MeV Mott polarimeter

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We report on the design and performance of a Mott polarimeter optimized for a nominal 5-MeV electron beam from the Continuous Electron Beam Accelerator Facility (CEBAF) injector. The rf time structure of this beam allows the use of time of flight in the scattered electron detection, making it possible to cleanly isolate those detected electrons that originate from the scattering foil, and resulting in measured scattering asymmetries which are exceptionally stable over a broad range of beam conditions, beam currents, and foil thicknesses. In two separate series of measurements from two different photocathode electron sources, we have measured the Mott scattering asymmetries produced by an approximately 86% transversely polarized electron beam incident on ten gold foils with nominal thicknesses between 50 and 1000 nm. The statistical uncertainty of the measured asymmetry from each foil is below 0.25%. Within this statistical precision, the measured asymmetry was unaffected by ± 1 -mm shifts in the beam position on the target foil, and by beam current changes and dead-time effects over a wide range of beam currents. The overall uncertainty of our beam polarization measurement, arising from the uncertainty in the value of the scattering asymmetry at zero foil thickness as determined from our fits to the measured asymmetries versus scattering foil thicknesses, the estimated systematic effects, and the (dominant) uncertainty from the calculation of the theoretical Sherman function, is 0.61%. A simulation of the polarimeter using GEANT4 has confirmed that double scattering in the target foil is the sole source of the dependence of the measured asymmetry on foil thickness, and gives a result for the asymmetry versus foil thickness in good agreement with both our measurements and a simple calculation. Future measurements at different beam energies and with target foils of different atomic numbers will seek to bound uncertainties from small effects such as radiative corrections to the calculation of the polarimeter analyzing power. A simultaneous high-precision measurement of the beam polarization with a different polarimeter, AESOP (Accurate Electron Spin Optical Polarimeter), under development at the University of Nebraska, clearly possible at the CEBAF accelerator, will allow a high-precision comparison of our measured asymmetries with theoretical calculations of the Mott analyzing power. Finally, the improved precision of the current Mott polarimeter along with similar improvements to other Jefferson Lab polarimeters warrants another precision comparison of all of these polarimeters when measuring a beam of the same polarization.

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I. INTRODUCTION

Soon after the publication of Dirac's revolutionary equa-39 tion for the electron, Mott calculated the elastic scattering 40 of electrons by the Coulomb field of the nucleus in this 41 new formalism [1]. His motivation was to determine whether 42 the anticipated polarization of the scattered electron, pro-43 duced by spin-orbit coupling and in principle measurable in 44 a double-scattering experiment, could be used to determine 45 the magnetic moment of the free electron. At that time, the 46 then unusual g factor of 2 for the electron was both inferred 47

from measurements of the fine structure of atomic spectra 48 and predicted by Dirac's equation. It was understood at the 49 time that the uncertainty principle precluded the separation of 50 free-electron spins with static electromagnetic fields, and thus 51 a direct measurement of the electron magnetic moment. 52

Mott's solutions for the spin-flip and non-spin-flip scatter-53 ing amplitudes are conditionally convergent series in which 54 pairs of terms very nearly cancel, requiring the calculation 55 of many terms to obtain reasonably precise values for the scattering cross section and scattered beam polarization. Al-57 though various mathematical transformations were employed 58 to reduce the complexity of the calculations, they remained tedious (see Ref. [2] and references therein). Before the advent 60 of digital computers, calculated values for the cross section 61 and polarization were restricted to a limited number of elec-62

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tron energies and a 90° scattering angle. The first extensive
computer calculations of the cross section were published
in 1956 by Doggett and Spencer [3], and by Sherman [4],
who also calculated the scattered beam polarization, which
is transverse to the plane of scattering. Since that time, the
analyzing power of Mott scattering has been known as the
Sherman function.

Several early attempts to demonstrate electron polarization 70 in a double-scattering experiment gave negative or incon-71 clusive results prior to the first successful measurement by 72 Shull et al. [5]. As Mott scattering was the only demonstrated 73 method for producing polarized electrons at the time, exper-74 iments using them were uncommon. One early application 75 was a measurement of the free-electron g factor with 0.5% 76 precision, satisfying Mott's original motivation (though not 77 in the way he envisioned) [6]. Following the experimental 78 demonstrations of parity violation in the weak interactions 79 in 1957, Mott polarimeters, coupled with electrostatic spin 80 rotators, were developed in a number of laboratories to mea-81 sure the longitudinal polarization of beta decay electrons. This 82 led to a much-improved understanding of the experimental 83 technique, and to several well-designed polarimeters [7–9]. 84 The development of polarized electron sources began in the 85 late 1950s, and required polarimetry to quantify and improve 86 their performance [10,11]. Mott scattering at modest energies 87 was universally employed for these studies. All of these early 88 Mott polarimeters operated at energies well below 1 MeV. The 89 experimental challenges, and the problems in computing the 90 effective Sherman function at these relatively low energies, 91 are decidedly different than those encountered at few-MeV 92 energies, and will not be discussed here. 93

Mott polarimetry at energies well above 1 MeV was first 94 employed in a search for possible time-reversal violation in 95 the beta decay of ⁸Li [12,13]. The success of this experiment 96 led some of its participants, with collaborators at the Mainz 97 Microtron (MAMI) accelerator at Mainz, to make detailed 98 measurements of the analyzing power of ²⁰⁸Pb foils at 14 MeV 99 [14,15]. Their measurements were the first to convincingly 100 show the reduction in analyzing power from the nuclear size 101 effect, in agreement with the calculations of Unginčius et al. 102 [16]. These measurements are consistent, within their approx-103 imately 3% statistical uncertainty, with the dependence of 104 the analyzing power on target thickness arising entirely from 105 double scattering in the target foil, with no net polarization 106 dependence in the second scattering. These double-scattering 107 events must belong to one of two categories, viz., (1) a first 108 scattering very close to 90° followed by a second scattering 109 making the remainder of the total large scattering angle or 110 (2) a first relatively large-angle scattering followed by a sec-111 ond relatively small-angle scattering completing the net large 112 scattering angle (or vice versa). The very thin target foils, 113 and the strong dependence of the differential cross section 114 on angle, effectively restrict events from other than these two 115 classes from significant contributions at few-MeV energies. 116 Only events from category 2 above have useful analyzing 117 power. 118

Detection of Mott-scattered electrons at a few MeV for precision electron transverse polarization measurement is not experimentally simple, as a quick examination of the relevant cross sections and analyzing powers reveals. Scattering foils 122 with high atomic number, Z, must be used to provide a large 123 spin-orbit effect. The analyzing power is greatest at large 124 scattering angles, while the cross section drops dramatically 125 at larger scattering angles-facts which become ever more 126 pronounced with increasing electron energy. As a result, for 127 every large-angle scattering event providing useful polariza-128 tion information, a much larger number of electrons scattered 129 at smaller angles are also generated. If one detects only elec-130 trons independent of their origin, it is essentially impossible 131 to assure that a detected electron originated from a single or 132 double large-angle scattering in the target foil, as opposed to a 133 scattering in the target foil followed by scattering from the ap-134 paratus walls, etc. Since each scattering is primarily elastic or 135 quasielastic, the scattered electron energy is not a very useful 136 discriminant, compared with the percent level energy resolu-137 tion of commercial scintillating materials often employed for 138 detection of MeV energy electrons (see Sec. VII). Thus MeV 139 energy Mott scattering asymmetry measurements generally 140 include an uncertain and potentially significant contamination 141 from the detection of electrons which did not arise from a 142 single or double elastic scattering in the target foil, and which 143 have a very different scattering asymmetry. 144

With the high average current available from contemporary 145 polarized electron sources, precision experimental study of 146 Mott polarimetry at accelerator energies in the MeV range 147 becomes practical. Beams from these accelerators have rf 148 time structure, offering the prospect of time-of-flight (TOF) 149 discrimination against electrons that do not originate from 150 the primary scattering foil. The rf time structure and high 151 average beam current make continuous precision monitoring 152 of the beam current and position on the target foil possible. 153 Optical transition radiation (OTR) provides a visible signal 154 with a nonsaturating intensity directly proportional to the local 155 current density incident on the scattering foil, and can be 156 measured continuously for each polarization state during a 157 polarization measurement. Finally, the scattering foils may be 158 considerably thicker than those used at lower energies without 159 introducing overwhelming plural-scattering problems. 160

Along with these experimental advantages, calculation of 161 the Sherman function with good precision at MeV energies 162 is also practical. This calculation, and a discussion of the 163 many small effects that must be considered, are thoroughly 164 discussed in Sec. III [17,18]. Screening effects are very small 165 at few-MeV energies, while the energy is still low enough 166 that nuclear size effects are also quite small [16,19]. Each 167 of these effects can be calculated with ample precision at 168 the beam energies in question, and contribute very little to 169 the uncertainty in the calculated Sherman function. Exchange 170 scattering is no greater than $\approx 0.1\%$, and inelastic scattering 171 in the target foil makes a negligible contribution. The two 172 leading-order radiative corrections, vacuum polarization and 173 self-energy, each of order $\alpha(\alpha Z)$, increase with both Z and en-174 ergy and are difficult to calculate. They are, however, believed 175 to be of comparable magnitude and opposite sign, as discussed 176 later, leading to some cancellation. The vacuum polarization 177 contribution can be calculated in a reasonable approxima-178 tion, and is $\approx 0.4\%$ at our 5-MeV beam energy [17,18]. 179 The total radiative corrections give the largest contribution 180 to the theoretical uncertainty in the Sherman function in the few-MeV energy range, and are estimated to be no greater than $\approx 0.5\%$. By measuring the Mott asymmetry from foils of several different atomic numbers and at several different energies it may be practical to place meaningful bounds on this theoretical uncertainty.

These favorable experimental and theoretical considera-187 tions led us to develop a Mott polarimeter capable of high 188 statistical precision measurements, which was optimized for a 189 5-MeV electron beam, the nominal value at the Continuous 190 Electron Beam Accelerator Facility (CEBAF) injector. The 191 5-MeV polarimeter we describe here has been in use for over 192 20 years, and has proven to be a readily available, easily 193 used, and reliable monitor of beam polarization in the low-194 energy region of the injector. For beam energies now reaching 195 11 GeV, the beam polarization is not measurably degraded 196 during multiple acceleration passes through the full CEBAF 197 accelerator, and remains entirely in the horizontal plane in 198 both the polarized injector and the experimental halls, despite 199 the intervening vertical bends to separate and recombine the 200 beams from different passes through the linear accelerators 201 [20]. Thus the polarization measured in the low-energy region 202 of the injector is directly relevant to the polarization measured 203 at the final energy in the experimental halls over the full 204 energy range of the present accelerator. 205

Since our original development of this polarimeter, sig-206 nificant improvements to the shielding, detectors, electron-207 ics, time-of-flight system, and beam dump have been made, 208 resulting in the current version of the polarimeter presented 209 below. A very early result reported asymmetry measurements 210 from foils of three different Zs (29, 47, and 79) in reasonable 211 agreement with expectations, as well as OTR measurements 212 showing that the beam profile was independent of the beam 213 polarization to a high degree [21]. Detailed measurements 214 of a beam with constant polarization at three different beam 215 energies (2.75, 5.0, and 8.2 MeV) made with the original 216 polarimeter with the addition of time-of-flight background 217 rejection have been presented, along with fits to the asym-218 metry versus target foil thickness at each energy using a 219 semiempirical model based on Wegener's study of the double-220 scattering problem [22,23]. The entire three-energy data set 221 was fit very well with this model, as shown in Fig. 1, and 222 is consistent with the polarization at all three beam energies 223 being the same within about 0.3%. It is worth noting that 224 foil thicknesses spanning a factor of 100, from 0.05 to 5 μ m, 225 were used in these measurements. Using an unpolarized beam, 226 it was determined that the instrumental asymmetry of the 227 polarimeter was $(4 \pm 6) \times 10^{-4}$. Finally, it should be noted 228 that no radiative corrections were included in the computation 229 of the Sherman function at these three energies. Given the 230 dependence of the leading-order radiative corrections on en-231 ergy, this result provides strong circumstantial support that the 232 net effect of these corrections largely cancels, as theoretically 233 anticipated. 234

One other polarimeter operating in the MeV range at an accelerator has been reported [24]. This device was operated between 1 and 3.5 MeV at the MAMI microtron accelerator at Mainz. It employed two double-focusing spectrometer magnets followed by scintillation detectors, with a fixed scattering



FIG. 1. Asymmetry vs foil thickness measured at three different energies with the original version of the polarimeter. The fits to the three data sets (measured scattering asymmetry vs target foil thickness) are based on the semiempirical model developed by Steigerwald [22]. The fit intercept at zero foil thickness, along with the theoretically calculated Sherman function, then determines the beam polarization.

angle of 164°, corresponding to the maximum analyzing
power at 2 MeV. They reported a reproducibility better than
1% in their asymmetry measurements, and they believe they
reach an absolute accuracy for the measured polarization of
about 1%.240
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II. MOTIVATION AND METHODS

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The motivation for our MeV Mott polarimetry studies has 246 been to reduce the uncertainty in the measured polarization 247 of longitudinally polarized electron beams used for parity-248 violation studies at CEBAF. This is because uncertainty in the 249 beam polarization is the dominant uncertainty in the measured 250 parity-violating asymmetry in the scattering of longitudinally 251 polarized electrons from nuclear or electron targets. The high-252 precision Mott polarimeter described here not only provides 253 an independent measurement of the beam polarization from 254 the injector, but is a very useful instrument to normalize 255 the polarization measured by various polarimeters in the 256 experimental halls [25]. A meaningful reduction in the un-257 certainty of the electron-beam polarization will directly im-258 pact the physics interpretation of high-energy parity-violation 259 measurements. 260

In this paper we have employed methods to test and 261 improve both the accuracy and precision of the measured 262 beam polarization. The accuracy was improved by performing 263 theoretical calculations of the Sherman function, applying 264 statistical analyses to the analyzing power dependence on 265 polarimeter target thickness, and developing GEANT4 simu-266 lations to model and validate the analyses. The precision of 267 the polarimeter was investigated by detailed examination of 268 the dependence of the measured physics asymmetry on the 269 detector signals that are recorded to isolate the polarization 270 dependent Mott elastic signal, as well as a number of poten-271 tially important systematic effects. 272 298

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For a given beam polarization the measured experimental 273 asymmetry is proportional to the analyzing power of the 274 polarimeter. Theoretically, the analyzing power of Mott scat-275 tering from a single atom is known as the Sherman function. 276 Experimentally, in a real target foil, an electron may scatter 277 from more than a single atom, leading to a lower analyzing 278 power known as the effective Sherman function. The usual 279 way to determine the effective Sherman function for a par-280 ticular foil thickness and unknown polarimeter is to measure 281 the asymmetry for several foil thicknesses and extrapolate to 282 the zero-thickness single-atom value. The extrapolated asym-283 metry in conjunction with the theoretical Sherman function 284 is then used to determine the beam polarization and also 285 calibrate the effective Sherman function of each target foil 286 tested. 287

Data obtained over two run periods (referred to as runs 1 288 and 2) were used for this paper. The two runs were performed 289 six months apart, each run employing a similar but physi-290 cally different photocathode to produce the polarized beams. 291 Systematic studies of possible sensitivities of the results on 292 various beam parameters were performed during both run 293 periods. In the sections that follow, the purpose and methods 294 are discussed for each significant aspect of the measurements, 295 and the corresponding systematic and statistical uncertainties 296 associated with each are analyzed. 297

III. CALCULATION OF THE THEORETICAL SHERMAN FUNCTION

For electron scattering at few-MeV energies, the scattering 300 potential is modified from the Coulomb field of a point 301 nucleus by four effects. In order of importance for our case, 302 these are (i) the finite size of the nucleus, (ii) screening of 303 the nuclear Coulomb field by the atomic electrons (essentially 304 only those in the K shell), (iii) the exchange interaction 305 between the incident electron and the atomic electrons, and 306 (iv) inelastic scattering of the incident electron on the atomic 307 electrons. Each of these effects reduces the Sherman function 308 below that calculated for a point nucleus. These four effects 309 were calculated with the code ELSEPA [26,27], which does 310 relativistic partial-wave calculations of the differential cross 311 section and spin-polarization functions with state-of-the-art 312 potentials. In addition to these effects, bremsstrahlung and 313 QED radiative corrections must also be considered. We dis-314 cuss the impact of each of these effects below. Consider-315 able detail on the calculation of the Sherman function for 316 our experiment is given in a recent paper by Roca-Maza 317 [18]. 318

For calculations of the effect of nuclear size on the po-319 larization functions and differential cross section, the nuclear 320 charge density was modeled by a two parameter Fermi func-321 tion. As the de Broglie wavelength of a 5-MeV electron 322 (226 fm) is very large compared to the rms charge radius 323 of ¹⁹⁷Au (5.437 fm in the two parameter Fermi function 324 model), greater detail for the nuclear charge distribution is 325 safely neglected (cf. Figs. 1 and 7 in Ref. [18]). Indeed, the 326 Sherman function calculated with the two parameter Fermi 327 function agrees with that calculated using a multiparameter 328 self-consistent mean-field model of the nuclear charge distri-329

bution to within 0.1% in the region of interest. For 5-MeV electrons on ¹⁹⁷Au, the nuclear size effect reduces the Sherman function of a point nucleus by 1.4%, with an uncertainty less than 0.1%.

To calculate the effects associated with atomic electrons, 334 the most accurate electron densities obtained from self-335 consistent relativistic Dirac-Fock calculations have been used 336 [28]. For the calculation of exchange scattering, the Furness-337 McCarthy exchange potential was used [29]. Inelastic scatter-338 ing was calculated using a potential proposed by Salvat [30]. 339 The effects of screening, exchange, and inelastic scattering 340 on the Sherman function in our kinematic region are all very 341 small. Specifically, for 5-MeV electrons on ¹⁹⁷Au at 172.6°, 342 screening is about 0.02%, and exchange is about 0.01%. 343 Inelastic scattering is 0.03% for 1-MeV electrons, and is 344 expected to be smaller at higher energies. The uncertainties 345 in each of these corrections are no greater than 10% of the 346 corrections. The details are covered in Ref. [18]. 347

We are unaware of any complete calculation of the two 348 lowest-order radiative corrections to Mott scattering, vac-349 uum polarization, and self-energy, each of order $\alpha(\alpha Z)$. The 350 vacuum polarization correction can be calculated with the 351 aid of the Uehling potential, as has been done recently by 352 Jakubassa-Amundsen [31]. As the Uehling potential has the 353 same sign as the Coulomb potential, the vacuum polarization 354 effect increases the analyzing power. At our 5-MeV energy, 355 the calculated effect is +0.39%. The size of this correction 356 increases with energy. 357

While the lowest-order self-energy terms have not been 358 calculated for Mott scattering, a subset of these terms has been 359 calculated for the related process of radiative electron capture 360 by a bare heavy nucleus, which is the time-reversed analog 361 of the photoelectric effect [32]. As with the Mott calcula-362 tion, the vacuum polarization terms were evaluated with the 363 aid of the Uehling potential. The self-energy correction was 364 calculated only for the part involving the bound-state electron 365 wave function, omitting the part involving the continuum-state 366 wave function. The calculations were done for three incident 367 heavy-ion (U^{92+}) energies. In all cases, the magnitude of the 368 corrections increased with energy, the vacuum polarization 369 terms were positive, and the self-energy terms calculated were 370 negative and about a factor of 3 larger than the calculated vac-371 uum polarization terms. This gives some cancellation between 372 the vacuum polarization and self-energy terms for the total 373 first-order radiative corrections. Given the similarity of the 374 vacuum polarization and self-energy effects in both radiative 375 electron capture and Mott scattering, it is widely believed that 376 these two terms will be of opposite sign and similar magnitude 377 in Mott scattering. 378

There is also a correction due to bremsstrahlung. One 379 calculation of this correction at several energies between 128 380 and 661 keV, and at five angles in 30° steps to 150°, has been 381 reported [33]. The calculated correction increased the mea-382 sured polarization at all points. The correction decreased with 383 energy for the central angles and increased with energy at both 384 forward and backward angles. The increase was more pronounced at forward angles than backward angles. The cor-386 rection calculated at 661 keV and 150° was +1.18%. These 387 calculations are not useful for making any projection about the 388



FIG. 2. Polarization determination for several energies using two different fit functions (excerpted from Ref. [24]). Uncertainties $(\pm 2\sigma)$ are from the fit only.

³⁸⁹ bremsstrahlung corrections at our beam energy and scattering
³⁹⁰ angle, but it appears possible to calculate this correction for
³⁹¹ our conditions, using the complex expressions presented in the
³⁹² paper. If this correction remains positive with our kinematics,
³⁹³ this will counter the anticipated net negative effect of the
³⁹⁴ first-order radiative corrections.

The 197 Au nucleus has a spin of 3/2 and a relatively small 395 magnetic moment of ≈ 0.147 nm. Jakubassa-Amundsen [31] 396 has calculated the effect of magnetic scattering in the case of 397 Pb²⁰⁷, with its much larger magnetic moment of 0.593 nm. 398 and has shown that it is completely negligible below energies 399 of $\approx 100 \text{ MeV}$ and angles less than $\approx 178^{\circ}$ [31]. We therefore 400 believe that magnetic scattering is negligible for Au¹⁹⁷ with 401 our kinematics and make no correction for the effect. Finally, 402 we have made no correction for recoil effects, which we 403 believe to be small. The β of the recoiling gold nucleus is 404 $\approx 0.5 \times 10^{-5}$ for 5-MeV incident electrons. 405

Two experiments have previously reported Mott scattering 406 polarization measurements over a range of energies between 407 1.0 and 8.2 MeV. The first of these reported measurements 408 at three energies between 2.75 and 8.2 MeV, with a range 409 of foil thicknesses spanning a factor of 100, from 50 nm to 410 5 μ m [22]. These data were fit with a single semiempirical 411 function based on Wegener's study of double scattering [23]. 412 The results, shown in Fig. 1, show the same polarization at 413 all three energies within about 0.3%. These results included 414 no corrections for QED radiative effects or bremsstrahlung. 415 The second measurement covered the energy range between 416 1.0 and 3.5 MeV and showed a polarization consistent with 417 a constant value to within about 0.5%, as shown in Fig. 2 418 [24]. Again, no corrections for QED radiative effects or 419 bremsstrahlung were made. 420

These two experiments, using different polarimeters and conducted by different groups at different laboratories, present strong circumstantial evidence that the total effect of QED radiative corrections, bremsstrahlung, and recoil is no larger than about 0.4% over the full energy range measured. There is good reason to believe that the vacuum polarization correction, known to be positive, is a fraction of the self-energy 427 correction, and there is some evidence that the bremsstrahlung 428 correction may have the same sign as the vacuum polarization 429 term over this kinematic range. The vacuum polarization 430 correction calculated with the aid of the Uehling potential is 431 known to increase significantly with energy over the range 432 in question, and the self-energy terms calculated for radiative 433 recombination also increase with energy. It therefore appears 434 that the net effect of these corrections nearly cancels over 435 the full energy range measured. The QED corrections are 436 proportional to Z, and it has been demonstrated practical 437 to measure Mott scattering with different Z foils with our 438 polarimeter. In the future, such measurements may lead to 439 improved limits on the total magnitude of these corrections. 440

Our estimate is that the net effect of the QED corrections, bremsstrahlung, and recoil is negligible, with an uncertainty of about 0.4%. For a nominal electron-beam kinetic energy of 5.0 MeV and a scattering angle of 172.6° , the theoretically calculated Sherman function is 0.5140 ± 0.0026 , having increased the total uncertainty to 0.5%.

IV. POLARIZED ELECTRON INJECTOR

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The CEBAF polarized electron injector comprises several 448 subsystems, including a dc high voltage electron gun with a 449 photoemission cathode, a laser system for illumination of the 450 photocathode, a group of electromagnetic elements to orient 451 the spin of the electron beam, several rf cavities to temporally 452 shape the individual electron bunches and accelerate them to 453 several MeV, a number of conventional steering and focusing 454 magnets, and beam diagnostic elements which allow us to 455 establish and maintain the desired beam conditions. A plan 456 view of the injector, from the electron gun to downstream of 457 the Wien filter spin orientation section, is shown in Fig. 3. 458

The inverted-insulator dc high voltage electron gun has 459 a load lock to allow exchange of photocathodes without 460 breaking the ultrahigh vacuum in the gun [34]. The photo-461 cathode is a strained multilayer GaAs – GaAs_{1-x} P_x structure 462 which delivers $\approx 86\%$ longitudinally polarized electrons when 463 illuminated at normal incidence by 100% circularly polarized 464 light of near-band-gap photon energy [35]. Any small residual 465 linear polarization of the optical beam does not result in any 466 polarization of the electron beam. Under normal CEBAF op-467 erating conditions, the photocathode is illuminated with laser light from three rf gain-switched diode lasers, each delivering 469 a pulse train at 499 MHz, which is one-third of the 1497-MHz 470 fundamental rf frequency of the CEBAF accelerator [36]. For 471 the work reported here, only a single laser was used. This 472 laser was operated on the 16th subharmonic of 499 MHz, 473 producing a train of electron bunches at a 31.1875 MHz, and 474 thus providing a separation of 32.0641 ns between bunches. 475 Producing an optical pulse train at this low frequency was ac-476 complished by a digital laser gain-switching technique, which 477 produced optical pulses largely free of secondary pulses [37]. 478 The fundamental laser wavelength is 1560 nm, which was 479 frequency doubled to 780 nm, providing maximum electron 480 polarization from the photocathode. The linear polarization of 481 the doubled laser beam was converted to circular polarization 482 with a Pockels cell which rapidly reverses the beam helicity. 483



FIG. 3. This plan view of the first part of the CEBAF injector highlights the polarized photogun followed by the electromagnetic elements that determine the spin direction of the beam. The orientation of the electron polarization is longitudinal as the beam exits the photogun.

A high-quality zeroth-order mica half-wave plate before the
 Pockels cell allows the sense of the circular polarization, and
 hence the electron-beam polarization, to be reversed while
 leaving the Pockels cell voltages unchanged.

Jefferson Lab polarized electron experiments generally 488 require longitudinally polarized electrons. There is a very 489 large polarization precession in the horizontal plane of the 490 CEBAF accelerator between the polarized electron source 491 and the experimental targets, requiring the polarization of the 492 beam exiting the electron injector to be properly oriented to 493 give maximum longitudinal polarization at the experiment. 494 This orientation is done by two Wien filters and two nom-495 inally identical solenoids between them. Small quadrupoles 496 allow correction of the electron-optical astigmatism of the 497 Wien filters. This scheme allows the beam exiting the second 498 Wien filter to have any spin orientation while keeping the 499 beam properly focused. The Wien filters are described in 500 detail in Grames et al. [25]. They are capable of providing 501 a 90° spin rotation to a 130-keV electron beam, the current 502 electron gun operating voltage. The two solenoids between 503 the Wien filters allow reversal of the beam polarization 504 without altering the focusing through the injector, which is 505 506 valuable for understanding polarization associated systematic effects, particularly in experiments such as parity-violation 507 studies, which must measure very small asymmetries. The 508 complete polarization orientation system, including the de-509 tails of its electron optics, is described in Grames et al. 510 [38]. 511

Magnetic solenoids with their magnetic-field axis colinear 512 with the beam axis both rotate any transverse component of 513 electron spin passing through them about the beam axis (leav-514 ing any longitudinal component undisturbed) and focus the 515 beam. The spin rotation is proportional to the magnetic-field 516 integral of the solenoid, while the focusing is proportional to 517 the integral of the square of the field through the solenoid. 518 A compound solenoid with a pair of magnetically separated 519 equal and opposite excitation coils (a so-called counterwound 520 or Stabenow configuration) produces a net beam focusing 521 from the net square of the field integral, but no net spin 522 rotation from the net zero-field integral. All solenoids in the 523 CEBAF injector following the Wien filter section are of this 524 type. This assures that the spin orientation established in the 525 Wien filter section is maintained through the injector. 526

The two Wien filters and the associated solenoids orient the electron spin for all CEBAF experiments, as well as providing spin orientation reversals for systematic error cancellations. 529 We conducted two independent series of Mott polarization 530 measurements from two different photocathode sources (runs 531 1 and 2). In the run 1 measurements, the vertical Wien filter 532 oriented the electron spin vertical, and the two solenoids 533 rotated the spin to the horizontal direction. This provided an 534 electron beam maximally polarized in the horizontal plane at 535 the Mott polarimeter, and thus nominally gave a maximum "up-down" asymmetry and a zero "left-right" asymmetry in 537 the polarimeter detectors. In run 2, the vertical Wien filter 538 again oriented the electron spin vertical, but the two solenoids 539 were set to only focus the beam, without polarization rotation, 540 and thus gave a maximum left-right asymmetry with a zero 541 up-down asymmetry. In both runs the second horizontal Wien 542 filter remained unpowered. 543

The electrons for the Mott experiment are accelerated 544 first to 500 keV by a normal conducting accelerating cavity 545 and then by two five-cell superconducting (SRF) accelerating 546 cavities designed to maximally accelerate electrons moving 547 at the velocity of light. For the Mott measurements, these 548 cavities produced a beam of 5-MeV nominal energy, accel-549 erating electrons from $\beta = v/c$ of 0.86 to $\beta = 0.996$. Since 550 $\beta < 1$, care must be taken to assure that the phase of the rf 551 power to the SRF cavities, which are designed for accelerating 552 $\beta = 1$ beams, produces both a high-energy gain and a minimal 553 energy spread. 554

The beamline between the SRF cavities and the Mott po-555 larimeter is shown in Fig. 4. The magnets through this section 556 are conventional quadrupoles, air core steering correctors, and 557 a dipole. These magnets do not have any significant effect 558 on the polarization orientation. The dipole is used to deflect 559 the beam to a spectrometer at -30° , to the Mott polarimeter 560 at -12.5° , or to a well-instrumented beamline leading to 561 various other injector energy experiments at 25°. Following 562 the two experimental runs, the vertical bending component 563 of the magnetic field through this region, typically $\approx 0.5 \,\text{G}$, 564 was measured. This information, coupled with details of the 565 corrector fields, quadrupole strengths, and the centering of the 566 beam as it passed through the quadrupoles and the position 567 monitors, led to a detailed calculation of the beam kinetic 568 energy entering the Mott polarimeter. The resulting beam 569 energies and uncertainties for the two runs are described in 570 detail in a Jefferson Lab technical note [39]. The beam kinetic 571 energies were 4.806 \pm 0.097 MeV for run 1, and 4.917 \pm 572 0.013 MeV for run 2. 573

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FIG. 4. Plan view of the injector illustrating the rf accelerating cavities, the 12.5° beam line through the Mott polarimeter, the spectrometer beam lines at -30° and 25° , and the straight beam line leading to the rest of the CEBAF injector including an rf cavity beam current monitor (BCM) and a Faraday cup (FC).

The beam emittance was measured following run 1 by 574 the quadrupole scan method using the first quadrupole in the 575 beam line and a downstream wire scanner. The horizontal 576 normalized rms emittance was about 0.56 μ m, and the vertical 577 normalized rms emittance was about 0.4 μ m. These emit-578 tances, though small, reflect the relatively large illuminated 579 area of the photocathode as used in a recent parity-violation 580 study [40]. Given these small emittances, they were not re-581 measured in run 2. These emittances resulted in beam sizes 582 of typically ≈ 0.5 -mm rms at the Mott scattering foil, and 583 similarly small diameters throughout the entire beam line. 584

V. DESIGN OF THE POLARIMETER

The polarimeter vacuum chamber, shown in Fig. 5, is com-586 posed of three segments—a scattering chamber containing the 587 target foils, apertures, and detector ports; an extension section 588 providing a vacuum pump port; and a long drift chamber 589 ending in a beryllium and copper beam dump structure. The 590 polarimeter is connected directly to a beam port 12.5° off 591 the main accelerator beam line, with no intervening vacuum 592 windows. The beam is steered to the polarimeter by a dipole 593 magnet. When not in use, the polarimeter is isolated with a 594



FIG. 5. Elevation view of the Mott polarimeter, including the beam line from the dipole magnet which steers the beam into the polarimeter.



FIG. 6. Sherman function for three electron-beam kinetic energies.

⁵⁹⁵ beam line vacuum valve. Vacuum in the chamber is maintained below a nominal pressure of $\approx 10^{-7}$ Pa by several deionized ion pumps and a non-evaporable getter (NEG) pump. The internal surfaces of the chambers have a 12.7-mm-thick aluminum liner downstream of the target foils to reduce both backscattered electrons and the photon background in the detectors.

The scattering chamber has four detector ports, each cen-602 tered on a scattering angle of 172.6° and separated by 90° 603 in azimuth, with two in the horizontal plane and two in 604 the vertical plane, allowing simultaneous measurement of 605 both transverse components of the beam polarization. Four 606 internal knife-edge apertures of 4.87-mm diameter are pre-607 cisely machined in a 25.4-mm-thick aluminum plate, centered 608 on a 25.4-mm-diameter aperture to pass the incident beam. 609 This plate is mounted in turn on a 12.7-mm-thick aluminum 610 plate which covers very nearly the entire cross section of 611 the scattering chamber. The solid angle subtended by each 612 aperture is 0.23 ms. Using precision survey techniques, the 613 25.4-mm-thick plate was positioned so the four apertures were 614 centered on the 172.6° scattering angle lines between the 615 center of the scattering foil and the detector packages. The 616 5.0-MeV Sherman function for a point nucleus was originally 617 calculated to be maximum at the 172.6° angle. This angle is 618 somewhat greater when the nuclear size effect is included. 619 Recent calculations place the Sherman function maximum, 620 corrected for the nuclear size, at about 173.0°. It is worth not-621 ing that the Sherman function is within 0.995 of its maximum 622 value in this case over about 1.8°. The individual apertures 623 noted below, in each channel, span about 0.9°. Scattered elec-624 trons that pass through an aperture enter a detector package 625 through a 50- μ m aluminum window, immediately followed 626 by 9.7-mm-diameter aperture in a 12.7-mm-thick aluminum 627 plate centered on the 172.6° scattering angle. Figure 6 shows 628 the Sherman function for three electron-beam kinetic energies 629 [18]. 630

The target ladder is mounted on a bellows sealed translation mechanism with 600 mm of travel, which is driven by a stepper motor. It has 16 target foil mounting positions, each with a 25.4-mm-diameter clear aperture. One of these is left open intentionally, and a second contains a chromox beam viewscreen, leaving 14 positions available for scattering foils. Fourteen gold foils were installed, although four of these



FIG. 7. Mott detector assembly illustrating each of the collimators, scintillators, and phototubes which comprise the coincident detection of a scattered electron.

foils had nonstandard mountings and were not used for the 638 measurements reported here. The target ladder assembly is 639 thoroughly described in a JLAB technical note [41]. Details of 640 the target foils are discussed in the Appendix. Finally, a port 641 with an optical window is located on the side of the chamber 642 behind the target foil plane, allowing the target foil to be 643 viewed by a polished stainless-steel mirror. OTR propagating 644 backward at about 167° provides a visible image, viewed by 645 a CCD camera, of the beam incident on the scattering foil. 646 This provides an accurate, nonsaturated real-time image of the 647 beam profile at the target foil. 648

A 2.5-m section of a 20-cm-diameter aluminum vacuum 649 tube terminating in a beam dump follows the vacuum exten-650 sion section. The dump is an 18.4-cm-diameter, 6.35-mm-651 thick disc of Be metal, affixed to a water-cooled reentrant 652 copper flange structure by screws. Beryllium offers excellent 653 thermal conductivity, and a low ratio of radiative to collisional 654 electron energy loss. The use of Be offers high beam power 655 handling capability, and minimizes both electron backscat-656 tering and photon production. Operation with 75- μ A beam 657 current (375-W beam power) has been conducted with this 658 dump, which is designed to operate with a 1-kW beam power 659 limit. 660

Figure 7 shows one of the four identical detector pack-661 ages. Each package contains two plastic scintillation detectors 662 behind a lead and an aluminum collimator. The first " ΔE " 663 detector is a 1.0-mm-thick, 25.4-mm square plastic scintillator, while the second "E" detector is a 76.2-mm-diameter 665 by 62.6-mm-long plastic scintillator. The ΔE scintillator is 666 optically connected to a 25.4-mm-diameter phototube (Hama-667 matsu R6427) by an acrylic lightguide glued to both the 668 scintillator and the phototube, while the E scintillator is di-669 rectly glued to the face of a 76.2-mm phototube (Hamamatsu 670 R6091). The surfaces of the E scintillator were painted with a 671 diffuse reflector to improve the optical photon transport to the 672 photomultiplier cathode. The entire four detector package was 673 enclosed in at least 10-cm-thick lead shielding constructed 674 from standard $51 \times 102 \times 203$ -mm lead blocks. 675

VI. DATA ACQUISITION SYSTEM

The electronic signal processing circuitry for the ΔE and E signals of one of the four arms of the data acquisition E signals of one of the four arms of the data acquisition E signals of one of the four arms of the data acquisition E signals of one of the four arms of the data acquisition E signals of one of the four arms of the data acquisition E signals of one of the four arms of the data acquisition E signals of one of the four arms of the data acquisition E signals E sis signals E signals E signals E signals

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FIG. 8. Electronic signal processing of the ΔE and *E* signals (the "left" of four detector arms).

system (DAQ) is shown in Fig. 8. The photomultiplier high 679 voltages for each of the four ΔE and E detectors were set to 680 produce nominal -200-mV signal pulses. A Mott scattered 681 electron deposits about 10 keV in the ΔE scintillator, and the 682 remainder of the energy in the E scintillator. The ΔE and 683 E signals are sent to linear fanouts. Copies of each signal 684 are sent to a multichannel flash analog to digital converter 685 (FADC) and to constant fraction timing discriminators. The 686 discriminator outputs are sent to both a scalar (S1) and an 687 AND logic module to generate a ΔE -E coincidence for that 688 detector arm. The ΔE detector signal has a faster rise time 689 than the E detector signal, so a delay was added to the ΔE 690 signal line to improve the timing jitter of the coincidence 691 signal. The ΔE -E coincidences for each of the four arms (L, 692 R, U, and D) are sent to two scalers, S1 and S2. The S1 scaler 693 counts only when the beam polarization is stable between 694 helicity reversals and is tagged by the sign of the polarization. 695 The S2 scaler is free running and counts whenever the DAQ 696 is running. 697

The four ΔE -E coincidence signals are combined in an OR logic module. Two outputs from this module are read out by scalers S1 and S2 to count the total number of events in the four detector arms. Another output triggers the DAQ event read out.

During run 1, we began with a conservative low discriminator threshold (-25 mV) for the *E* detectors. Detailed studies showed that we could raise these discriminator thresholds considerably, thus reducing the counting rate and dead time meaningfully, without impacting the results. This higher threshold (-100 mV) was used for the second half of run 1 and throughout run 2.

During run 2, a hardware time-of-flight veto was added 710 (see Fig. 8) to reduce the background events associated with 711 the beam dump. The timing veto signal with a width of 12 ns, 712 synchronized to the 31-MHz laser-rf signal, was adequate 713 to eliminate electrons backscattered from the dump from 714 reaching the scattering foil and subsequently scattering into 715 a detector arm. In this way, we eliminated this contribution to 716 the DAQ dead time and were thus able to increase the effective 717 event rate from the scattering foil. 718

We used a virtual machine environment (VME)-based data acquisition system. The VME crate contained the S1 and S2 scalars, the FADC and time to digital converter (TDC) modules, and a system trigger interface and distribution module. The helicity control board is located in



FIG. 9. A typical time-of-flight distribution of Mott coincidence triggers. Mott events from the scattering foil appear at \approx 54 ns, while electrons backscattered from the beam dump are detected at \approx 66 ns. Data from run 1 with the 355-nm foil.

a separate electrically isolated VME crate distant from the detector electronics and DAQ. A thorough description of the scintillation detectors, detector electronics, DAQ, and helicity control system is given in a JLAB technical note [42].

Measurement of the TOF distribution of coincidence 729 events was done using two channels of the TDC. The TDC 730 common start signal is generated by a Mott detector trigger. 731 One stop signal is generated by a suitably delayed Mott 732 detector trigger, and the other stop signal is from the 31-MHz 733 laser-rf signal which defines the beam pulse. The difference 734 between these two TDC channels generates the TOF distri-735 bution unaffected by any jitter in the generation of the TDC 736 common start signal. The TDC has a full scale of 134 ns, and 737 a resolution of 34 ps/channel. A typical TOF distribution is 738 shown in Fig. 9. 739

The standard deviation of the TOF distribution around the 740 elastic peak is 0.73 ns. The time for a speed of light particle 741 to move from the scattering foil to the dump is 6 ns, and 742 thus a dump peak is detected 12 ns after the elastic peak. 743 When applied, the timing veto removes events between 62 744 and 74 ns, which includes events associated with the beam 745 dump. The remaining events that occur in the TOF distribution 746 arise from electrons scattering from vacuum chamber surfaces 747 which reach the detectors out of time with the desired Mott 748 events. 749

The FADC is a 12-bit analog-to-digital converter (ADC) that samples at 250 MHz. Eight FADC input channels with 750 a -500-mV full range are used for the ΔE and E photomultiplier signals. For every Mott scattering event, 50 samples from each of the eight FADC channels, equally spaced in time, are read out. The first ten samples in the E signal occuring before the Mott event arrives are used to calculate an average 750



FIG. 10. Typical histograms of the (a) E and (b) ΔE detectors. The E threshold was set to -25 mV for these data and no timing cut was applied. Data from run 1 with the 355-nm foil.

pedestal of the FADC. A value proportional to the total energy deposited in the *E* detector is then calculated by summing the pedestal subtracted signal over the remaining 40 ADC samples. Examples of representative histograms are shown in Fig. 10. TOF histograms for each detector are also generated as in Fig. 9.

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VII. DATA REDUCTION

For each individual Mott measurement the DAO generates 764 a raw data file, which is decoded into a ROOT tree [43]. The 765 Mott analysis code consists of three loops which are executed 766 sequentially. In the first loop, the time-of-flight and energy 767 spectra are fit in order to isolate elastic events from the target 768 foil. In the second loop, these events are sorted by their 769 beam helicity to compute the experimental asymmetries and 770 determine helicity averaged rates. In the third loop, scaler 771 data are used to determine the integrated beam current charge 772 asymmetry and DAQ dead time. 773

A. Loop 1: Identifying Mott scattered coincidence events

The elastic peak of each detector's TOF spectrum is fit with 775 a Gaussian using the default ROOT TH1 class χ^2 least-squares 776 fitting routine that uses MINUIT and the MIGRAD minimizer. 777 The fit is restricted to the 49- to 55-ns range, shown as the 778 solid curve in Fig. 9. Note that in this figure the TOF veto has 779 not been applied, so events originating from the beam dump 780 are also present, centered at approximately 66 ns. From this fit, 781 the time window for Mott scattering events from the target foil 782 is taken to be -2σ to $+2\sigma$ of the mean Gaussian fit, shown as 783 the hatched area in Fig. 9. 784

The four *E* detector spectra, after applying the TOF cut, were then normalized to place the Mott peak of each detector in a standard channel—in our case channel 8000. This was done by linearly shrinking or expanding the raw spectra. In all cases this was a very small change, $\approx 4\%$ in the largest case. The results are shown in Fig. 11, showing that the four *E* detector spectra are quite similar. This normalization allows us to standardize the cuts to the energy spectra. 789 790 791 792 793

The four normalized and TOF-cut *E* detector spectra are then each fit with a Gaussian. The fit is restricted to ± 500 channels about the central peak bin. Again, the default ROOT TH1 fitter is used. A fit to a left detector energy spectrum is shown in Fig. 12. A "good" elastic scattering event has been determined to lie between -0.5σ and $+2\sigma$ as shown as the hatched area in Fig. 12.



FIG. 11. The superimposed normalized energy spectra of the four E detectors, after the timing cut was applied. Data from run 2 with the 355-nm foil.



FIG. 12. A Gaussian fit (solid line) to an energy spectrum of the E detector used to define the events used to calculate a Mott scattering asymmetry. Data from run 1 with the 355-nm foil.

Figure 13 shows a contour plot of energy versus time of flight for all Mott events from a particular single Mott measurement, with the energy and TOF cuts shown. The choice of both the TOF and energy cuts is explained in detail next.

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B. Loop 2: Computing helicity correlated asymmetries

Establishing the beam helicity and transmitting this information to the Pockels cell high-voltage driver and the Mott DAQ is done with the helicity control board. The helicity control board generates a 0.5-ms "T-settle" signal which indicates when the Pockels cell high voltage is changing between 809 states, followed by a 33.33-ms "T-stable" signal indicating 810 that the Pockels cell voltage, and thus the beam helicity, is 811 stable. Mott events are tagged as good when they occur during 812 the T-stable times. Beam helicities are generated in quartet 813 patterns of either + - - + or - + + -, with the quartet 814 pattern selected randomly. Each of the four entries in a guartet 815 pattern is composed of a single 0.5-ms T-settle time and a 816 single 33.33-ms T-stable time. 817

With final histograms for the *E* detectors and the TOF 818 spectra, we calculate the helicity correlated experimental 819 asymmetries using the cross-ratio method [44]. The cross-820 ratio method cancels to all orders the relative variations in 821 detector efficiencies and solid angles of the two detector arms, 822 and any variation in beam current that might exist between the 823 two helicity states. With "L+" and "R+" referring to the events 824 within specified TOF and energy cuts in a pair of opposing 825 detectors for positive incident beam polarization (L^+ and R^+), 826 and similarly for negative polarization (L^{-} and R^{-}), the cross 827 ratio r is 828

$$r = (L^{+}R^{-}/L^{-}R^{+})^{1/2}$$

and the quantity N is

$$N = (1/L^{+} + 1/L^{-} + 1/R^{+} + 1/R^{-})^{1/2}$$

The asymmetry is then given by

$$A = (1 - r)/(1 + r)$$

with a statistical uncertainty

$$dA = rN/(1+r)^2.$$

We conducted an extensive study of the effect of varying the energy and TOF cuts on the resulting asymmetry value. The asymmetry showed only a very small dependence on 834



FIG. 13. Energy-time coincidence plots show the distribution of events from a single Mott measurement in (a) run 1 with the 355-nm foil where no hardware TOF veto was applied and (b) run 2 with the 355-nm foil where this hardware veto was applied. In each case the dashed lines indicate the applied TOF and energy cuts that were used to select the events for the calculation of the asymmetry.

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the TOF cuts. TOF cuts larger than $\pm 2\sigma$ had essentially no 835 effect on the resulting asymmetry, while $\pm 1\sigma$ gave somewhat 836 smaller asymmetries (though well within $\pm 1\sigma$ of the maxi-837 mum asymmetry), likely due to the fact that the TOF peak 838 in each detector arm occurred at fractionally different TOF 839 bins due to small cable length differences. With the TOF cut 840 settled at $\pm 2\sigma$, we binned the TOF-cut asymmetries in 0.5σ 841 energy bins between -5σ and $+5\sigma$. The asymmetry within 842 each 0.5σ slice was calculated for the Padé (0,1) and Padé 843 (1,1) functions, described below. 844

The pulse height spectrum in the *E* detector spectra (of 845 Fig. 12) is not Gaussian over the full range of the peak. 846 This is primarily because there are mechanisms that generate 847 real or apparent energy loss, but none that generate energy 848 gain. So, for example, imperfect light collection from the 849 scintillator, bremsstrahlung, or Compton scattering leading 850 to undetected photon energy or electron (or positron) escape 851 from the scintillator may all contribute to peak broadening on 852 the low-energy side of the peak. While GEANT4 simulations of 853 the detector package were performed to validate these mech-854 855 anisms, we have not attempted to precisely model the full energy spectrum for the purpose of defining the analysis. In-856 stead extensive examination of the energy spectra with various 857 functional forms (e.g., Gaussian or Lorentzian) led to the use 858 of energy cuts between -0.5σ and $+2.0\sigma$. We further exam-859 ined these cuts by systematically shrinking or enlarging them 860 in 10% steps up to 30% and noting the effect these changes 861 had on the uncertainty in the asymmetry. In all cases, at the 862 statistical expense of eliminating events, our choice of cuts led 863 to the smallest uncertainty on the asymmetry, and did not bias 864 the scattering asymmetry. A systematic uncertainty of 0.1% is 865 assigned to the energy cut. Thorough details of the analysis 866 study are described in a JLAB —technical report [45]. 867

The cross-ratio method can also be used to check for any 868 instrumental asymmetries. For example, if r were instead 869 defined as $(L^+L^-/R^-R^+)^{1/2}$, then the asymmetry calculated 870 would indicate how different the right detector is from the 871 left detector (e.g., detector solid angle, detection efficiency, 872 or discriminator threshold). Alternatively if r were defined as 873 $(L^+R^+/L^-R^-)^{1/2}$ the calculated asymmetry would indicate a 874 difference in the beam between the two helicity states (e.g. 875 beam current or target thickness variation). The detector and 876 beam instrumental asymmetries for both run 1 and run 2 were 877 less than 1%, affirming the advantageous use of the cross-ratio 878 method to calculate the Mott asymmetries. 879

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C. Loop 3: Computing current dependencies

A fundamental frequency rf cavity (labeled BCM in Fig. 4) 881 was cross calibrated against a precision Faraday cup fur-882 ther down the primary beam line and used to noninvasively 883 monitor the beam current during each scattering asymmetry 884 measurement. The beam generated rf power from the cavity, 885 proportional to the square of the beam current, was processed 886 to provide a voltage signal proportional to the instantaneous 887 beam current. This signal could be cross calibrated against a 888 precision Faraday cup further down the primary beam line. 889 Conversion of this voltage signal to frequency provided a 890

pulse train that was counted to give the integrated beam current over the duration of a single Mott measurement.

As noted earlier, the cross-ratio method of calculating scattering asymmetries is insensitive to any variation in beam current that might be present between the two helicity states, a metric termed "charge asymmetry." However, as a precautionary measure, the charge asymmetry on the electron beam was minimized by fine tuning the Pockels cell voltages. Measured charge asymmetries were consistently small—typically below 10^{-3} —and are not used in further analysis.

Finally, we calculate the number of Mott triggers passing 901 the TOF and energy cuts for the four detector arms, inde-902 pendent of helicity, for each scattering foil, normalized to the 903 average beam current on the particular foil. These rates were 904 corrected for both electronic dead time and DAQ dead time 905 [42]. The average rate from the up and down detectors for 906 run 1 and from left and right detectors for run 2 was used 907 in the target thickness extrapolation. The details of the rate 908 calculations and uncertainties are given in Ref. [45]. 909

VIII. BEAM SYSTEMATICS

We have quantitatively examined a number of additional 911 effects that might, in principle, affect our measured asymme-912 tries. These include the reversal of the beam polarization ef-913 fected by inserting a properly oriented half-wave plate before 914 the Pockels cell, the temporal stability of the measured asym-915 metry during the target thickness extrapolation measurements 916 that occurred over roughly a day of data taking, the motion 917 of the beam spot on the target foil, variation in the beam spot 918 size at the target foil, variations in the beam energy or energy 919 spread, and the electronic dead time over the range of beam 920 currents used. 921

A. Asymmetry dependence on laser polarization and wave-plate reversal

In setting up the laser system for the polarized source, we measured the circular polarization of the optical beam after the Pockels cell both with and without the half-wave plate, and for both Pockels cell voltages. Each of these four measurements gave a circular polarization of greater than 99.8%. These polarization numbers are very stable over extended periods of time (months).

Data from each scattering foil were accumulated in an even 931 number of single Mott measurements of nominally equal inte-932 grated beam current—half with the insertable half-wave plate 933 in, and half with the plate out. In run 1, the weighted average 934 of the measured asymmetries with the wave plate out divided 935 by that for the wave plate in was 1.0022 ± 0.0020 , and in run 936 2 it was 1.0017 ± 0.0021 . The primary effect of the half-wave 937 plate is the reversal of the sense of circular polarization of 938 the light illuminating the photocathode, and thus the beam 939 polarization, while leaving all else nominally unchanged. The 940 insertable half-wave plate essentially allows the elimination of 941 any electronic pickup effect in the detector electronics associ-942 ated with the reversal of the Pockels cell high voltage. While 943 this is an important feature for parity-violation experiments 944 where very small asymmetries are measured, with our very 945



FIG. 14. Mott asymmetry vs radial displacement from the center of the target foil using the nominal (a) 1- μ m and (b) 0.225- μ m foils. The solid lines show the average value of all measured points, while the dotted region shows a +/- 1 σ band about this average.

large asymmetries, the use of the wave plate is not expected 946 to have any significant effect. We have made no correction to 947 our physics asymmetry results for any difference between the 948 wave plate out and wave plate in. In our data analysis, we treat 949 the wave plate in and out asymmetries equally (with the appro-950 priate sign). Overall, we estimate that the circular polarization 951 of the optical beam is 0.998 ± 0.001 . Since the Mott asym-952 metry is calculated using both helicities and any difference in 953 polarization does not cancel in the cross-ratio method, a sys-954 tematic uncertainty of 0.1% was assigned due to the different 955 laser polarization between the + and - helicity states. 956

B. Asymmetry dependence on beam position and beam size at the scattering foil

We measured the scattering asymmetry as a function 959 of beam position on the nominal 1- and 0.225- μ m target 960 foils during run 1. For each foil, we moved the beam to a 961 total of six noncentered locations, spanning a radial distance 962 of $\approx 1 \text{ mm}$ from the foil center. The image position was 963 verified by observing the beam spot with the OTR signal 964 from the foil. The details are described in a JLAB technical 965 note [46]. The results are consistent, within their statistical 966 uncertainties, with all measured points representing the same 967 value. The results are shown in Fig. 14. Realistically, any 968 beam motion on the target foil is much smaller than the 1-mm 969 displacements measured. This is the result of the high level 970 of stabilization of all active beam line elements (magnets 971



FIG. 15. Mott asymmetry vs beam size. The solid line shows the average value of all points with beam spot sizes no greater than 1.0-mm FWHM, while the dotted region shows a $+/-1\sigma$ band about this average.

and rf cavities). Magnet currents and rf cavity amplitudes and phases are all controlled to a high degree by feedback stabilized power sources. The actual beam motion measured in the beam line to the Mott polarimeter, using microwave beam position monitors, is about $50-\mu m$ rms and the most likely source of beam motion is the effect of small stray ac magnetic fields in the low-energy region of the injector. 972

We also measured the asymmetry as a function of beam spot size (see Fig. 15), finding it to be independent for beam sizes less than 1-mm full width at half maximum (FWHM). Given the measured insensitivity of the asymmetry to beam steering, this result is expected.

C. Asymmetry dependence on beam energy, energy spread, scattering angle, and acceptance

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The magnitude and stability of the beam energy and energy 986 spread are determined almost exclusively by the rf phases and 987 amplitudes of the two superconducting accelerating cavities. 988 The remainder of the injector energy is determined by the 989 130-keV electron gun voltage (stability of $\approx 1 \times 10^{-4}$) and 990 field strength of a normal conducting cavity, which provides 991 \approx 400 keV of energy gain. The amplitude and phase of the 992 fields in all the rf cavities are controlled with precision rf 993 control modules. For the superconducting cavities the cavity rf phase is controlled to less than 0.25° of the 1497-MHz 995 phase over periods of days, and the amplitude is held to within 996 0.00045 rms of the set value [47] over a similar time period. 997 These very tight tolerances assure that the beam energy and 998 energy spread are stable during operation. Typical results are 999 an energy spread of less than 4 keV in the 5-MeV region of 1000 the injector. 1001

As was pointed out earlier, the beam kinetic energies were 1002 4.806 ± 0.097 MeV for run 1, and 4.917 ± 0.013 MeV for 1003 run 2. The theoretical Sherman function at these energies (and 1004 a scattering angle of 172.6°, weighted by the Mott differential 1005 cross section and averaged over the 0.9° angular acceptance) 1006 is 0.514 ± 0.001 for both run 1 and run 2, resulting in a 1007 systematic uncertainty of 0.2%. The energy spread of the 1008 beam, other than being accounted for in the optical setup of 1009



FIG. 16. Mott asymmetry vs beam current. The solid line shows the average of all measured points, while the dotted region shows a $+/-1\sigma$ band about this average.

the beam spot size at the polarimeter target, is inconsequentialto the scattering asymmetry.

D. Asymmetry dependence on electronic dead time

During run 1, we explored the effect of electronic dead 1013 time on our asymmetry measurements at five different average 1014 beam currents ranging from 0.245 to 4.3 μ A incident on 1015 a 1- μ m foil—the thickest foil used in our measurements— 1016 with dead time varying from 3 to 43% over this current 1017 range. All our measurements of asymmetry and counting rate 1018 versus target thickness were done with beam currents well 1019 within this range. The results are shown in Fig. 16. The five 1020 measurements are all within their statistical uncertainty of 1021 representing the same average value, a confirmation of the 1022 fact that common electronic dead time does not affect the 1023 asymmetry calculated with the cross-ratio method. We have 1024 thus made no correction to our physics asymmetry results for 1025 an electronic dead-time effect and no systematic uncertainty 1026 was assigned. On the other hand, a small correction to the 1027 counting rates in each detector arm was made, arising from 1028 1029 the dead time associated with DAQ readout, as described in Ref. [45]. 1030



E. Dependence of asymmetry stability over time

During run 1 and run 2, we repeated the asymmetry mea-1032 surement of the 1- μ m foil after each target foil measurement, 1033 for a total of 42 measurements. In total, these measurements 1034 address the long term stability of the electron beam and 1035 Mott apparatus. The distribution of these repeated asymmetry 1036 measurements for run 1 and run 2 is shown in Fig. 17. Each 1037 measurement using the same $1-\mu m$ gold foil and a beam 1038 current of 1.0 μ A yields a statistical uncertainty of about 1039 0.21% in about 10 min. The rms width of these distributions is 1040 very close to the statistical accuracy of a single measurement. 1041 This shows that the relative contribution of the overall system-1042 atic uncertainty to any of the ten scattering asymmetries we 1043 measured for the target thickness extrapolation is negligible. 1044 It is notable that these measurements demonstrate the stability 1045 of both the electron beam and polarimeter over time scales 1046 longer than one day; specifically 26 h (run 1) and 27 h (run 2). 1047



FIG. 17. Distribution of asymmetry values of the stability runs. (a) Stability measurements during run 1 (both low and high discriminator thresholds). (b) Stability measurements during run 2. The rms width of the distribution is very close to the statistical uncertainty of the single Mott measurement.

In summary, the effects we have examined indicate that any systematic uncertainties in our measured asymmetry are contributing a total of 0.24% to the measured beam polarization.

IX. TARGET THICKNESS EXTRAPOLATION

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The Sherman function S is calculated for single elastic scat-1052 tering from the nucleus, with corrections for the small effects 1053 due to the atomic electrons, as described in Sec. III. This value 1054 of S applies to an experiment with a zero-thickness target foil. 1055 To assign a beam polarization, an effective analyzing power 1056 that depends on target foil thickness must be determined from 1057 scattering asymmetries A(t) measured for a range of target 1058 thicknesses t. The A(t) measurements extrapolated to zero 1059 thickness A(0) are used to assign the beam polarization by the 1060 relationship $P = A(0)/S(0) \equiv A(0)/S$. Once the value of the 1061 beam polarization is known the analyzing power of any foil, 1062 known as the effective Sherman function, may be determined 1063 from S(t) = A(t)/P. 1064

Gold foils over a broad range of nominal thickness from 50 to 1000 nm were purchased from the Lebow Corporation. We independently measured the thickness of each foil using field emission scanning electron microscopy (FESEM). Although we refer to the foils by their respective nominal thickness as 1069



FIG. 18. Fits to the measured asymmetry vs foil thickness for run 1 (a) and run 2 (b), for the allowed Padé functions PA(01), PA(11), and PA(20), and for asymmetry vs relative rate for run 1 (c) and run 2 (d) and allowed Padé functions PA(11) and PA(02).

determined by the vendor, we use in our experimental analysis 1070 and theoretical models the values we determined by the FE-1071 SEM method (summarized in the Appendix). The measured 1072 foil thicknesses are generally within 5% of those reported 1073 by the manufacturer. Measuring the scattering asymmetry 1074 for each foil thickness to high statistical precision (less than 1075 0.25%) required from less than 1 h using the thickest foil to 1076 many hours for the thinnest. Only the statistical uncertainties 1077 of the measured asymmetries were included in the target 1078 thickness extrapolation. We could have included any of the 1079 relative systematic uncertainties but these were consistent 1080 with zero. The way we took the data on the different foils en-1081 sured no changes to the beam or polarimeter and the stability 1082 measurements taken during run 1 and run 2 show no relative 1083 systematic uncertainties (within the statistical precision). 1084

Historically, and at lower energies less than 1 MeV (and 1085 typically 100-200 keV) where multiple and plural scatterings 1086 are more significant, the target thickness extrapolation has 1087 been performed by choosing one of a variety of empirical 1088 or model driven functional forms which lead to systematic 1089 uncertainties at the 1% level [9,48,49]. At higher energies, as 1090 is the case of this polarimeter, it is reasonable to assume that 1091 single and double scattering account essentially for all of the 1092

measured scattering asymmetry as the cross section falls as 1093 the energy is increased greater than 1 MeV. 1094

The dependence of the analyzing power on the single and double scattering will affect the rate at which the scattering asymmetry falls with increasing target thickness. For example, in the case where there is no polarization dependence in the second scattering the asymmetry as a function of target thickness is of the form 1009

$$A(t) = A(0)/(1 + \beta t).$$

If instead the second scattering also contributes an (albeit 1101 small) polarization dependence, the asymmetry as a function 0f target thickness becomes 1103

$$A(t) = A(0)[(1 + \alpha t)/(1 + \beta t)].$$

In this paper, rather than limiting the possible functions to those expected, we have systematized the A(t) fitting procedure using the method of Padé approximants to determine those rational functions which best describe the data [50].

A Padé approximant is the quotient of two power series, 1108 which in our case are 1109

$$A = A(0) \frac{(1 + a_1t + a_2t^2 + a_3t^3 + \dots + a_mt^m)}{1 + b_1t + b_2t^2 + b_3t^3 + \dots + b_nt^n}.$$

	PA(mn)	a_0	a_1	a_2	b_1	b_2	Reduced $\chi 2$	
Run 1 $A(t)$	PA(01)	44.06(10)			0.31(01)		1.2	
	PA(20)	44.08(13)	-13.8(1.0)	3.5(1.2)			1.4	
	PA(11)	44.12(14)	3.8(5.7)		0.41(16)		1.29	
Run 2 $A(t)$	PA(01)	44.06(11)			0.31(01)		1.19	
	PA(20)	44.10(14)	-14.0(1.0)	3.9(1.2)			1.35	
	PA(11)	44.16(15)	5.7(5.9)		0.47(16)		1.23	
$\operatorname{Run} 1 A(R)$	PA(11)	44.09(11)	0.10(02)		$4.54(47) \times 10^{-3}$		1.34	
	PA(02)	44.03(11)			$2.14(08) \times 10^{-3}$	$-3.03(47) \times 10^{-6}$	1.61	
$\operatorname{Run} 2A(R)$	PA(11)	44.14(13)	0.12(02)		$5.03(55) \times 10^{-3}$		1.38	
	PA(02)	44.07(13)			$2.26(10) \times 10^{-3}$	$-3.48(53) \times 10^{-6}$	1.69	

TABLE I. Fit parameters for zero foil thickness extrapolations vs either thickness or rate, including reduced χ^2 values.

In our analysis, we examined Padé approximants with m1110 ranging from 1 to 3 and *n* ranging from 0 to 2, increasing 1111 the order of the fitting function until a statistical F test [51] 1112 indicates that larger values of m and/or n are not justified. 1113 The "F test" measures the impact of including additional 1114 higher-order Padé terms on the χ^2 value of the resulting fit. 1115 All fits that passed the F test were then judged by their reduced 1116 χ^2 . Reduced χ^2 values larger than 2 indicate a less than 2% 1117 likelihood of accurately representing the data, and lead to the 1118 rejection of the associated PA(m,n). 1119

¹¹²⁰ Plots showing the allowed Padé solutions of both A(t) for ¹¹²¹ the two experimental runs are shown in Figs. 18(a) and 18(b), ¹¹²² followed by a table giving the Padé function parameters and ¹¹²³ the reduced χ^2 values for the fits to the data.

Alternatively, one can also consider the measured asym-1124 metry A as a function of the relative rate, R, averaged from 1125 both detectors, corrected for dead time, and normalized to 1126 the measured beam current [52]. The advantage of doing this 1127 is that the number of counts is very large, and thus should 1128 generally lead to fits with smaller statistical uncertainty. The 1129 total uncertainty on the relative rate was about 2% and is 1130 a combination of the statistical uncertainty and systematic 1131 uncertainties due to the beam current measurement and dead-1132 time correction. 1133

Plots showing the allowed Padé solutions of both A(R) for the two experimental runs are shown in Figs. 18(c) and 18(d) and the fit results are shown in Table I. The values for A(R)at R = 0 and A(t) at t = 0 are essentially equal within a small fraction of their fit uncertainty. Use of Padé approximants, the *F* test, and the reduced χ^2 ¹¹³⁹ test indicates the best fits in both runs are to the *A*(*t*) data and ¹¹⁴⁰ by the function PA(01). It is noteworthy that PA(01) and the ¹¹⁴¹ next best fit PA(11) are the two functions described above that ¹¹⁴² reflect the expected contributions of both single and double ¹¹⁴³ scattering in the measured scattering asymmetry. ¹¹⁴⁴

The A(0) results of all of the successful Padé functions 1145 presented in Table I are graphically represented in Fig. 19. 1146 That all are in good agreement to well within 1 σ demonstrates 1147 the challenge that remains to a priori analytically forecast 1148 the only correct function. However, based on the statistical 1149 analysis discussed above we can argue the best fit to our data 1150 is the A(t) function PA(01), giving 44.06(10) for run 1 and 1151 44.06(11) for run 2, and a corresponding relative uncertainty 1152 of 0.25% in the determination of A(0). 1153

From an examination of the fits to the four groups of data 1154 listed in Fig. 18, the data points for the 482- and 215-nm target 1155 foils are the largest outliers from the fit. We thus examined 1156 fits to the normalized counting rate versus the foil thickness to 1157 check for anomalies. These fits were forced through R(0) = 01158 at t = 0, and can be compared to the GEANT4 simulations dis-1159 cussed in the next section. The data for rate versus thickness 1160 are plotted in Fig. 20, with coefficients in Table II; the fits are 1161 very good, and no anomalies are apparent. 1162

Finally, using the values of A(0) and dA(0) determined 1163 from run 1 and run 2, divided by the Sherman function of 1164 0.514 calculated in Sec. VIII, gives beam polarizations of 1165 85.72 \pm 0.19% for run 1 and 85.72 \pm 0.21% for run 2. It is 1166 interesting to note that these very similar results are from two 1167







FIG. 20. Normalized counting rate vs foil thickness for (a) run 1 and (b) run 2.

different photocathodes cut from a single wafer, indicating the
 excellent uniformity of the growth of this complex semicon ductor structure.

1171 X. GEANT4 SIMULATION OF THE POLARIMETER

A GEANT4 [53] model of the polarimeter was constructed 1172 to simulate the scattering rate and asymmetry as a function 1173 of target thickness, motivated by Wegener's argument [23] 1174 which concludes that single and double scattering essentially 1175 account for all of the observed dependence of the analyzing 1176 power on target thickness. Further, we anticipate that our data 1177 can be well simulated with this model, which is strongly 1178 supported both by our Padé approximant analysis of our 1179 asymmetry measurements and for our energy range by the 1180 results of the Mainz experiment [15]. 1181

Clearly, single scattering will have a counting rate pro-1182 portional to the scattering foil thickness, and an asymmetry 1183 independent of the foil thickness, while the double-scattering 1184 rate will depend on the square of the foil thickness, and 1185 also have an asymmetry independent of the foil thickness. 1186 Thus we can write the scattering rate into the left (L) and 1187 right (R) detectors (or up and down detectors) for a beam of 1188 polarization $\mathbf{P} = P_0 \mathbf{y}$ as a function of the foil thickness *t*: 1189

$$R_{L1} = a_1^{sim} t(1 + P\varepsilon_1), \quad R_{R1} = a_1^{sim} t(1 - P\varepsilon_1),$$

$$R_{L2} = a_2^{sim} t^2 (1 + P\varepsilon_2), \quad \text{and} \ R_{R2} = a_2^{sim} t^2 (1 - P\varepsilon_2),$$

where the subscripts 1 (2) refer to single (double) scattering, and the *a* and ε parameters are the simulated scattering rates and analyzing power coefficients for the two processes. Using the common definitions for the measured scattering rate and 1193 asymmetry, one finds 1194

and

$$A^{\text{sim}} = ([R_{\text{L}1} - R_{\text{R}1}] + [R_{\text{L}2} - R_{\text{R}2}]) / ([R_{\text{L}1} + R_{\text{R}1} + R_{\text{L}2} + R_{\text{R}2}]).$$

 $R_{\text{tot}}^{\text{sim}} = 1/2[R_{\text{L1}} + R_{\text{R1}} + R_{\text{L2}} + R_{\text{R2}}],$

These lead to expressions for the predicted scattering rate 1196 and asymmetry: 1197

$$R^{\text{pred}}(t) = a_1 t + a_2 t^2$$

and

$$A^{\text{pred}}(t) = P(a_1\varepsilon_1 + a_2\varepsilon_2 t)/(a_1 + a_2 t)$$

Using only quantities derived from our simulations, the 1199 predicted effective Sherman function is 1200

$$S^{\text{pred}}(t) = (a_1\varepsilon_1 + a_2\varepsilon_2 t)/(a_1 + a_2 t).$$

In our simulations of the polarimeter, the relevant geometry 1201 and material properties of the detector package and scattering 1202 chamber were used. The theoretically calculated values of 1203 the cross section $(d\sigma/d\Omega)$, Sherman function (S), and spin-1204 transfer functions (T, U) as defined in Ref. [10] were also 1205 used. Aside from these terms, the GEANT4 electromagnetic 1206 physics package was used. The initial electron distribution 1207 was defined with momenta in the longitudinal direction ($\mathbf{p} =$ 1208 $p_0 \mathbf{z}$), and polarization in the vertical direction ($\mathbf{P} = P_0 \mathbf{y}$). The 1209 electron beam at the target foil was defined as a transverse 1210

TABLE II. Rate vs thic	ckness fits for runs 1 an	d 2.
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		a_0	a_1	a_2	b_1	Reduced $\chi 2$
Run 1 R(t)	PA(11)	0	143.42(3.62)		-0.27(0.04)	0.39
	PA(20)	0	141.37(4.57)	51.42(8.76)		0.34
Run 2 R(t)	PA(11)	0	138.70(4.27)		-0.26(0.04)	0.50
	PA(20)	0	136.91(5.24)	47.54(9.98)		0.55

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Gaussian of 1-mm FWHM diameter and with a mean energy 1211 of 4.9 MeV and Gaussian energy spread of 150-keV FWHM. 1212 Although the measured values of the energy spread ($\approx 4 \text{ keV}$) 1213 are considerably smaller, we chose larger and more conser-1214 vative values in the simulation. Experimentally, we find both 1215 the measured scattering rates and calculated asymmetries to 1216 be insensitive to values less than those used in the GEANT4 1217 simulation. 1218

We used the method of rejection sampling [54] to determine the values for the asymmetries ε_1 and ε_2 from single and double scattering. In the single-scattering case, we used the following algorithm.

- (1) Choose a scattering position x_1 within the intersection of the beam and our scattering foil.
- (2) Choose a point \mathbf{x}_2 within the acceptance of the primary collimator.
- (3) Calculate $d\sigma/d\Omega(\mathbf{x_1}, \mathbf{x_2})$.

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(4) Rejection sample this value of $(\mathbf{x}_1, \mathbf{x}_2)$ against the calculated cross section. If accepted generate the event. If rejected, repeat the first three steps.

Implementing these steps in the simulation, the singlescattering parameter is

$$\varepsilon_1 = (N_{\rm L1} - N_{\rm R1})/(N_{\rm L1} + N_{\rm R1}) = -0.513 \pm 0.001,$$

in excellent agreement with the theoretical value of the singleatom scattering asymmetry of -0.514 ± 0.003 described in Sec. III, which provides important validation of the simulation algorithm.

¹²³⁷ For the case of double scattering, we used the following algorithm.

- (1) Choose a scattering position $\mathbf{x_1}$ within the intersection of the beam and our scattering foil.
 - (2) Choose a point \mathbf{x}_2 within the foil such that $|\mathbf{x}_2 \mathbf{x}_1| < 0.16$ mm. Beyond this distance an electron would have lost sufficient energy to fall outside of our cuts.
 - (3) Calculate $d\sigma_1/d\Omega_1(\mathbf{x_1}, \mathbf{x_2})$.
- - (5) Calculate $d\sigma_2/d\Omega_2(\mathbf{x_2}, \mathbf{x_3})$.
- 1248 (6) Rejection sample this value against 1249 $(d\sigma_1/d\Omega_1)(d\sigma_2/d\Omega_2)$. If accepted generate an 1250 electron at \mathbf{x}_2 towards \mathbf{x}_3 . If rejected repeat the 1251 first five steps.

Simulation of 10⁷ events at each foil thickness produces an asymmetry of

$$\varepsilon_2 = (N_{L2} - N_{R2})/(N_{L2} + N_{R2}) = -0.011 \pm 0.003.$$

The double-scattering simulation results for one detector are shown in Fig. 21. As anticipated, these results clearly show that the first scattering is in or exceptionally close to the plane of the foil, while the second scattering shows significant peaks at $90^{\circ} \pm 7.4^{\circ}$ to produce the required total scattering angle of 17259 172.6° for electrons to arrive at the detectors.

The rate coefficient for single scattering into the four detector channels was computed by a numerical integration over the initial parent phase space (x, y, z, E, θ , ϕ) without regard to the electron polarization. The result for the total 1263 single-scattering rate coefficient is 1264

$$a_1^{\text{sum}} = 198 \pm 1 \,\text{Hz}/(\mu \text{A} - \mu \text{m}).$$

Such an integration cannot be used to calculate the double-1265 scattering rate coefficient, as the phase space is significantly 1266 more complex, and the integration must be performed over 1267 more dimensions. Instead, a numerical Monte Carlo estimator 1268 was used to uniformly sample and integrate from the phase 1269 space of double-scattering events originating from the target 1270 foil and reaching the detector acceptance. The distance be-1271 tween the first and second scattering in the foil was restricted 1272 to be less than 160 μ m, corresponding to the distance in 1273 which an electron would lose 500 keV and thus fall outside 1274 the energy cuts we used. In practice, this cut did not have 1275 a significant impact on the result. Our result for the total 1276 double-scattering rate coefficient is 1277

$$a_2^{\text{sim}} = 62 \pm 15 \,\text{Hz}/(\mu \text{A} - \mu \text{m}^2).$$

With simulation results in hand for both the single- and 1278 double-scattering rates and asymmetries, we can compare with actual data, shown in Figs. 22(a)-22(d). In order to make 1280 a comparison between GEANT4 simulations with experimental 1281 results it is necessary to relieve the stringent energy cuts that 1282 are applied in the experimental data reduction (see Figs. 11 1283 and 12) which throw out some fraction of good events. While 1284 less important for the computed asymmetry, this is especially 1285 necessary when comparing the calculated simulation rate with 1286 a corresponding experimentally measured rate. 1287

The simulated asymmetries, which are insensitive to these 1288 details, are in quite good agreement with the measured asym-1289 metries. Although there is some variation between the simu-1290 lated and experimental counting rates, these very likely arise 1291 from our estimation of the total counting rate, with large 1292 uncertainty in the estimation of a background subtraction 1293 which was made of the *E* detector spectrum corresponding to 1294 otherwise good simulated events that were well outside of the 1295 stringent energy cuts that we applied in Secs. VI-IX. Overall, 1296 we conclude that the GEANT4 simulation of the polarimeter 1297 gives quite a good description of its performance. 1298

XI. CONCLUSIONS AND FUTURE PLANS

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The primary conclusion from our measurements and analysis is that electron polarimetry based on Mott scattering in the few-MeV range has reached a level we believe is well below 1% uncertainty.

Our polarimeter design is optimized to isolate electrons 1304 which only scatter from the target foil. The use of a coinci-1305 dence ΔE -E detector and measurement of both the energy and 1306 timing of the scattered electrons allows for careful isolation 1307 of elastic events that carry the full asymmetry of the analyzed 1308 beam. The use of the super-ratio method makes the computed 1309 asymmetry insensitive to beam intensity and detector solid 1310 angles. Systematic studies of the DAQ and of dependence 1311 on the meaningful beam properties demonstrate these effects 1312 contribute less than 0.24% to the measured asymmetry. 1313

The target thickness extrapolation, a questionable uncertainty owing to the challenges associated with knowledge of



FIG. 21. The results for the simulation of double-scattering events into one detector arm show the (a) cross section and (b) scattering angle of the first scattering within the plane of the target foil and the (c) cross section and (d) scattering angle for the second scattering towards the Mott detector. The phi angle was also simulated, but does not alter the simulation results above.

the physical dependence, has been especially well charac-1316 terized in this paper. Extensive measurements and statistical 1317 analysis have demonstrated knowledge of the zero-thickness 1318 foil analyzing power with a precision of $\approx 0.25\%$. While 1319 the calculation of the theoretical Sherman function remains 1320 the large contribution to the absolute uncertainty the modern 1321 calculations presented here predict this value convincingly 1322 at a level of $\approx 0.5\%$. Consequently, we have demonstrated 1323 the capability to measure the electron polarization at a beam 1324 energy with a total uncertainty $\approx 0.6\%$ (see Table III). 1325

The statistical precision of measured scattering asymme-1326 try for each target foil was about 0.25% and can easily be 1327 improved beyond this level in practical periods of time or by 1328 operation at higher beam current. A GEANT4 model was de-1329 veloped that predicts the dependence of the analyzing power 1330 on target foil thickness in good agreement with our measure-1331 ments. While the fact that the leading-order OED corrections 1332 and the real bremsstrahlung correction are not fully calculated 1333

at MeV energies is displeasing, it is also true that these small 1334 corrections all show a significant dependence on both energy 1335 and Z, allowing meaningful bounds on the total contribution 1336 of these terms to be experimentally established. The vacuum 1337 polarization and real bremsstrahlung corrections appear to be 1338 of the same sign, and there is good reason to believe that the 1339 self-energy correction is of the opposite sign, thus offering 1340 some degree of cancellation. Two previous measurements, 1341 each covering a significant and different range in energy and 1342 using very different polarimeters, have made no correction for 1343 the sum of these effects, and yet showed agreement well below 1344 the 1% level over the energy ranges measured. We have made 1345 initial asymmetry measurements at three different Z values 1346 (29, 47, and 79) and at energies between 2.75 and 8.2 MeV. 1347 These measurements can be done with much greater precision, 1348 and a serious study of the Z and energy dependence of the 1349 analyzing power should readily yield meaningful limits on 1350 the total of these small corrections. It would also be useful to 1351



FIG. 22. Measured counting rates compared to the GEANT4 simulation for (a) run 1 (U/D) and (b) run 2 (L/R), and the measured asymmetry for (c) run 1 and (d) run 2, all vs FESEM measured foil thickness.

extend the bremsstrahlung calculations of Johnson *et al.* [33]to our kinematic region.

As strong and as well supported as the above statements 1354 are, no one would accept the precision claimed for this Mott 1355 polarimeter without clear and independent corroboration. This 1356 may be had by comparing measurements of the electron 1357 polarization obtained by independent polarimetry techniques. 1358 In 2000, this was first done at the $\approx 2\%$ level for the five pol-1359 arimeters at Jefferson Lab [25]. These measurements are made 1360 possible by the fact that polarization placed in the horizontal 1361

TABLE III. Uncertainty budget for the 5 MeV Mott polarimeter.

Contribution to the total uncertainty	Value	
Theoretical Sherman function	0.50%	
Target thickness extrapolation	0.25%	
Systematic uncertainties	0.24%	
Energy cut (0.10%)		
Laser polarization (0.10%)		
Scattering angle and beam energy (0.20%)		
Total	0.61%	

plane in the injector remains in the horizontal plane after pas-1362 sage through the full multipass CEBAF accelerator permitting 1363 measurement of the beam polarization at both the injection 1364 energy and high energy by simply sweeping the polarization 1365 through 180° in the horizontal plane at the injector. These 1366 first measurements clearly demonstrated that the claimed 1367 analyzing powers for the various polarimeters were not in 1368 agreement at the 1-2% level, and that one polarimeter was 1369 in more serious disagreement. Since that time all the high-1370 energy polarimeters have been upgraded [48,55–58] and have 1371 improved their systematic and statistical precision. Moreover, 1372 an additional Compton polarimeter has been installed in hall 1373 C. Thus we are at a point where another multipolarimeter 1374 comparison, at the 0.5% or better level, is warranted. This 1375 statement is supported by a recent review of precision electron 1376 polarimetry which demonstrates that $M\phi$ ller polarimeters now 1377 reach precisions of 0.8 to 0.9% while Compton polarimeters 1378 reach $\approx 0.6\%$ uncertainties at few-GeV energies [48]. 1379

A different approach involves making an absolute measurement of the electron-beam polarization from the same photocathode and laser illumination system used for the Mott polarization measurements. This is made possible by the fact that the photocathode used in the CEBAF injector can be re-1380

moved from the electron gun and transported under ultrahigh 1385 vacuum to an optical polarimeter, AESOP (Accurate Electron 1386 Spin Optical Polarimeter). The AESOP method is being de-1387 veloped at the University of Nebraska in collaboration with 1388 Jefferson Lab [59,60]. In this method, a polarized electron 1389 excites a noble gas atom to a triplet state by spin exchange. 1390 The polarization of the light emitted along the axis of the 1391 initial electron-spin polarization in the decay of the atom to 1392 a lower triplet level is observed. The spin orientation of the 1393 incident electron results in the partial circular polarization of 1394 the decay photon through spin-orbit coupling in the excited 1395 atomic state. If the excited atomic state is well L-S coupled, 1396 the circular polarization can be directly related to the electron 1397 polarization without the need for dynamical calculations. The 1398 measurement of the Stokes parameters of the decay radiation 1399 thus provides an absolute calibration standard for electron 1400 polarization. While this method relies on a high-precision 1401 measurement of the Stokes parameters of the decay photon, 1402 it appears possible to make an electron polarization measure-1403 ment with an absolute precision of $\approx 0.5\%$ by this technique. 1404

There are changes to the present polarimeter that could de-1405 liver meaningful reductions in the uncertainty of the measured 1406 polarization. Redesigning the detector package for better light 1407 collection could significantly reduce the width of the elastic 1408 peak in the energy spectrum, resulting in smaller and less un-1409 certain cuts in separating the elastic-scattering events from the 1410 lower-energy background. It is worth considering replacement 1411 of the plastic scintillator of the E detector with a higher-quality 1412 crystalline scintillator. A higher-density scintillator would 1413 allow a reduction in its physical size, which would give some 1414 reduction in E detector backgrounds, but simulations to vali-1415 date this idea are necessary. It is also practical to reduce the 1416 transverse size of the ΔE detector, and possibly its thickness, 1417 again reducing background counts. These changes would also 1418 prove helpful in operation with higher beam currents. 1419

Operation at higher beam currents would be useful in 1420 obtaining even-higher-precision polarization measurements. 1421 The introduction of a beryllium beam dump was helpful to 1422 reduce background counting rates. The use of a relatively 1423 thin Be liner inside the beam pipe leading to the beam dump 1424 might be prohibitively costly, but would likely result in further 1425 background reduction. A portion of the beam dump tube clos-1426 est to the dump plate could be enlarged, and the dump plate 1427 could be moved further from the detectors. These changes can 1428 be explored in simulations before implementing them, but it 1429 seems clear that improvements are practical. 1430

Changing the length of the beam dump tube would also be 1431 beneficial. We made an unfortunate choice of the length of the 1432 beam dump tube, as it places the dump plate 12 ns away from 1433 the scattering foil, while the beam pulse repetition rate is an 1434 integer multiple of 2 ns. The concern is that electrons origi-1435 nating from an earlier 499-MHz beam pulse reflecting from 1436 the dump plate may arrive at the target foil in time with a new 1437 beam pulse reaching the target foil. However, in analyzing our 1438 data collected with a 31.1875-MHz beam pulse repetition rate, 1439 meaning a pulse spacing longer than the dump plate spacing, 1440 we observed that the addition of TOF analysis made only 1441 a small improvement, and only in the case of the thinnest 1442 target foils. This is because with all but the thinnest of foils 1443

the background events that arrive at the target foil are likely to have had inelastic energy losses from multiple surfaces and be removed through energy cuts. Thus, this polarimeter calibration, despite the fact that it was done at 32 MHz, is still applicable to 499 MHz when the 1- μ m foil is used. With a nominal electron-beam kinetic energy of 4.9 MeV, the effective Sherman function for the 1- μ m foil is 0.3921.

Moving the dump plate an additional 15 cm further from 1451 the scattering foil would not totally eliminate this background, 1452 but would very substantially reduce it. Some combination of 1453 enlarging the beam dump tube for a fraction of its length 1454 closest to the dump plate (with the addition of appropriate 1455 shielding), moving the dump plate an odd integer multiple of 1456 15 cm further away from the scattering foil, and to the extent 1457 feasible adding Be liners to the dump tube would give a very 1458 significant reduction in the backgrounds. 1459

Finally, some small additional improvements may come 1460 from improved knowledge of the thickness of the differ-1461 ent scattering foils. The foils for the Mainz polarimeter re-1462 ported in Ref. [24] were measured by α scattering, with a 1463 claimed precision of $\approx 3\%$. This is considerably better than 1464 the foil thickness uncertainty we measured by the FESEM 1465 technique ($\approx 5\%$), which in turn was a real improvement on 1466 the original thickness uncertainty quoted by the manufacturer 1467 of $\approx 10\%$. 1468

In conclusion, we have been able to demonstrate a mean-1469 ingful improvement in the uncertainty of the electron-beam 1470 polarization measured by few-MeV Mott scattering, to an 1471 accuracy $\approx 0.6\%$. The dominant uncertainty in our result 1472 arises from the imperfect knowledge of the Sherman func-1473 tion. The uncertainty in the Sherman function calculations 1474 and the uncertainty in the experimental measurement of the 1475 asymmetry and the extrapolation to zero thickness provide 1476 similar contributions to the accuracy of the measurement. 1477 The higher-order effects in the Sherman function calcula-1478 tion can be constrained. We have the capability to reduce 1479 this uncertainty significantly by measurements over a range 1480 of energies and from different Z scattering foils, and the 1481 experimental uncertainty can be improved with increased statistics and improved target thickness characterization. We 1483 believe that an overall uncertainty of electron polarization 1484 measurement below 0.5% will prove practical in the future. 1485 The possibility of making a separate, absolute measurement 1486 of the polarization from the photocathode and laser system 1487 used for beam generation with an AESOP polarimeter is being 1488 pursued. Moderate improvements to the scattering chamber 1489 downstream of the scattering foil will likely allow precision 1490 polarization measurement at much increased beam current. 1491 And, lastly, a new and improved precision comparison of 1492 the polarization measured by all the various polarimeters at 1493 Jefferson Lab seems important to the ultimate goal of demon-1494 strating polarization measurement at or below the 0.5% level 1495 accuracy desired by the next generation of parity-violation 1496 measurements. 1497

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1498

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FIG. 23. FESEM image of a nominal 625-nm-thick gold foil mounted on a silicon substrate (lower dark region). Many measurements of the thickness along this region indicate the foil is 561.2 ± 31.0 nm. Yellow lines with black labels denote the lines used for thickness analysis with ImageJ software [62].

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APPENDIX: THICKNESS MEASUREMENTS OF THE GOLD TARGET FOILS

Ten freestanding gold foils of varied thicknesses were mounted on the target ladder and used for this experiment. The 50-nm foils were constructed using a $50-\mu$ m Kapton sheet with a 10-mm-diameter aperture that supported the freestanding gold target foil. The other foils were all freestanding 1521 gold over a 25.4-mm-diameter circular aperture. All foils were 1522 manufactured by the Lebow Corporation from 99.99% pure 1523 gold. While Lebow does not measure the absolute thickness 1524 of the foils as delivered, they are guaranteed to be within 10% 1525 of the specified thickness, and uniform to 2% over the active 1526 area of the foil. Foils of a given thickness manufactured in 1527 a single batch (called "siblings") are guaranteed to have the 1528 same thickness to within 5%.

To obtain more accurate foil thickness values, we con-1530 ducted a series of measurements using FESEM [61]. The very 1531 high brightness of a field emission electron source makes 1532 it possible to obtain images with nanometer level precision. 1533 We used a Hitachi s-4700 FESEM at 15 kV. Magnifications 1534 between 10 000 and 150 000 were used depending on the 1535 foil thickness being measured. For the measurements, sibling 1536 foils of those used for the Mott measurements were mounted 1537 on a silicon substrate which was subsequently cleaved to 1538 expose a cross section of the foil. Although we believe this 1539 foil preparation process does not meaningfully change the 1540 apparent foil thickness at the location of the cleavage, we have 1541 not conducted detailed studies to verify this. A typical FESEM 1542 picture showing a gold foil on a silicon substrate is shown in 1543 Fig. 23. 1544

The determination of the foil thickness from the FESEM 1545 pictures was done with ImageJ software [62]. Generally, FE-1546 SEM images were made at a single location for each sample. 1547 The random uncertainty in the measurements was determined 1548 by measuring a number of different images of the same foil 1549 at the same position. Since these measurements should be 1550 identical, the variation is a good measure of the statistical 1551 uncertainty in the technique. 1552

Systematic uncertainties arise from the inherent resolution 1553 of the FESEM, from the variation in measured thickness 1554 in multiple analyses of the same image, and from the 5% 1555 possible variation between the sibling foil measured and the 1556 actual foil used in the Mott measurement. The largest of 1557 these is the uniformity of sibling foils; since the thickness 1558 measurement is a destructive testing technique and we cannot 1559 measure the samples on the target ladder, this sibling uncer-1560 tainty dominates the overall uncertainty for all but the 50-nm 1561 (thinnest) foil. The vendor and FESEM thickness and total 1562 uncertainty for each foils followed by the statistical image 1563 analysis uncertainty and the three contributions to systematic 1564 uncertainty are shown in Table IV. 1565

TABLE IV. Summary of purchased target foils and their FESEM measured thicknesses and corresponding uncertainty.

Lebow thickness (nm)	1000	870	750	625	500	355	225	50
FESEM thickness (nm)	943.7	836.8	774.5	561.2	482.0	389.4	215.2	52.0
FESEM uncertainty (nm)	59.8	44.2	41.9	31.0	27.7	22.1	11.7	4.7
Image analysis (nm)	29.0	7.1	9.1	8.0	9.7	4.5	1.9	2.3
FESEM resolution (nm)	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
Same image reanalysis (nm)	22.6	12.4	13.3	10.2	9.7	9.2	3.8	2.9
Lebow sibling 5% (nm)	47.2	41.8	38.7	28.2	24.1	19.5	10.8	2.6

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