**Basic cavity formulas**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *r/Q* | Geometric Shunt Impedance | Ω/m |  | *T* | Operational Temperature | K |
| *G* | Geometry Factor | Ω |  | *Rresid* | Residual Surface Resistance | Ω |
| *E* | Electric Field | V/m |  | *Q0* | Intrinsic Quality Factor |  |
| *L* | Electrical Length | M |  | *QFPC* | FPC coupling Factor |  |
| *ω0* | Cavity Frequency | S-1 |  | *QFP, Q2* | Field Probe Coupling Factor |  |
| *U* | Stored Energy | J |  | *RC* | Coupling Impedance | Ω |
| *rS* | Surface Resistance | Ω |  | *I* | Beam Current | A |
| *TC* | Critical Temperature | K |  | *IM* | Matching Current | A |
| *PX* | RF Power at Port X | W |  | *Pdisp* | Dissipated Power | W |
| *Pemit* | Emitted Power | W |  | *τ* | Decay Time | s |
| *R* | Shunt Impedance | Ω |  | *r* | Shunt Impedance per Unit L | Ω/m |





For a cavity with detune errors due to incorrect TDOFF and dead band and beam on crest.

Where is the time varying detune due to microphonics, is the sum of is the error in TDOFF in degrees, and is the tuner algorithm dead-band in degrees.

Klystron phase no beam.



Klystron phase with beam and a DC detune frequency.

On crest beam

where

Normally for all of the other ports except for port 1, which leads to:

Because and

Again all except for port 1, and this reduces to:

Or

**Optimized Loaded Q**

To determine the following start with.

And solve the following:

Matched loaded Q under all conditions.

Where

Matched Loaded-Q for accelerator operated on crest.

Matched Loaded-Q for accelerator operating off crest after transient loading.

It can be shown that:



For some reason Sergey (formally at INFN now at FRIB) set up his software with a Capacitance rather than shunt impedance or normalized shunt impedance.

Where the power is that exiting the port when the cavity has the stored energy U.

Define a variable *βX* as:

Where *Pemt* and *PFP* are the RF power that is passing out of the respective ports.

Multiplying equation (1) by *QL/Q0*

Power emitted at the FPC is given by

Consider a standing wave at the fundamental power port when the cavity is on tune. And recognizing that the RF power is given by.

Looking at the RF voltages at the fundamental power port:

Taking the square root of both sides

or

Or

For now we will define over coupling is when and under coupling is when and critically coupled when . Thus can also be written as:

Where where it is plus for over coupled and minus for under coupled. The following:

Substituting the equations for and leads to:

Recognizing that

It can be shown that the equation for *Q0* can be reduced to:

Introducing the concept of the easily measured reflection coefficient which for most of this document is referred to as .

One can show that

For

For

Or

Q of a port based on network analyzer measurements start with the following from page 17

Divide top and bottom by

Recognize that

Where is the power applied to the number 1 port. Thus

**CEBAF Cryomodule testing RF Performance Characterization**

Emitted Power Based Measurements

Starting Parameters:



Derivation of Performance Parameters:



**CEBAF Cryomodule testing RF Performance Characterization**

Emitted Power Based Measurements



**CEBAF Cryomodule testing RF Performance Characterization**

Q0 Measurements

CW measurements where Pdissipated is the average dissipated power measured calimetricly.



For pulsed operation the gradient is not constant throughout the measurement. In this case the field probe transmitted power is recorded as a function of time with at a sample interval Δ*t;* the gradient can be calculated using the transmitted power method; and Q0 is calculated as:



Where Pdissipated is the average dissipated power measured calimetrically, and T is the period of the pulses. It should be noted that the numerator in the above equation is used to account for the non square pulse shape. Values of Q0 calculated using this method will be different for different gradient pulse shapes or CW operations. If CW values are desired, it is best to make such measurements with pulse widths that are much greater than the cavity fill times.

To measure the dissipated power calimetrically one isolates the cryomodule from the helium supply and return lines and records the rate of rise of the pressure under three conditions. These are static heat load with RF and resistive heaters on; static plus a known resistive heat load applied to the bath; and static plus a unknown RF heat load due to the cavity losses. The dissipated power is then calculated using the following.



**Measurement Errors - Cryomodule:**

Starting Parameters:



Parameter Uncertainties:



**Measurement Errors - Cryomodule Continued**



Where  and  were entered values that were determined under different operating conditions and calibrations than the current measurement.



Where  and  were determined under same operating conditions and calibrations than the current measurement.



**CEBAF Vertical Pair Testing RF Performance Characterization**

Decay Measurement Formulas

Starting Parameters:



Performance Parameters:

Or in the general form

Where *PX* is the power exiting all of the other ports.



Decay Measurement Formulas Continued

The following implicitly assumes that *Q0* and the external-*Q* of all of the RF ports are independent of the stored energy, i.e. linear, flat *Q0* , and constant coupling factor for gradients at or below that of the starting point of the decay measurement; that the RF frequency is exactly that of the cavity resonance; and for error purposes that all devices within the power measurement path are linear and constant with respect to frequency and amplitude, e.g. no VSWR mismatch.

It can be shown that:

Which one can show is equivalent to (*Cβ* = +1 for over coupled):

CW Formulas

Starting Parameters:



Derivation of Performance Parameters:

Where *PX* is the power exiting all of the other ports.

CW Measurement Formulas Continued

Measurement Errors - Decay Measurement:

Starting Parameters:

 - Actual Measured CW RF power meter reading (Watts), just prior to turning the RF off for the decay measurement.

 - Sensitivity limit of power sensor used.

 - Factional uncertainty in absolute power measured. This includes uncertainties such as power meter absolute error of the reference power meter, non-linearity of the individual power meters during the calibration process as well as during the measurement process, errors introduced due to standing waves in the measurement circuit. Also included are errors introduced due to the frequency dependent directional coupler errors because to varying load impedances on the output of the high power directional coupler.

 Fractional uncertainty in the linearity of the power meter calibration

 - Fractional uncertainty in cable calibrations.

 - Error of a variable. The units are the same as the variable, i.e.  is the error in  in Watts.

Parameter Uncertainties:

Measurement Errors Continued - Decay Measurement:

Error in Tau due to mismatch looking back at the source. [1]

Where VSWR is the VSWR of the system looking from the cavity back to the source. Note that this value is also used in the CW measurements.

[1]J.P.Holzbauer, Yu.Pischalnikov, D.A.Sergatskov, W.Schappert, S.Smith "Systematic uncertainties in RF-based measurement of superconducting cavity quality factors" NIM-A, Volume 830, 11 September 2016, Pages 22-29

Measurement Errors - CW Measurement:

 Actual Measured CW RF power meter reading (Watts).

 Sensitivity limit of power sensor used.

is the power dissipated in the cavity walls.

Factional uncertainty in absolute power measured.

Fractional uncertainty in the linearity of the power meter calibration

 Error of a variable. The units are the same as the variable, i.e.  is the error in  in Watts.

 Uncertainty in  as determined from a decay measurement.

Coupling factor used for decay measurement

Measured incident and reflected power at the cavity input that was used for the decay measurement which was used to calculate

Parameter Uncertainties:

Where the primed values are those used to determine QFP (Q2), are the linearity errors associated with the instruments (typically 2%), and are the absolute calibration errors for the power measurements, (typically 7%).

Measurement Errors - CW Measurement Cont.:

Relative Gradient based on S11/S21 measurements