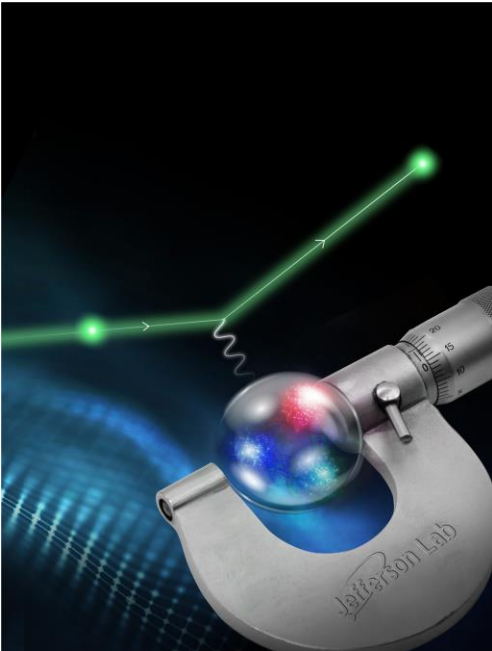


# PRadII - DRad Experiments Current Status and Plans



**Jingyi Zhou**

Duke University

For the PRad-DRad Collaboration

**CLAS Collaboration Meeting July 2023**

Acknowledgment: This work is supported in part by the U.S. Department of Energy under Contract No. DE-FG02-03ER41231.

# Outline

- Proton charge radius and PRad experiment
- PRad-II experiment (E12-20-004)
- Hidden Sector Particles/X17 search experiment (E12-21-003)
- DRad experiment (PR12-23-011)

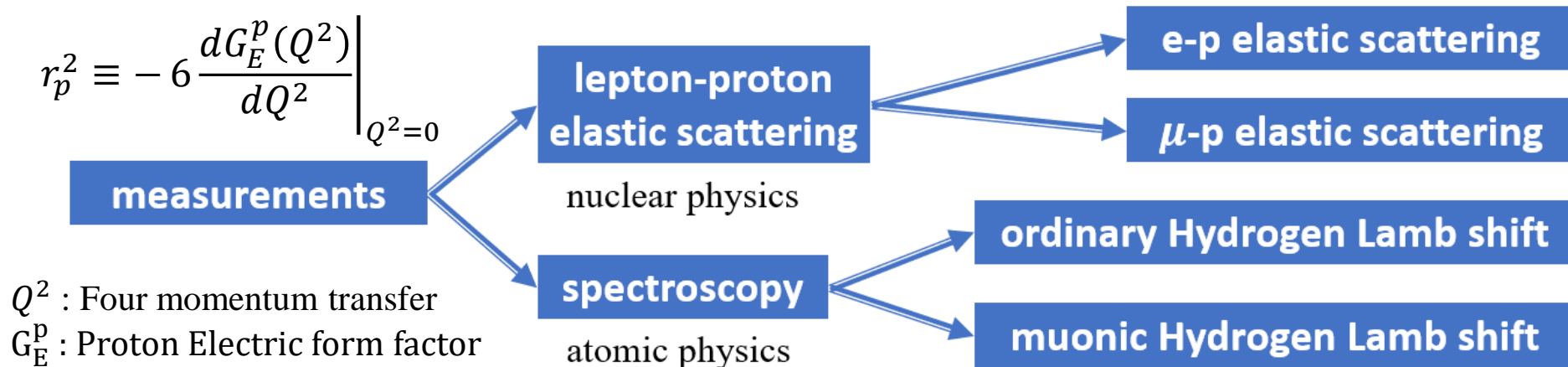


# Proton root-mean-square charge radius

- The proton is the primary, stable building block of nearly all visible matter in the Universe.

**Proton rms charge radius  $r_p$**  — an important quantity of the proton:

- Understand how **QCD** works in the non-perturbative region
- Important input to the bound-state **QED** calculations, the proton finite size contributes to the muonic H Lamb shift ( $2S_{1/2} - 2P_{1/2}$ ) by as much as **2%**
- Impacts the determination of the **Rydberg constant  $R_\infty$**



# Unpolarized e-p elastic scattering

- In the Born approximation (one photon exchange):

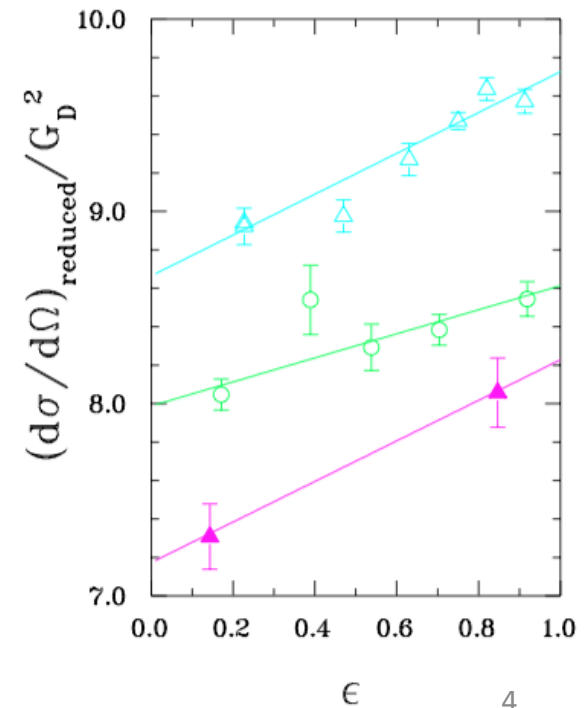
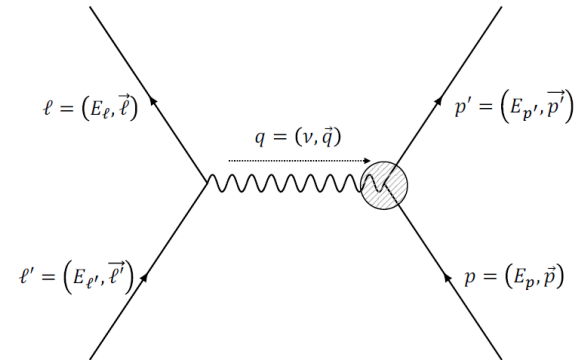
$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{Mott} \left( \frac{E'}{E} \right) \frac{1}{1 + \tau} \left( G_E^{p^2}(Q^2) + \frac{\tau}{\varepsilon} G_M^{p^2}(Q^2) \right)$$

$$Q^2 = 4EE' \sin^2(\theta/2)$$

$$\tau = Q^2/(4M_p^2) \quad \varepsilon = [1 + 2(1 + \tau) \tan^2(\theta/2)]^{-1}$$

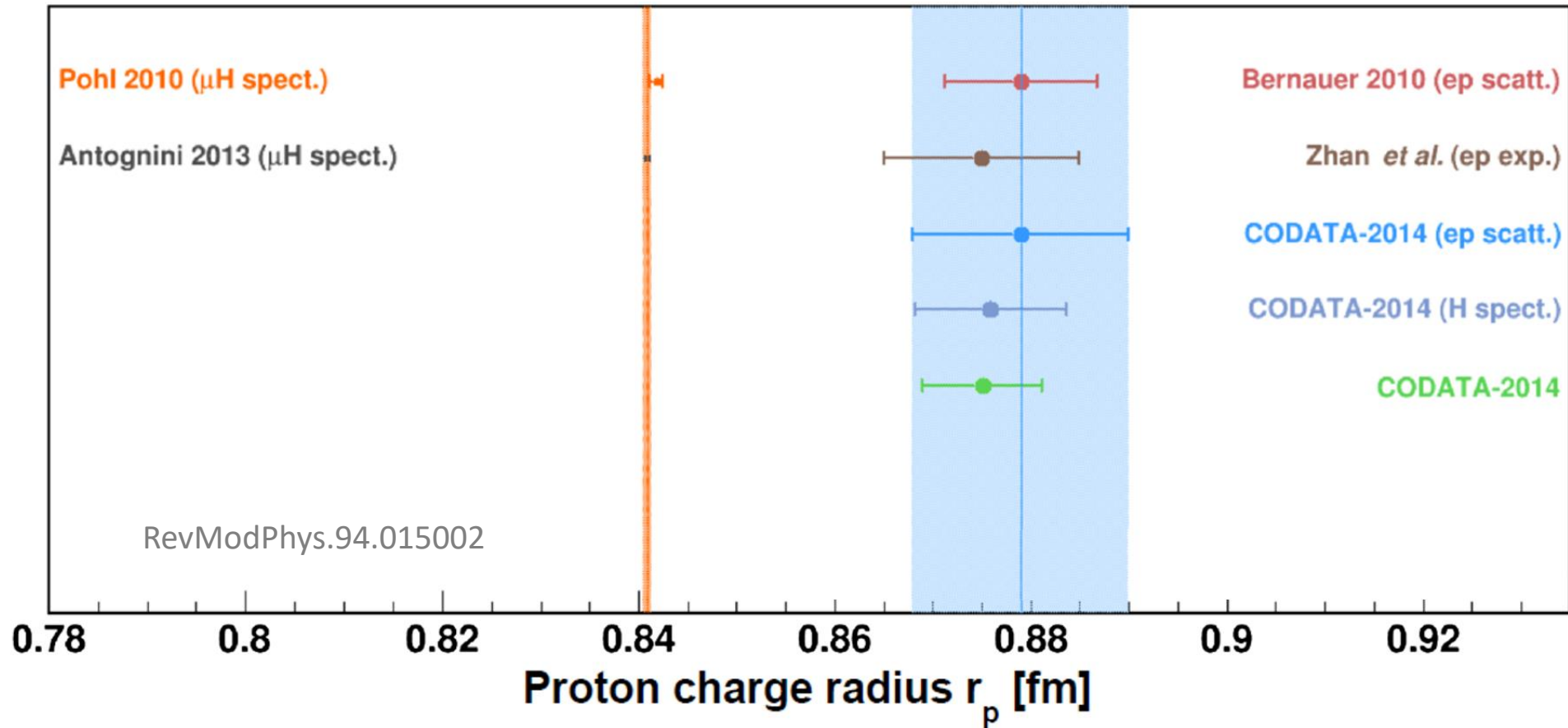
$$\left( \frac{d\sigma}{d\Omega} \right)_{reduced} = G_M^{p^2}(Q^2) + \frac{\varepsilon}{\tau} G_E^{p^2}(Q^2)$$

- $G_E^p$  and  $G_M^p$  can be extracted using **Rosenbluth separation**
- At very low  $Q^2$  region, cross section dominated by  $G_E^p$ , one may also extract  $G_E^p$  assuming  $G_M^p$  in certain form.



# Proton charge radius puzzle

~  $8\sigma$  discrepancy between muon and electron based measurements



Proton rms charge radius measured using

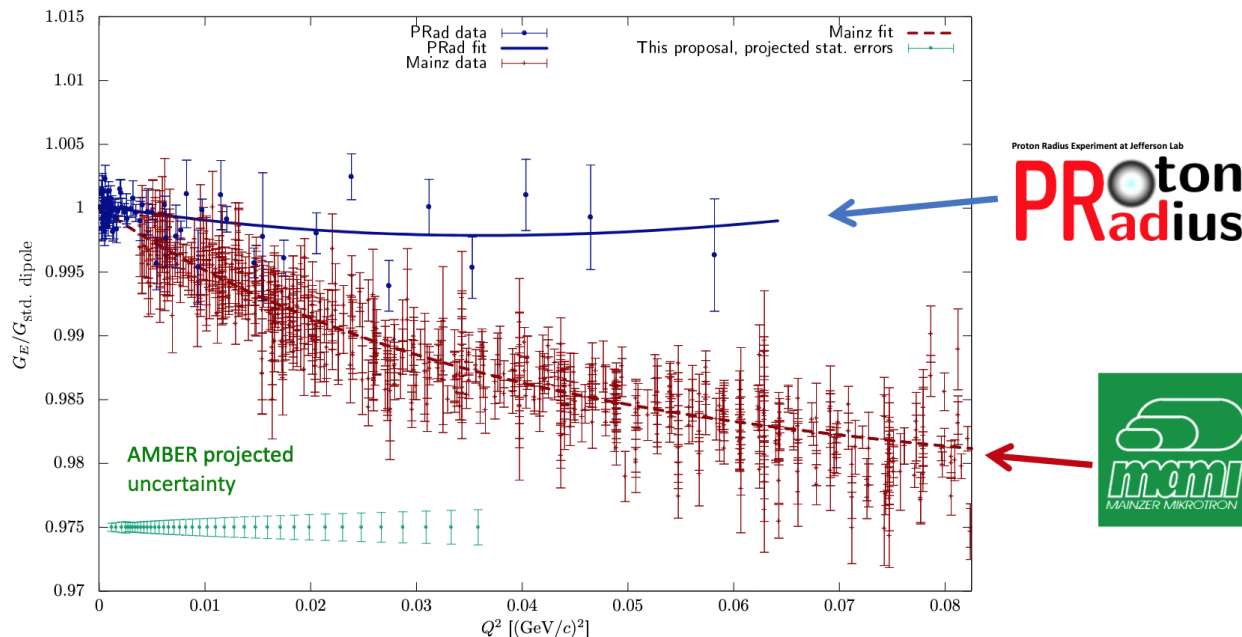
electrons:  $0.8770 \pm 0.0045$  (CODATA2010 + Zhan et al.)

muons:  $0.8409 \pm 0.0004$

# The PRad-II experiment

PRad result:  $r_p = 0.831 \pm 0.007(\text{stat.}) \pm 0.012(\text{sysm.})\text{fm}$  supports a smaller  $r_p$

- PRad has not reached its ultimate precision for this experimental technique
- Possible difference between proton radius from electronic vs. muonic system
- Need higher precision to investigate the discrepancy between PRad and MAMI form factor



- Based on the PRad experimental technique
- Three beam energies,  $E = 0.7, 1.4$  and  $2.1$  GeV to increase  $Q^2$  range
- Even lower  $Q^2 \sim 10^{-5}(\text{GeV}/c)^2$
- Upgrades to the original detectors, new detectors, new calculations...
- Overall uncertainty reduced by **3.8** times compared to PRad

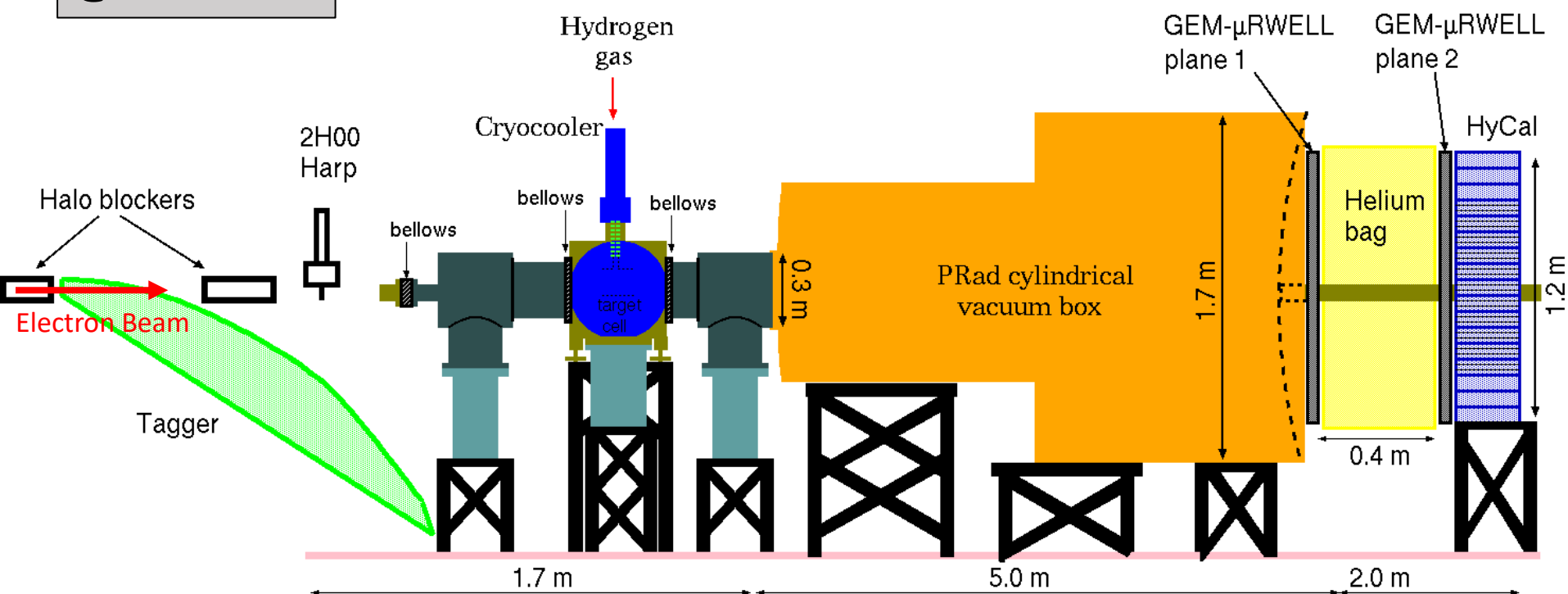
figure: J. Bernauer

# PRad-II experimental apparatus

- PRad-II experiment (E12-20-004), approved with highest "A" scientific rating among all proposals submitted in June 2020

@HallB JLab

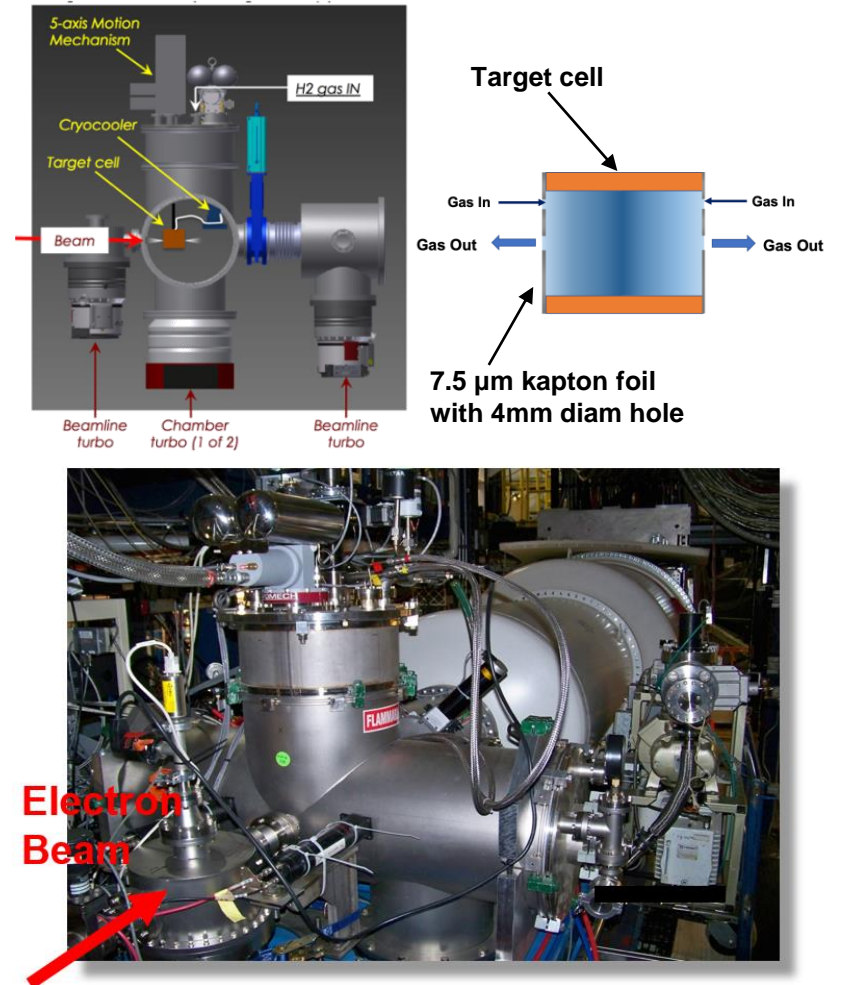
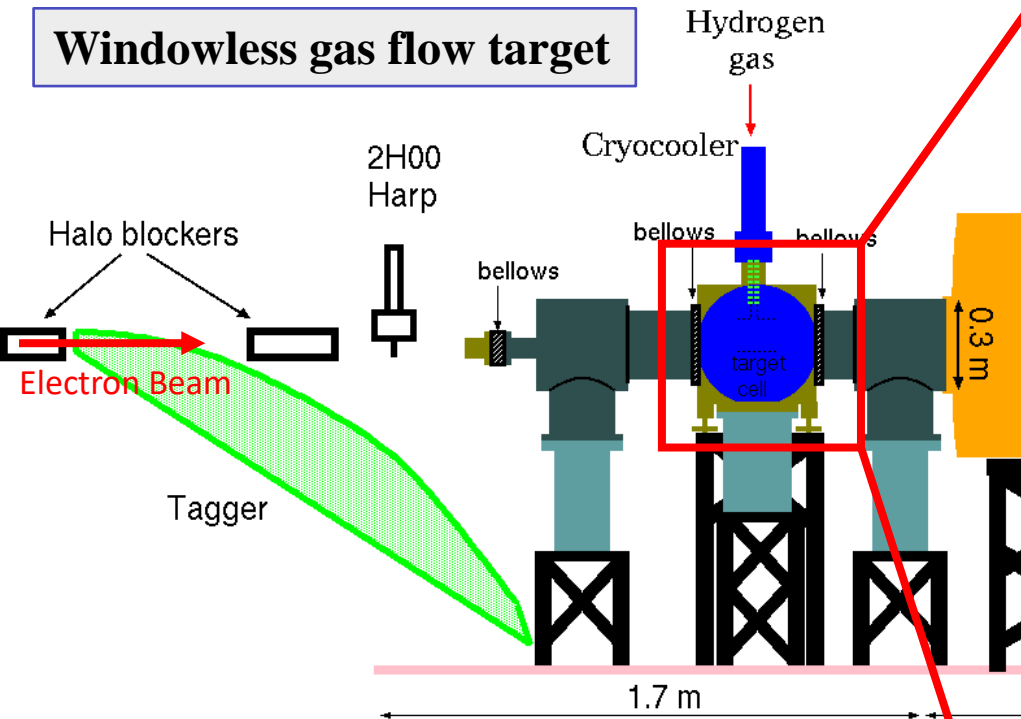
PRad-II Experimental Setup (Side View)





# PRad-II experimental apparatus

## Windowless gas flow target



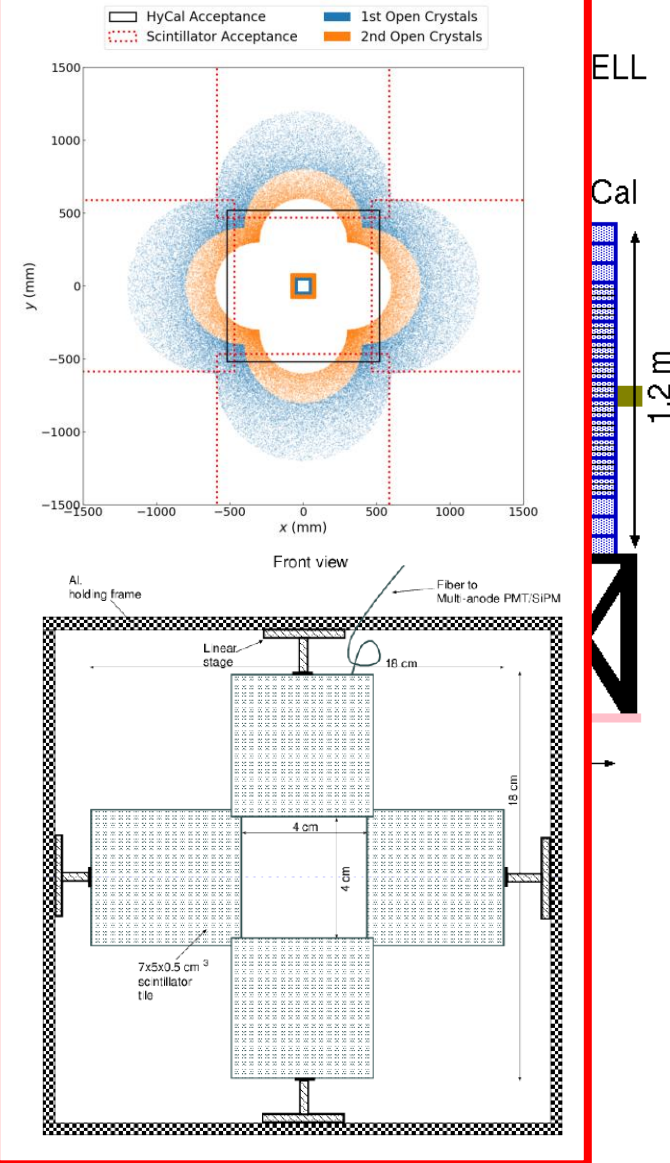
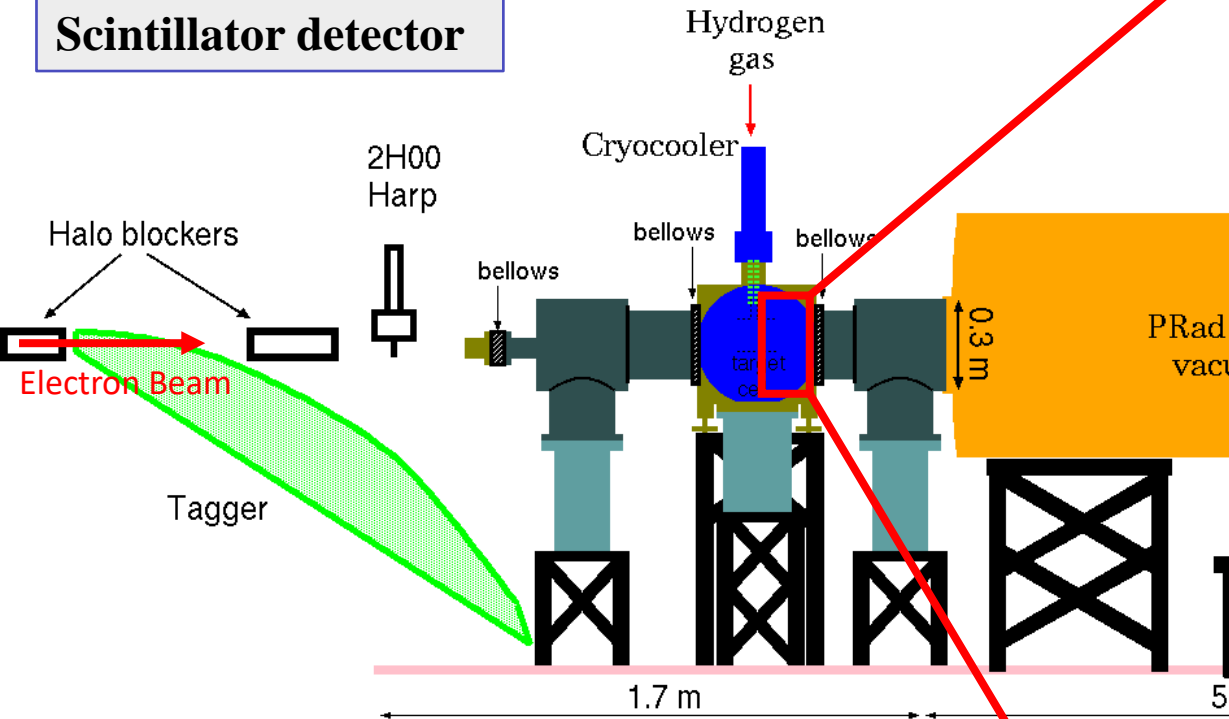
- 29 cm diam x 5.5 cm long target cell
- 4 mm diam holes open at front and back kapton foils, allows beam to pass through
- Target thickness:  $\sim 2 \times 10^{18}$  atoms / cm<sup>2</sup>
- remove major background source

*Nucl.Instrum.Meth.A 1003 (2021) 165300*



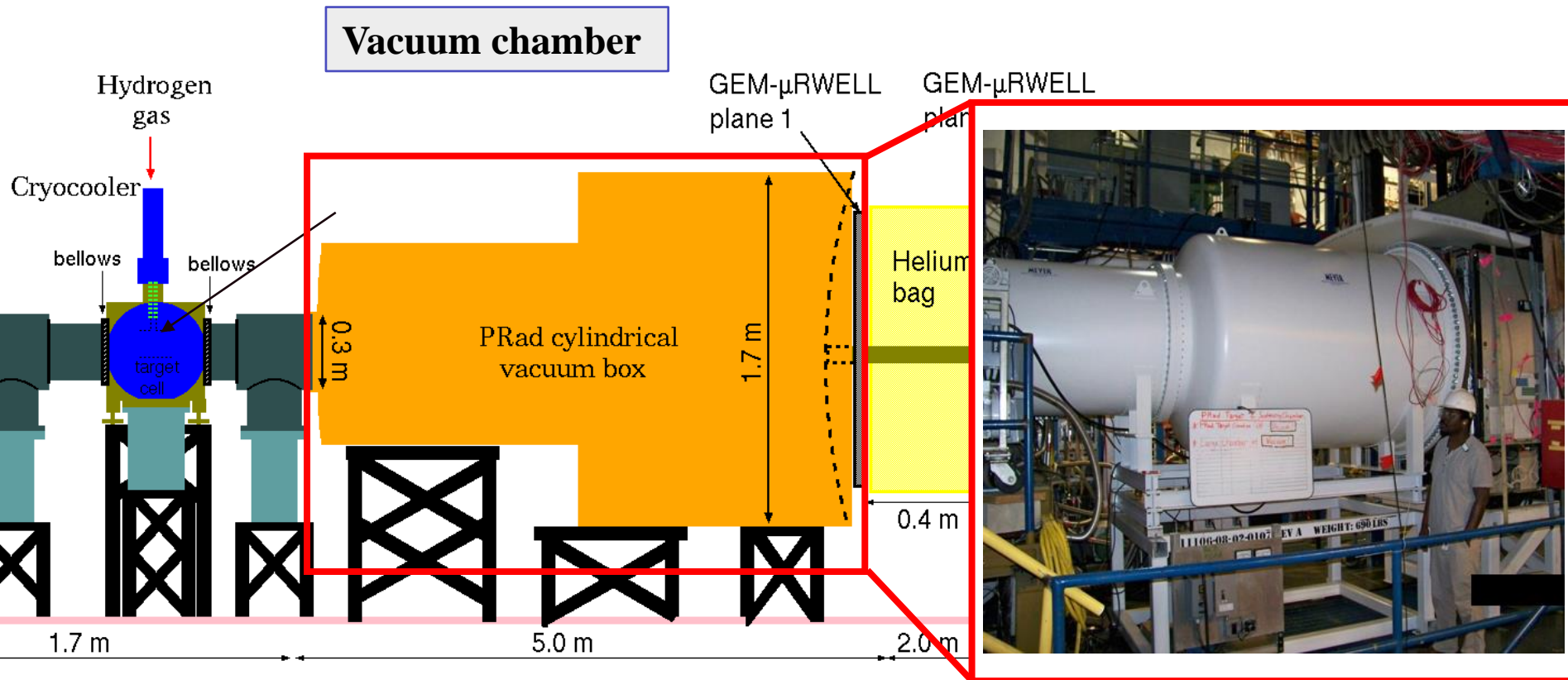
# PRad-II experimental apparatus

## Scintillator detector



- 25 cm downstream from the target center
- Four  $7 \times 5 \times 0.5 \text{ cm}^3$  tiles of plastic scintillators
- Each tile attached to a linear stage in x/y direction
- Identify e-p elastic events and Møller events at  $0.5 \sim 0.8$  degrees

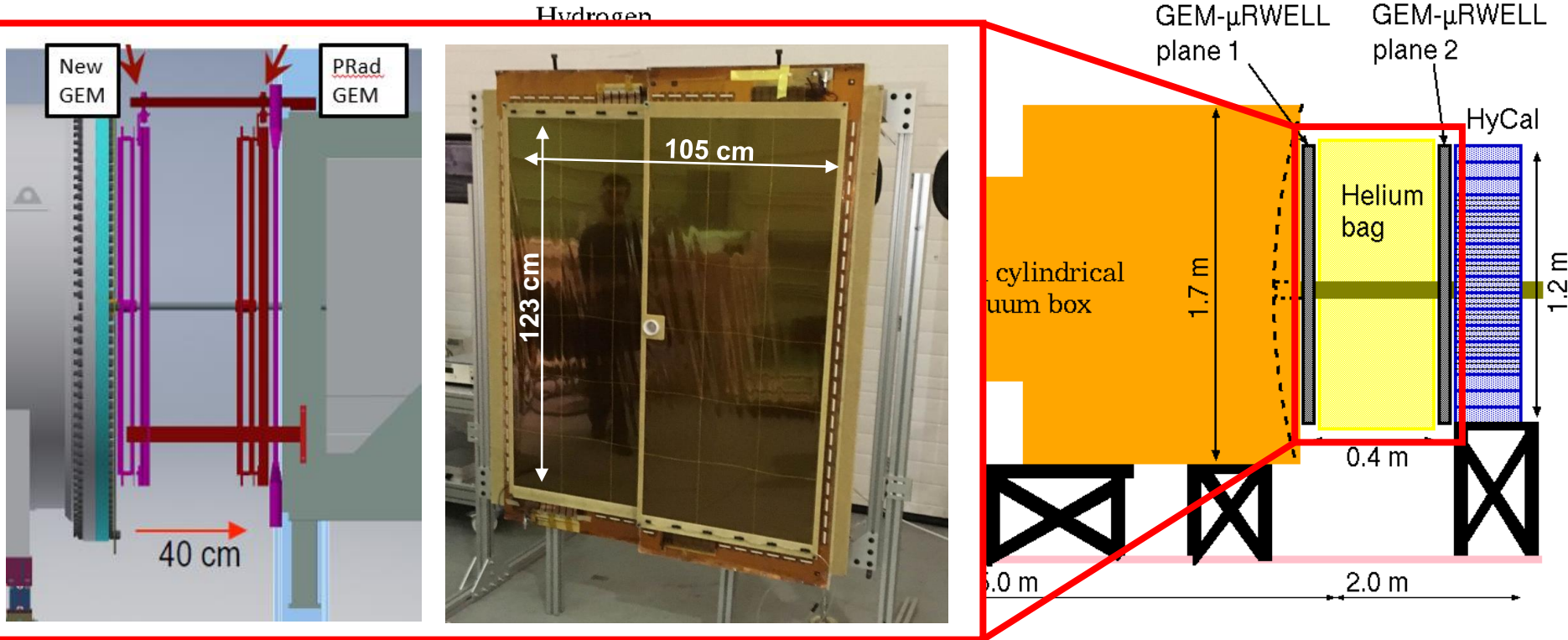
# PRad-II experimental apparatus



- 5 m long two stage vacuum chamber, further remove possible background source from the **electron multiple scattering**
- Vacuum chamber pressure: 0.3 mTorr

# PRad-II experimental apparatus

## Tracking detectors



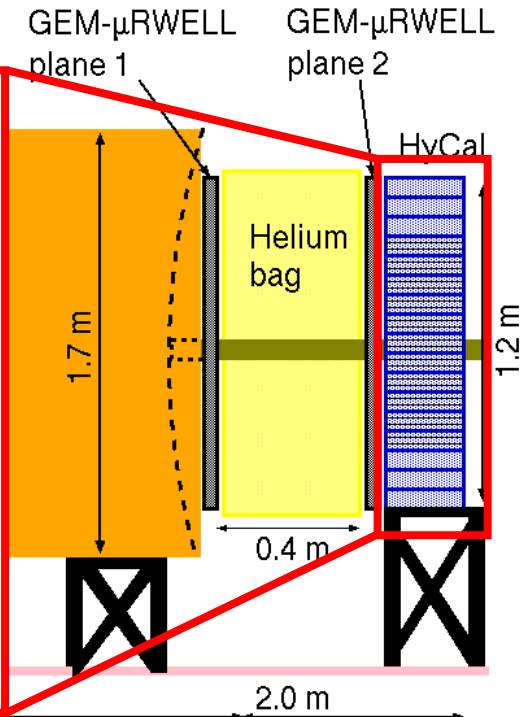
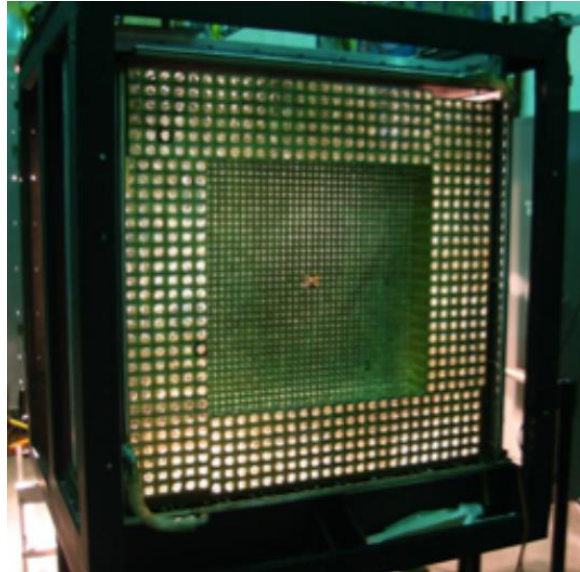
- Each GEM plane: two large area GEM chambers, small overlap region in the middle
- Provide excellent **tracking** for the scattered electrons
- Better control of **beam line background** from the upstream collimator, especially at very small angles (electron scattering angle less than 1 deg)

# PRad-II experimental apparatus

## Upgraded Hybrid Calorimeter (HyCal)

- High resolution and efficiency
  - 5.5 m from the target
  - Scattering angle coverage:  
~  $0.6^\circ$  to  $7.5^\circ$
  - Full azimuthal angle coverage
- PRad HyCal:
- Inner 1156  $\text{PbWO}_4$  modules
  - Outer 576 lead-glass modules

Hydrogen  
gas



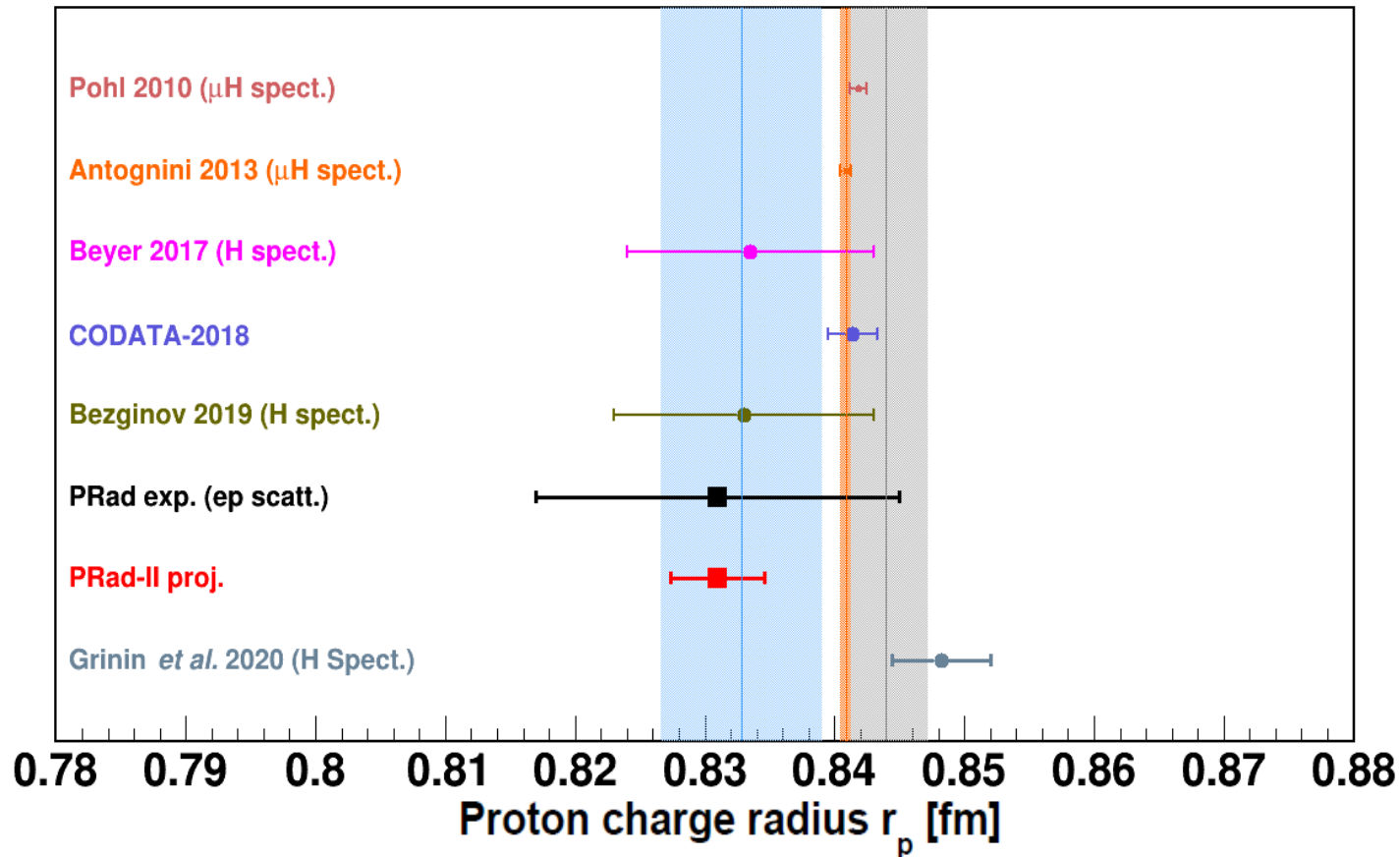
### Upgraded HyCal:

- Replace lead-glass modules with  $\text{PbWO}_4$  modules to have more uniform and better resolution, suppress inelastic contribution
- Convert to flash-ADC based readout to increase data taking rate



# PRad-II Projection

- The mentioned upgrades in hardware combines with the planned **NNLO radiative correction calculations** reduces the overall uncertainty by a factor of **3.8** compared to PRad
- Form factor measurements reach even lower  $Q^2 \sim 10^{-5}(\text{GeV}/c)^2$



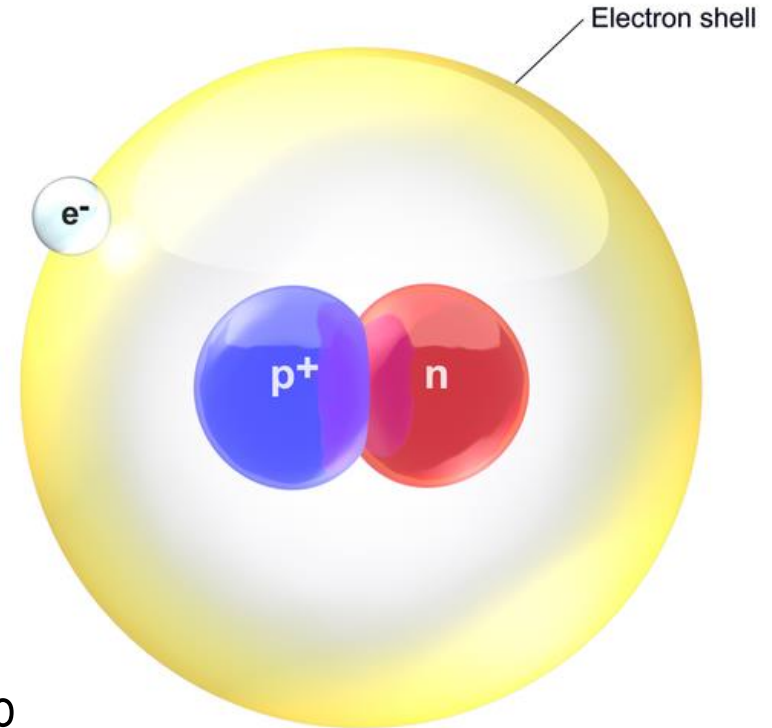
# Deuteron

- Excellent laboratory to study QCD in nuclei
  - The simplest and lightest nucleus in nature
  - The only bound two-nucleon system
  - Effective neutron target
  - Various theoretical calculations
- 
- **Deuteron rms charge radius:** an ideal observable to compare experiments with theories

$$r_d^2 \equiv -6 \left. \frac{dG_C^d(Q^2)}{dQ^2} \right|_{Q^2=0}$$

$Q^2$  : Four momentum transfer

$G_C^d$  : Deuteron charge form factor



**Hydrogen-2,  
deuterium**  
mass number: 2

# Unpolarized e-d elastic scattering

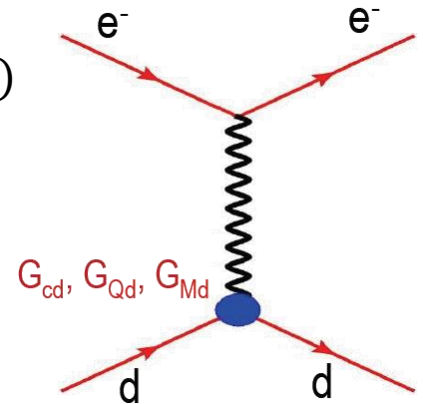
- In the Born approximation (one photon exchange):

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{Mott} \left[ A(Q^2) + B(Q^2) \tan^2 \frac{\theta}{2} \right] \quad Q^2 = 4EE' \sin^2(\theta/2)$$

A and B are structure functions related to the deuteron **charge ( $G_C^d$ )**, **magnetic ( $G_M^d$ )** and **quadrupole ( $G_Q^d$ )** form factors:

$$A(Q^2) = G_C^{d^2}(Q^2) + \frac{2}{3}\tau G_M^{d^2}(Q^2) + \frac{8}{9}\tau^2 G_Q^{d^2}(Q^2)$$

$$B(Q^2) = \frac{4}{3}\tau(1 + \tau)G_M^{d^2}(Q^2) \quad \tau = Q^2/(4M_d^2)$$

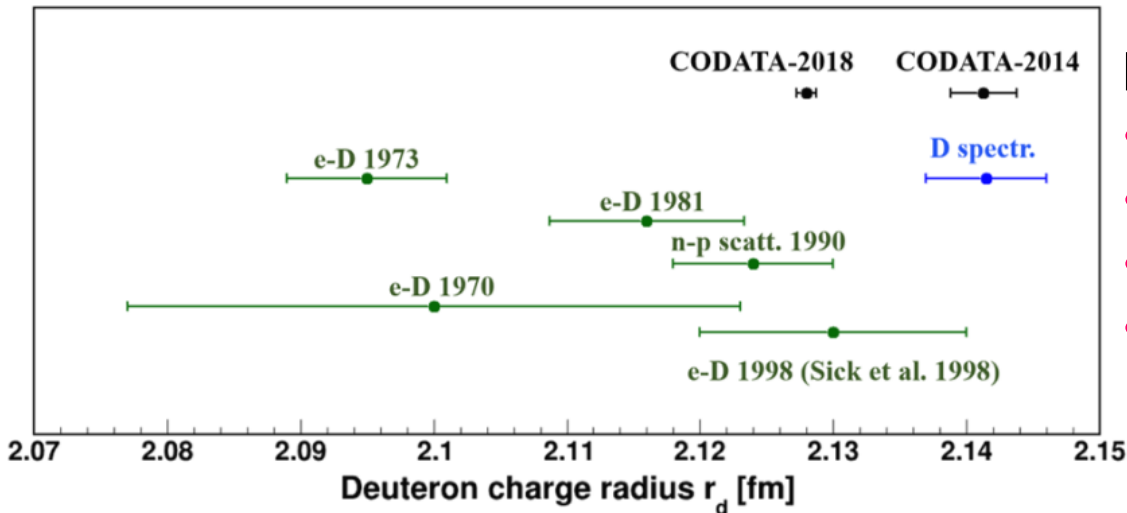


- At very low  $Q^2$  (DRad), cross section dominated by  $G_C^d$ , one may extract  $G_C^d$  by assuming  $G_M^d$  and  $G_Q^d$  in certain forms from parametrizations based on the data.
- The **rms charge radius** can be obtained from the slope of the charge form factor  $G_C^d$  at  $Q^2 = 0$ :

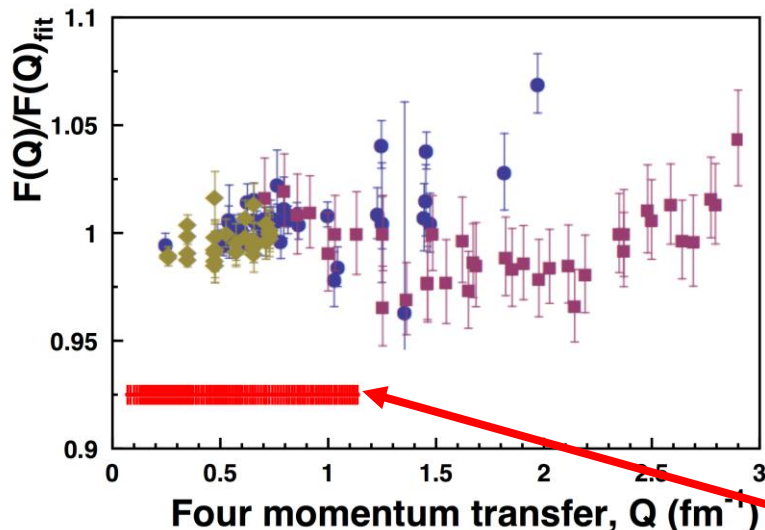
$$r_d^2 \equiv -6 \left. \frac{dG_C^d(Q^2)}{dQ^2} \right|_{Q^2=0}$$



# The deuteron charge radius from e-d scattering



I. Sick and D. Trautmann, NPA 637, 559 (1998)



Previous e-d scattering experiments:

- magnetic spectrometer method
- different types of targets
- normalized to  $e-p$  cross sections
- the most recent result in 1998 is a reanalysis of old data

◆ R.W. Berard et al. Phys. Rev. Lett. B47,355 (1973):

cooled H<sub>2</sub> and D<sub>2</sub> gas, measured ratio of  $ed/ep$  cross sections  
 $Q^2 = [4 \times 10^{-2} - 5 \times 10^{-2}] \text{fm}^{-2}$

● G.G. Simon et al. Nucl. Phys. A364, 285 (1981):

different gas and liquid targets:  
 $Q^2 = [4 \times 10^{-2} - 4] \text{fm}^{-2}$

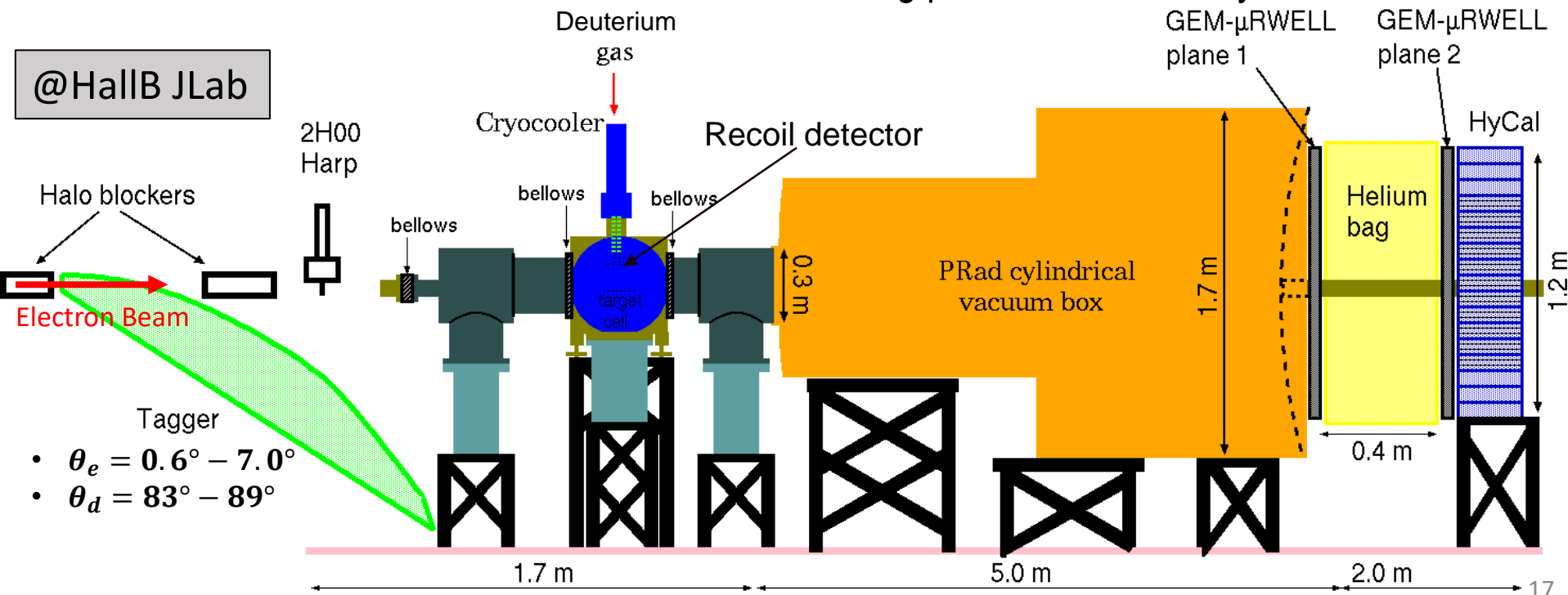
■ S. Platchkov, et al. Nucl. Phys. A510, 740, (1990):

different LH<sub>2</sub> and LD<sub>2</sub> targets  
 $Q^2 = [5 \times 10^{-2} - 20] \text{fm}^{-2}$

We propose a new **independent** method to measure  $e-d$  elastic cross sections with high precision

# The highlight of DRad proposal

- DRad proposal(PR12-23-011): **calorimetric** method with **windowless gas flow target** based on PRad-II experiment(E12-20-004)
- Measure  **$e$ - $d$**  elastic cross sections at very low  $Q^2$  range:  
 $[5 \times 10^{-3} - 1.3] \text{ fm}^{-2} / [2 \times 10^{-4} - 5 \times 10^{-2}] \text{ GeV}^2$
- Two beam energies,  $E = 1.1$  and  $2.2 \text{ GeV}$  to increase  $Q^2$  range
- A new two-layer **cylindrical recoil detector** for reaction elasticity
- Veto counters for timing (**PrimEx veto counters**)
- Simultaneous detection of  $ee \rightarrow ee$  Møller scattering process to control systematics



# Proposed DRad experiment apparatus

## Si-strip Cylindrical Recoil Detector inside the target cell

Detect recoil deuteron, proton background, provide information:

- Timing & Azimuthal angle
- Energy

Based on the CLAS12 Barrel Silicon Tracker (SVT)

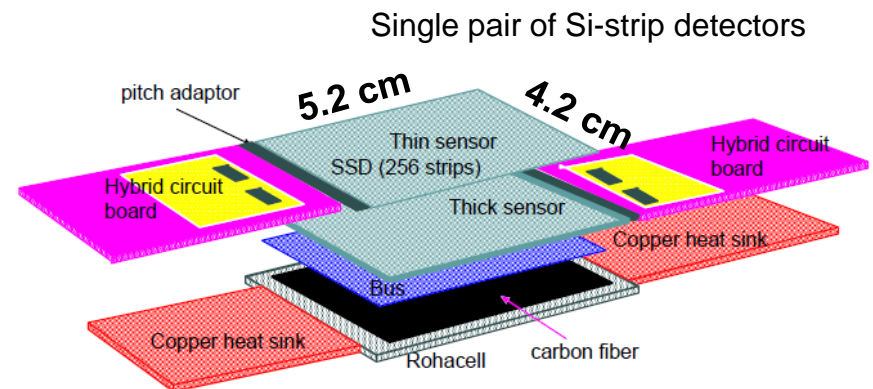
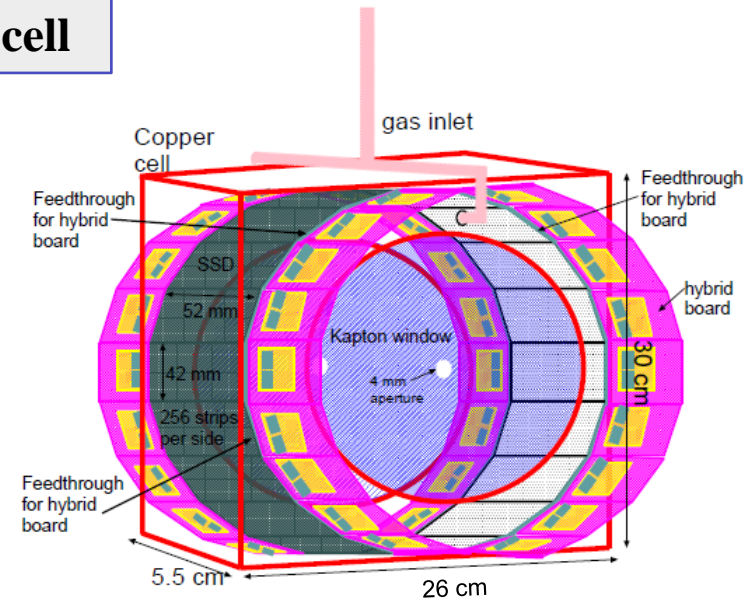
- 20 panels of twin, single sided Si-strip detectors ( $42 \times 52 \text{ mm}^2$ ), 20 sided polygon arrangement with around 13 cm radius
- Thicknesses:  
200  $\mu\text{m}$  (inner layer), 300  $\mu\text{m}$  (outer layer)
- 256 strips on each sensor:  
angular resolution 5 mrad ( $\phi$ ) 20 mrad ( $\theta$ )
- Inactive  $\text{SiO}_2$  layer can be as thin as 0.5  $\mu\text{m}$

CLAS12 Technical Design Report, 2008

([https://www.jlab.org/Hall-B/clas12\\_tdr.pdf](https://www.jlab.org/Hall-B/clas12_tdr.pdf));

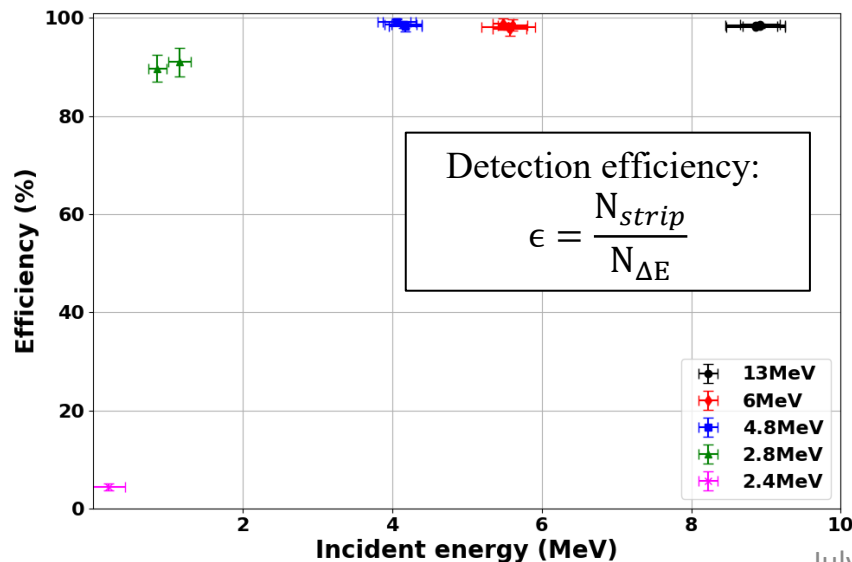
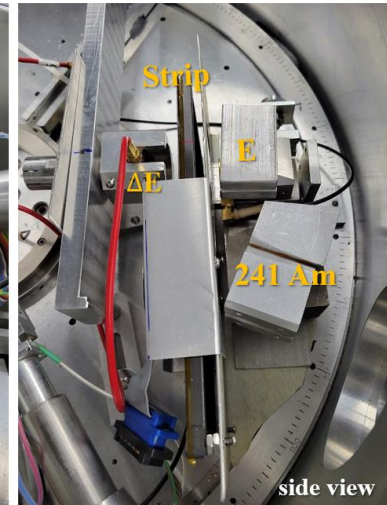
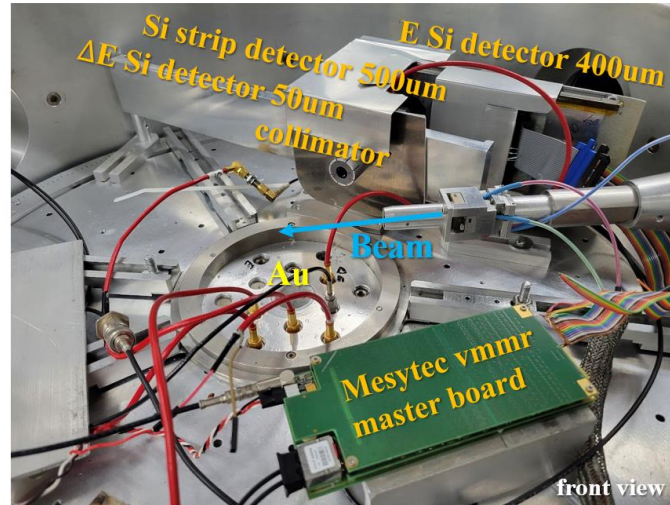
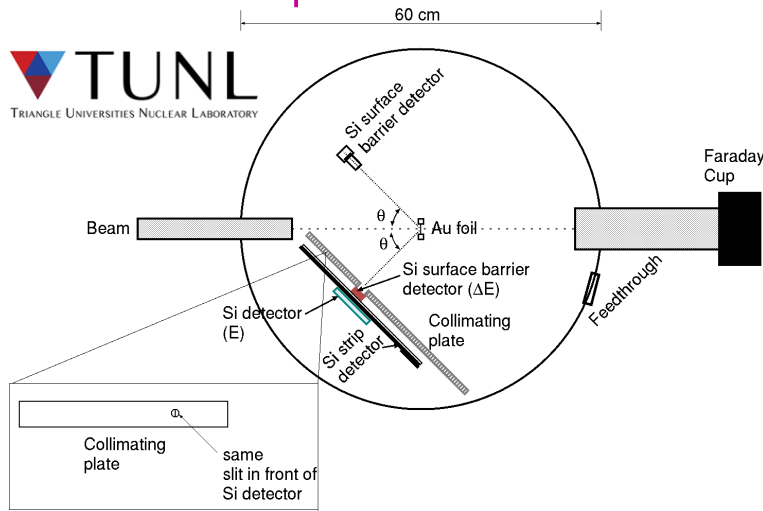
CLAS12 Detector documentation

(<http://clasweb.jlab.org/clas12offline/docs/detectors/html/svt/introduction.html>)

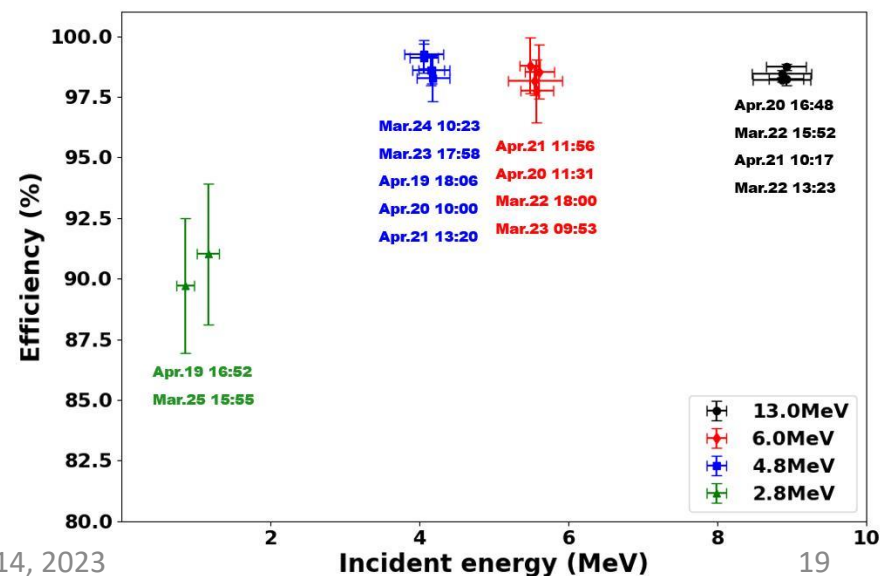


# The recoil detector calibration at TUNL

- The Si strip detector (SSD) can be calibrated using e-p elastic run on hydrogen and with the 2.5~13 MeV p/D beam from the Tandem accelerator at TUNL.



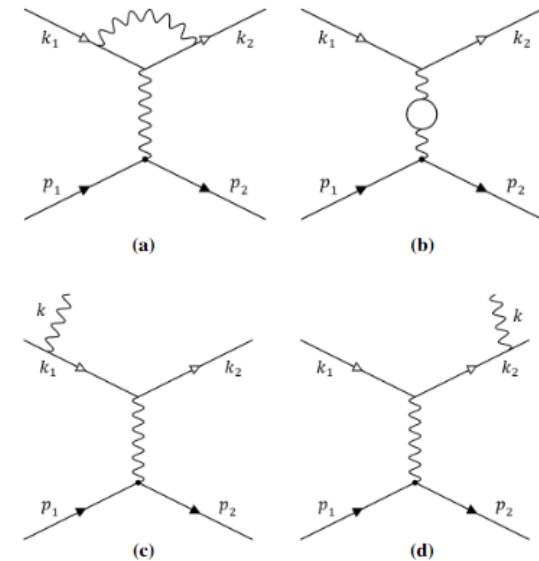
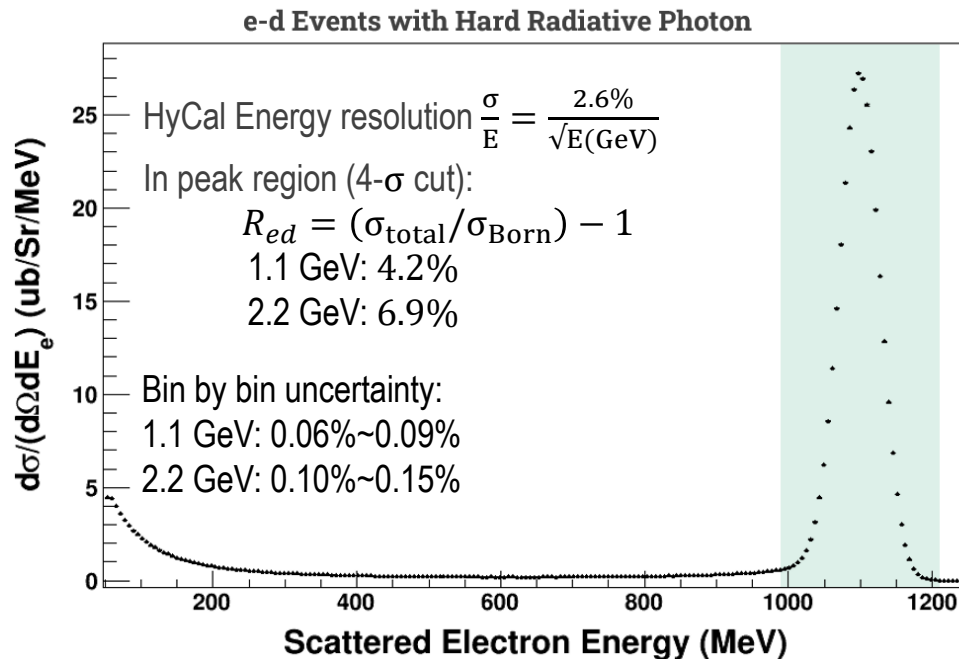
July 14, 2023





# Radiative Correction (RC) Calculations

- Complete **elastic e-d NLO cross section** including the **lowest order radiative corrections** beyond the ultrarelativistic limit has been calculated
- Based on the ansatz in the PRad RC calculation and used the Bardin-Shumeiko infrared divergence cancellation method [I. Akushevich et al. Eur. Phys. J. A 51.1(2015)]



**Fig. 2** Feynman diagrams from (a) to (d), describing the lowest-order QED RC contributions to the unpolarized elastic  $e + d$  scattering cross section: (a) vertex correction; (b) vacuum polarization; (c), (d) electron-leg bremsstrahlung.

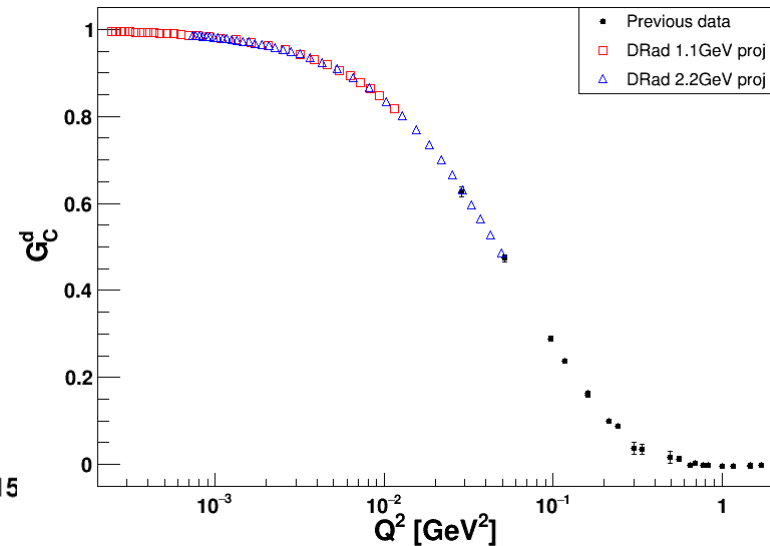
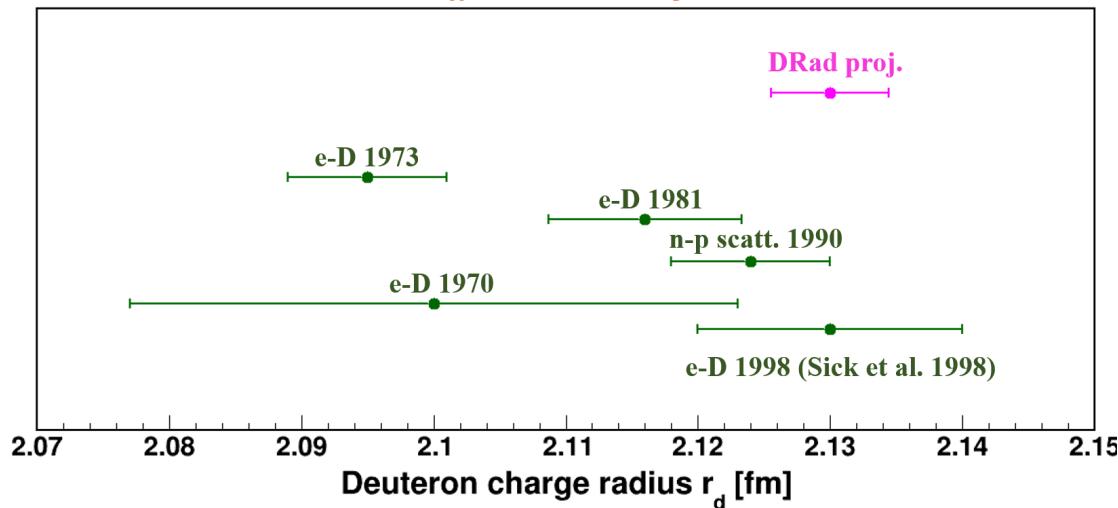
- An event **generator** is developed and the total correction to the elastic e-d Born cross section in the DRad kinematics is calculated
- The **uncertainty** of the NLO calculation is estimated, taking into account higher-order contributions, calculation assumptions, and differences between various recipes
- The paper is to be submitted to arXiv and European Physical Journal A

# DRad projection

Item	Uncertainty (%)
Event selection	0.110
Radiative correction	0.045
HyCal response	0.043
Geometric acceptance	0.022
Beam energy	0.008
Total correlated terms	0.13

Item	Uncertainty (%)
Statistical uncertainty	0.05
Total correlated terms	0.13
GEM efficiency	0.03
Inelastic e-d process	0.024
Efficiency of recoil detector	0.15
Total	0.21

Expected  $R_d$  uncertainty : 0.2%



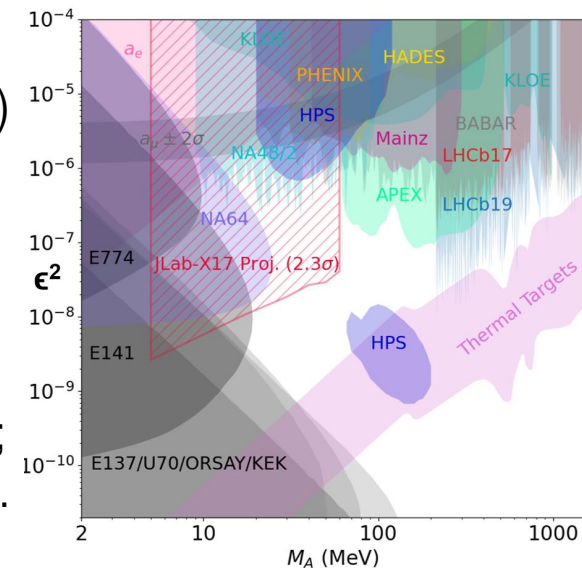
*The most precise single measurement from e-d elastic scattering*

# Hidden Sector Particles/X17 Search Experiment (E12-21-003)

- The experiment has two experimental objectives:
  - 1) Validate existence or establish an experimental upper limit on the electroproduction of the hypothetical **X17 particle** claimed in two **ATOMKI low-energy proton-nucleus experiments**.
  - 2) Search for “hidden sector” intermediate particles (or fields) in **[3 – 60] MeV** mass range produced in electron-nucleus collisions and detected in  $e^+e^-$  (or  $\gamma\gamma$ ) channels.
- The method:
  - “**bump hunting**” in the invariant mass spectrum over the beam background.
  - direct detection of all final state particles ( $e'$ ,  $e^+e^-$  and/or  $\gamma\gamma$ )  $\rightarrow$  full control of kinematics
- Electroproduction on heavy nucleus in forward directions:
 
$$e^- + \text{Ta} \rightarrow e' + \gamma^* + \text{Ta} \rightarrow e' + X + \text{Ta}, \quad \text{with} \quad X \rightarrow e^+e^- \text{ (with tracking)}$$

$$\text{and} \quad X \rightarrow \gamma\gamma \text{ (without tracking)}$$

in mass range of: **[3 - 60] MeV**,  
 target: Tantalum, ( $_{73}\text{Ta}^{181}$ ), 1  $\mu\text{m}$  ( $2.4 \times 10^{-4}$  r.l.) thick foil.
- All 3 final state particles will be detected in this experiment:
  - ✓ scattered electrons,  $e'$ , with 2 GEMs and  $\text{PbWO}_4$  calorimeter;
  - ✓ decay  $e^+$  and  $e^-$  particles, with 2 GEMs and  $\text{PbWO}_4$  calorimeter;
  - ✓ or decay  $\gamma\gamma$  pairs, with  $\text{PbWO}_4$  calorimeter (and GEMs for veto).





# Summary

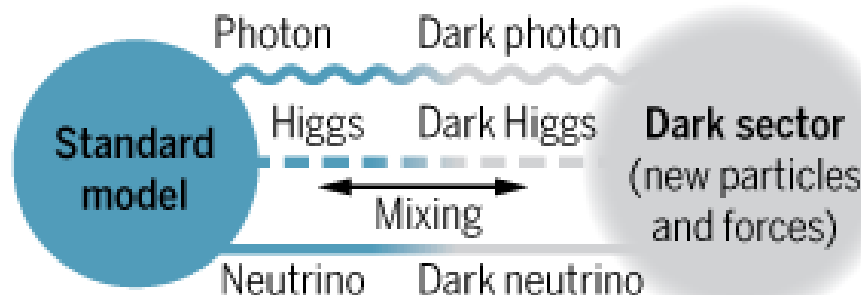
- PRad-II experiment:
  - Bring the best independent form factor measurement at low  $Q^2$  range
  - Address the FF difference between PRad and MAMI
  - reach lowest  $Q^2$  range  $10^{-5}\text{GeV}^2$
- Hidden Sector Particles/X17 search experiment:
  - Uniquely cost effective search for hidden-sector particles in 3~60 MeV mass range
  - Expect to run in 2025
- DRad experiment:
  - Perform the deuteron form factor measurement at very low  $Q^2$  range from cross section independently from the e-p cross section
  - Extract the deuteron charge radius with highest precision in e-d scattering

**Acknowledgment: This work is supported in part by the U.S. Department of Energy under Contract No. DE-FG02-03ER41231.**

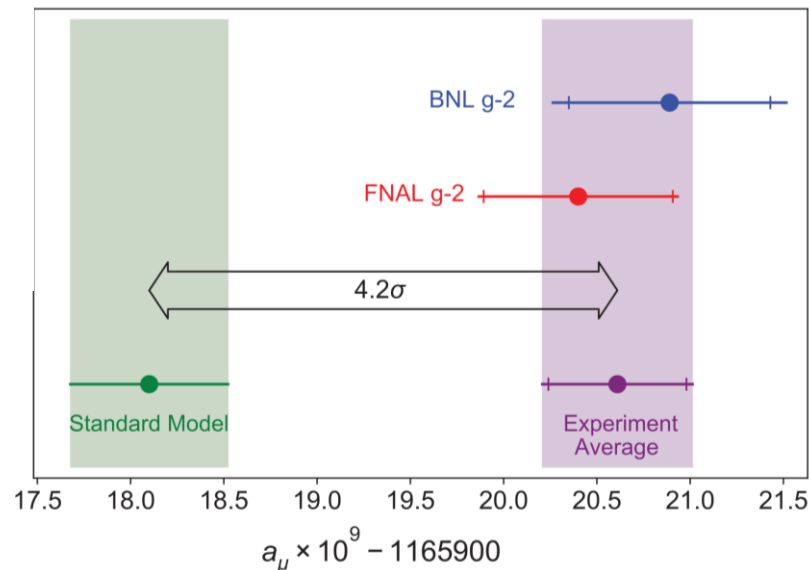
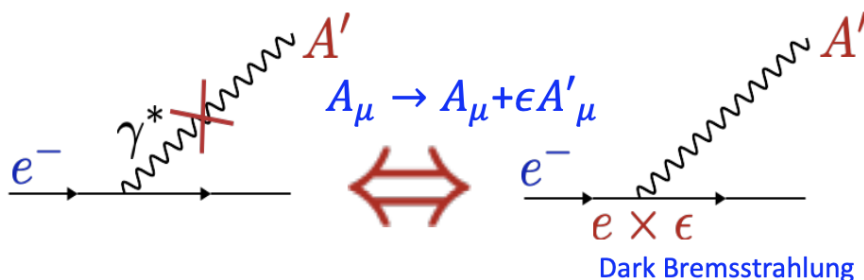
# Backup

# Hidden Sector Particles/X17

- The existence of the dark matter (DM) is well established by astronomical measurements
- No direct information about the **DM composition**



photon - dark photon mixing is equivalent to ordinary matter acquiring a milli-charge  $\epsilon e$



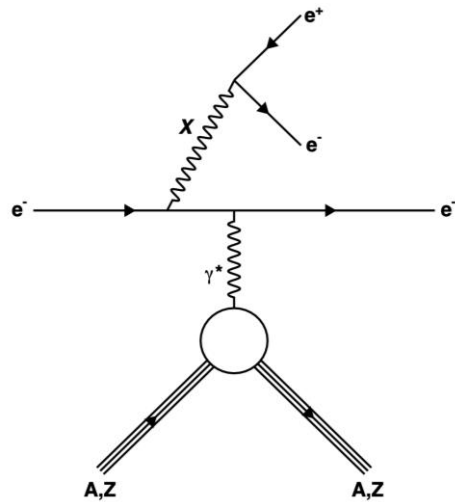
- Small-scale structure in astrophysical observations and the 4.2- $\sigma$  disagreement between experiments and the standard model prediction for the muon anomalous magnetic moment motivated new DM models and candidates, such as the **hidden sector DM** (HSDM) models in **1~100 MeV** mass region
- ATOMKI Beryllium Anomaly suggests a new particle with a mass of **16.84 MeV**, dubbed **X17**

# Hidden Sector Particles/X17 Search Experiment (E12-21-003)

Forward angle electroproduction on a heavy nucleus

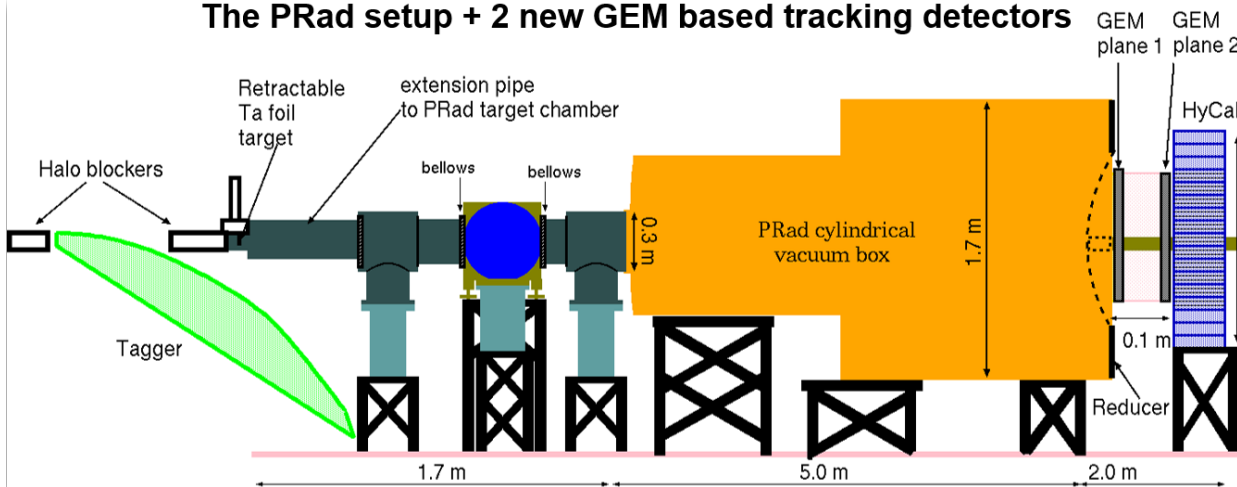
$$e^- + \text{Ta} \rightarrow e' + \gamma^* + \text{Ta} \rightarrow e' + X + \text{Ta}, \text{ with } X \rightarrow e^+e^-$$

All 3 final state particles will be detected in this experiment:

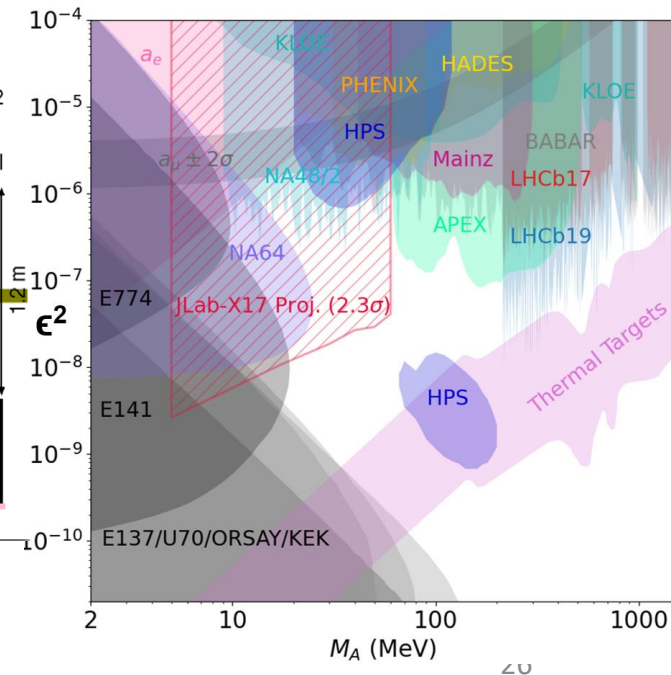


- The scattered electron and the pair produced  $e^+e^-$  will be detected using a pair of coordinate detectors and high resolution calorimeter
- As the only magnetic spectrometer free experiment at JLab it has an unique technique for background suppression
- Search for hidden-sector particles in **3~60 MeV** mass range

The PRad setup + 2 new GEM based tracking detectors



Will also be sensitive to the  $X \rightarrow \gamma\gamma$  channel but without optimal tracking



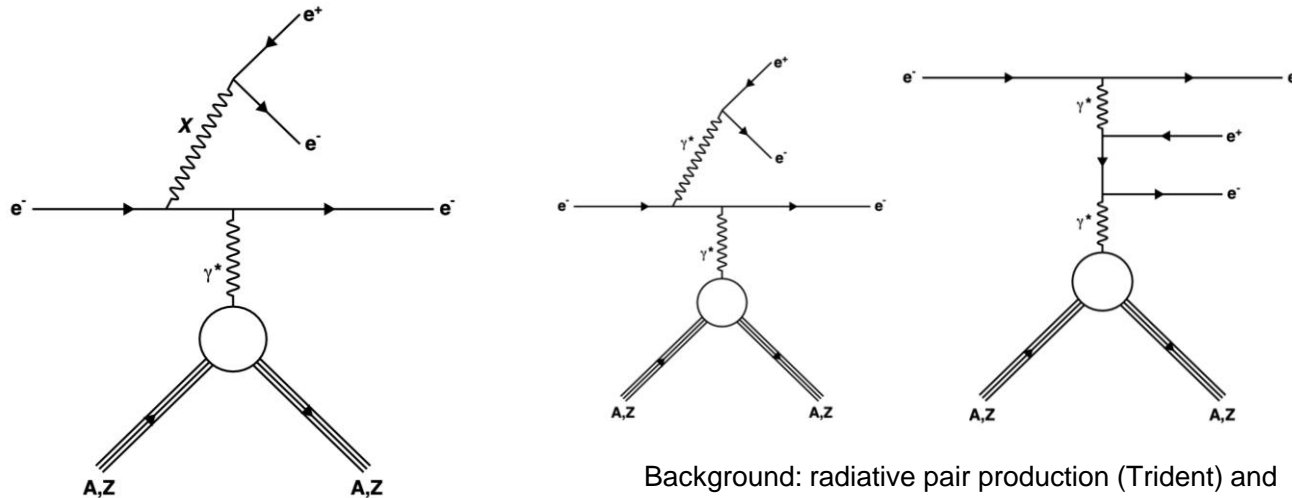
# Hidden Sector Particles Search Experiment

Forward angle electroproduction on a heavy nucleus

$$e^- + \text{Ta} \rightarrow e' + \gamma^* + \text{Ta} \rightarrow e' + X + \text{Ta}, \quad \text{with} \quad X \rightarrow e^+e^-$$

All 3 final state particles will be detected in this experiment:

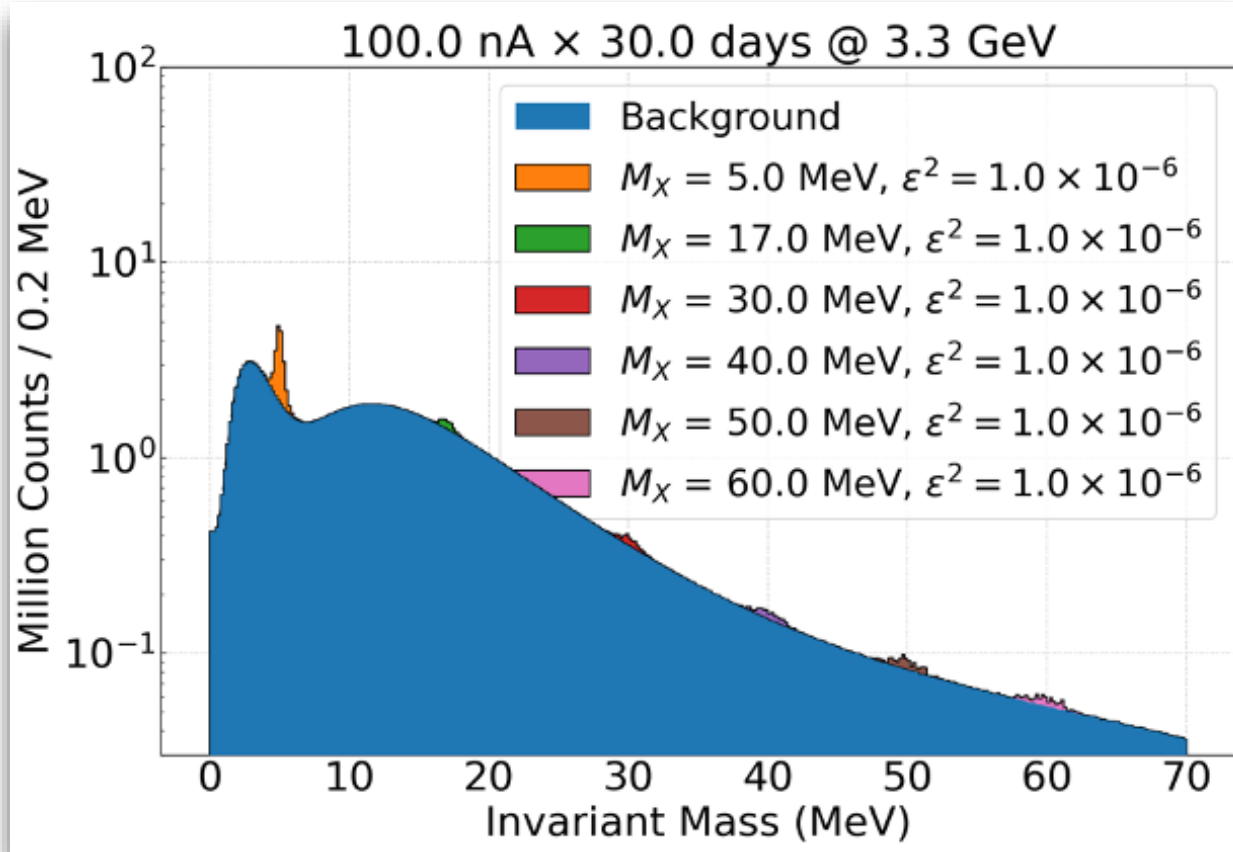
- The scattered electron and the pair produced  $e^+e^-$  – will be detected using a pair of coordinate detectors and high resolution calorimeter
- As the only magnetic spectrometer free experiments the JLab experiment has an unique technique for background suppression



Background: radiative pair production (Trident) and  
Bethe-Heitler process

Signal: Bremsstrahlung production of X

# Hidden Sector Particles Search Experiment

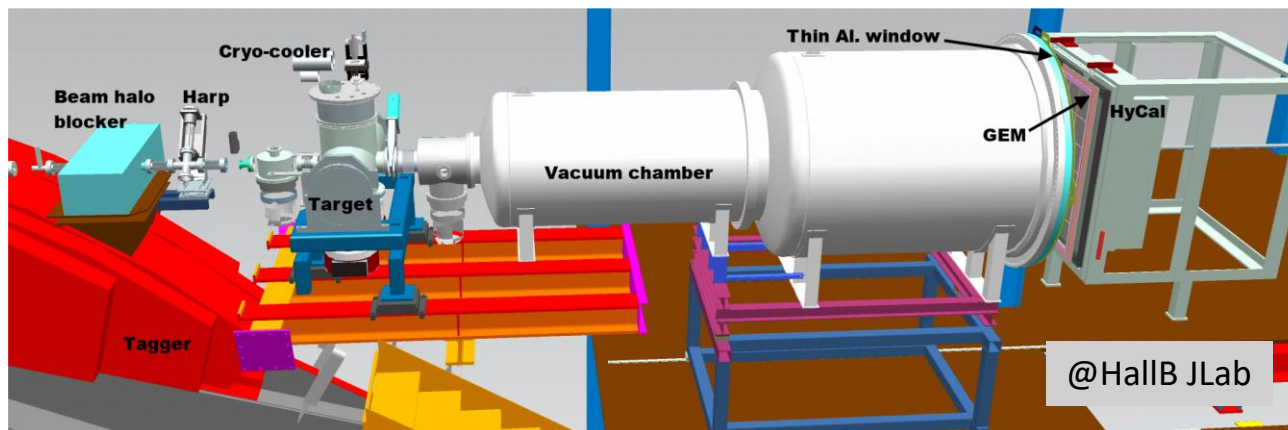


simulated background was **scaled**  
to 30 days of beam time

projected signal events  
with  $\epsilon^2 = 1.0 \times 10^{-6}$

# The PRad experiment overview

- Magnetic-spectrometer-free **calorimetric** method
- $E_{beam} = 1.1, 2.2 \text{ GeV}$ ,  $\theta' = 0.7^\circ \sim 7.0^\circ$
- Covers **two orders** of magnitude in low  $Q^2$  range **in one fixed setting**:  
 $[2 \times 10^{-4} \sim 6 \times 10^{-2}] (GeV/c)^2$

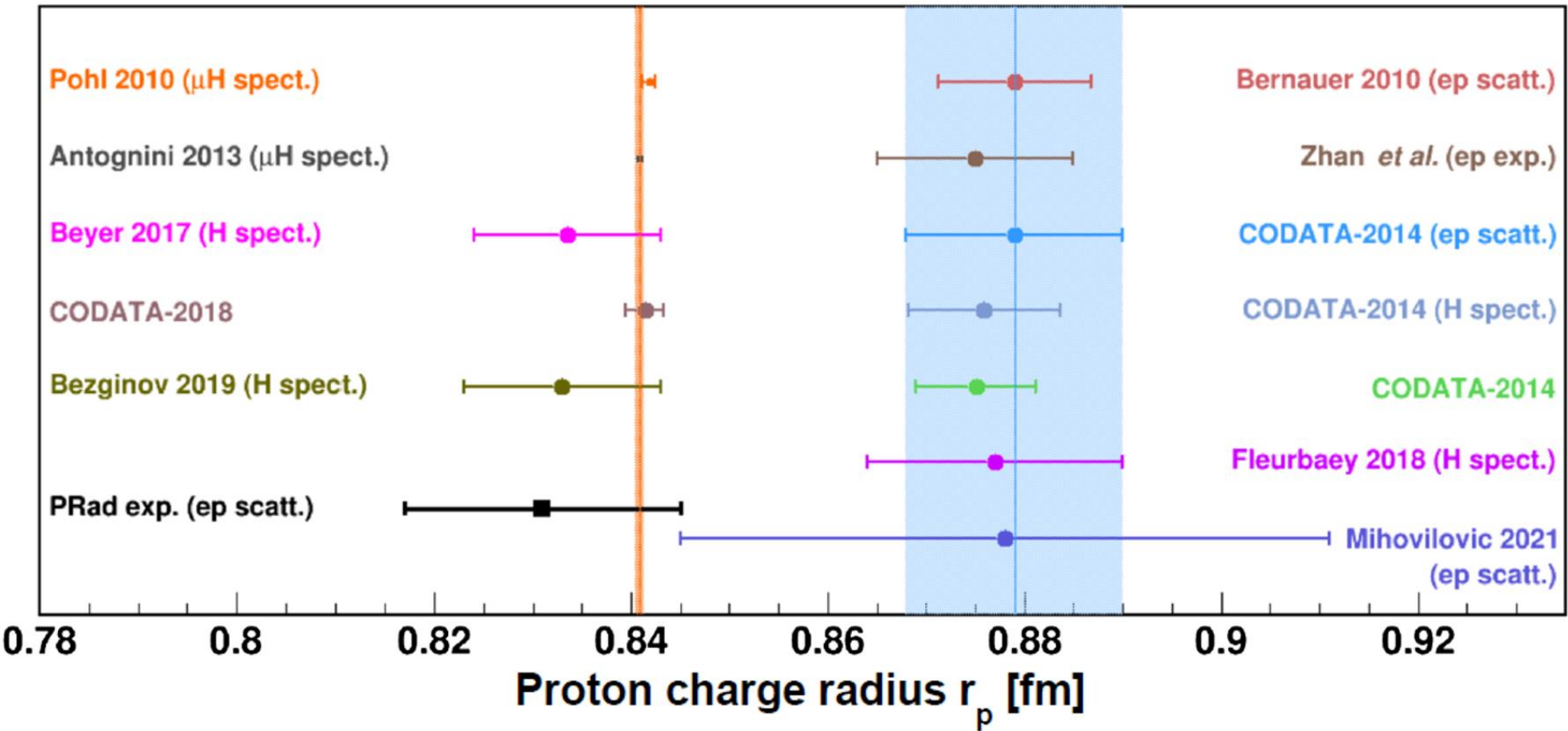


Xiong, W., et al., 2019, "A small proton charge radius from an electron-proton scattering experiment," Nature (London) 575, 147–150

- Unprecedented low  $Q^2$  ( $\sim 2 \times 10^{-4} (GeV/c)^2$ )
- A **windowless**  $H_2$  gas flow target removes major background source
- High resolution, large acceptance hybrid calorimeter detect and measure the electron energy
- Large area GEM detector for position measurement
- Simultaneous detection of  $ee \rightarrow ee$  **Møller** scattering process for normalization
- Extract the radius with precision from **sub-percent** cross section measurement



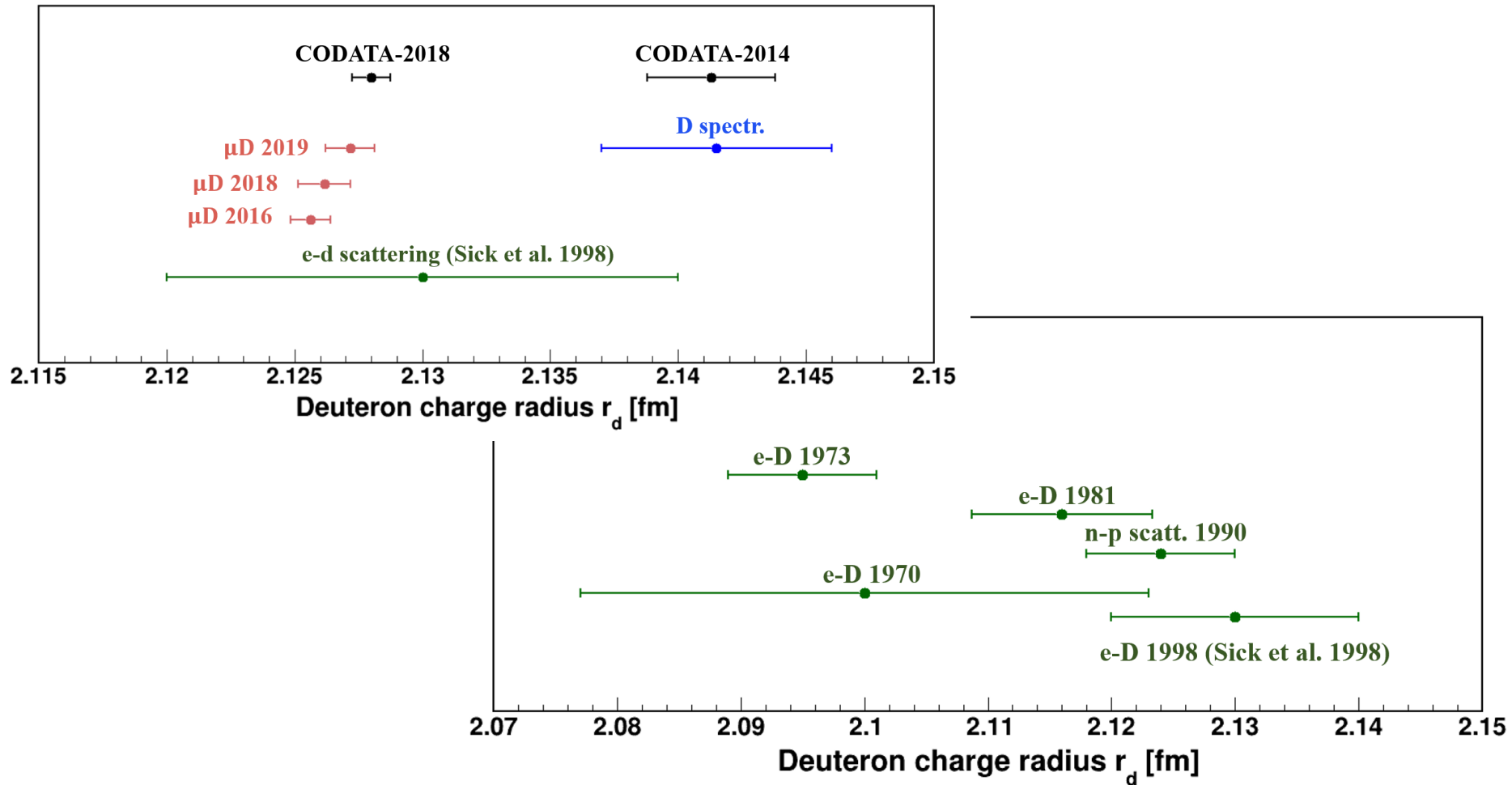
# The PRad-II experiment



RevModPhys.94.015002

# The deuteron charge radius puzzle

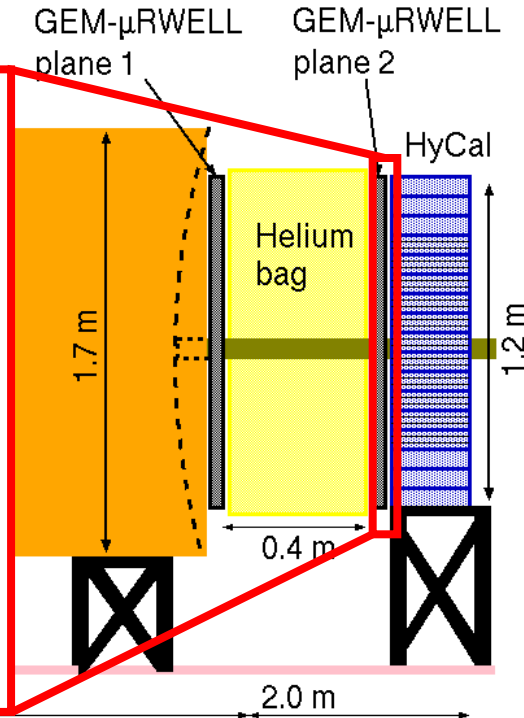
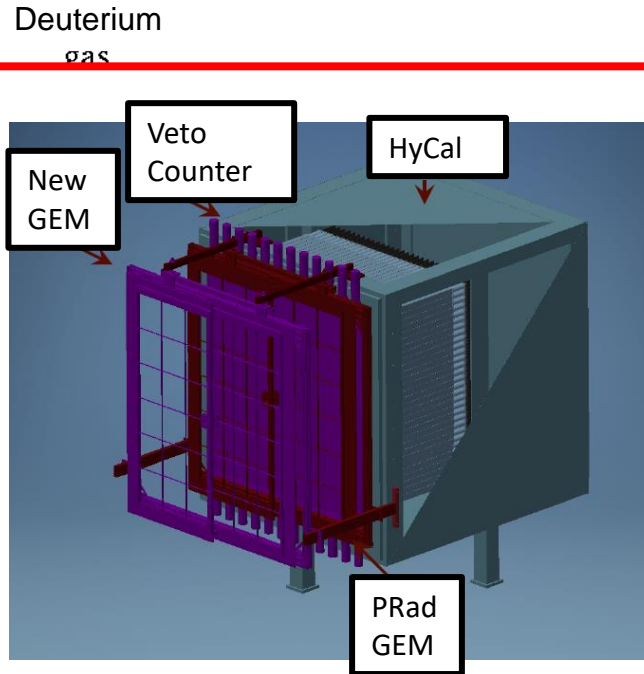
Independent of the famous “*Proton Charge Radius Puzzle*”



- $\sim 6\sigma$  discrepancy between  $\mu D$  spectroscopy results and CODATA-2014 value
- Uncertainties in previous e-d experiments are too large to resolve the puzzle

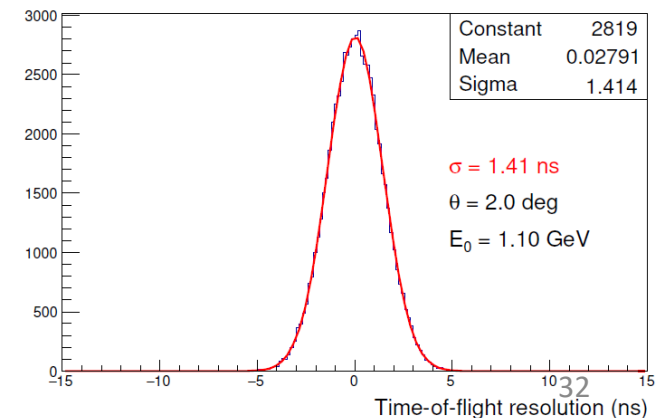
# DRad experiment apparatus

## Veto Scintillators



- The major background for the e-d elastic scattering is the e-d inelastic breakup process:  

$$e+d \rightarrow e+p+n$$
- Particle identification between deuteron and proton: measure the **time-of-flight difference** between the recoil detector and the HyCal

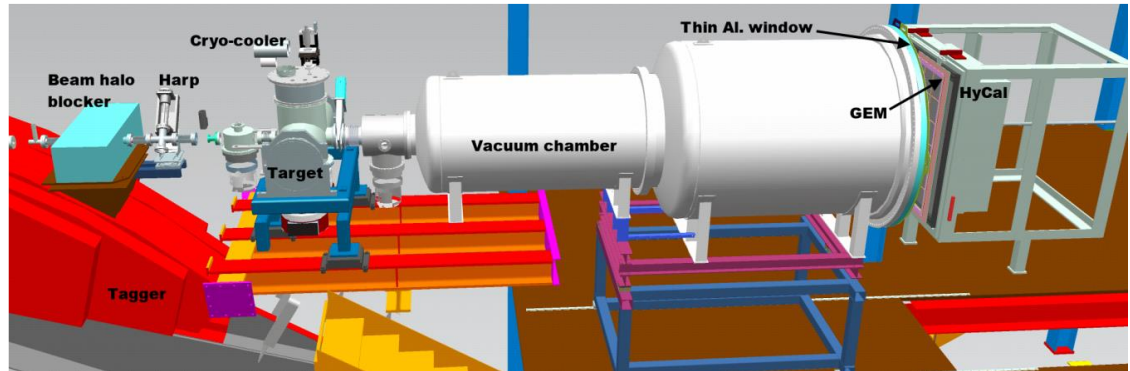


July 14, 2023

# The highlight of DRad experiment

DRad proposal: PR12-23-011

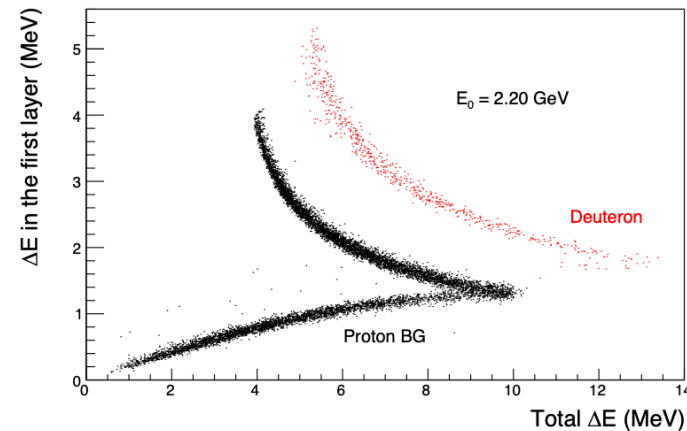
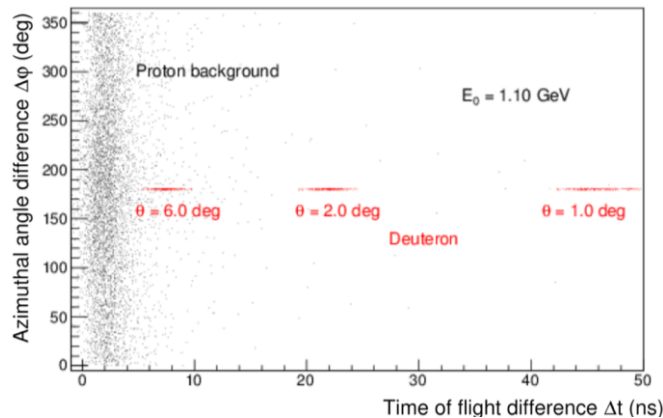
- Measure  $e$ - $d$  elastic cross sections at very low  $Q^2$  range:  
[ $5 \times 10^{-3} - 1.3$ ] fm $^{-2}$   
[ $2 \times 10^{-4} - 5 \times 10^{-2}$ ] GeV $^2$
- Two beam energies,  $E = 1.1$  and 2.2 GeV to increase  $Q^2$  range and control systematics.
- Experimental method based on PRad method and upgraded PRad-II (PR12-20-004):  
[W. Xiong et al. Nature 466 (2010) 213-216; H. Gao and M. Vanderhaeghen, Rev. Mod. Phys. **94**, 015002]
  - Magnetic-spectrometer-free **calorimetric** experiment;
  - **Windowless** deuterium/hydrogen gas flow target to reduce background;
  - Two planes of tracking detector for better scattered electron tracking (**PRad-II**);
  - Cylindrical recoil detector for reaction elasticity (**new**);
  - Veto counters for timing (**PrimEx veto counters**)
- That will allow:
  - Measure cross sections in one kinematical settings for a large  $Q^2$  range;
  - Simultaneous detection of  $ee \rightarrow ee$  Møller scattering process to control systematics;
  - Measure  $e$ - $d$  elastic cross section to **subpercent** precision



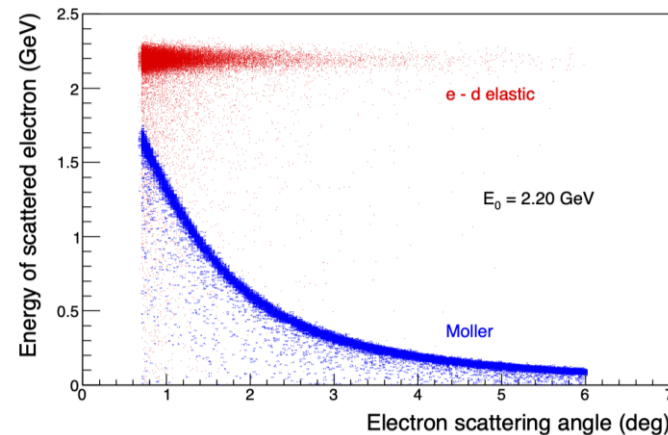
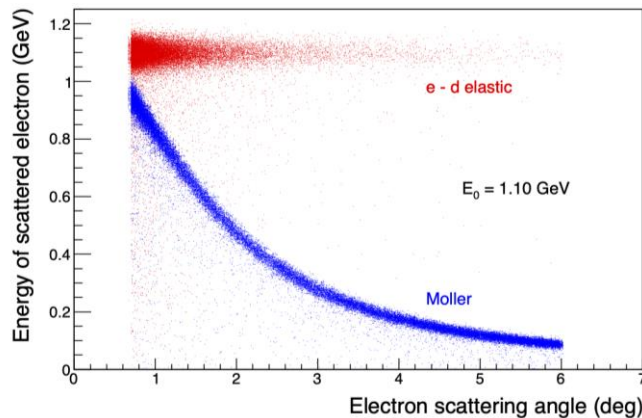
# PID and Event selection

*Comprehensive Geant4 simulation of the experiment was developed and used for studying the detection thresholds and backgrounds.*

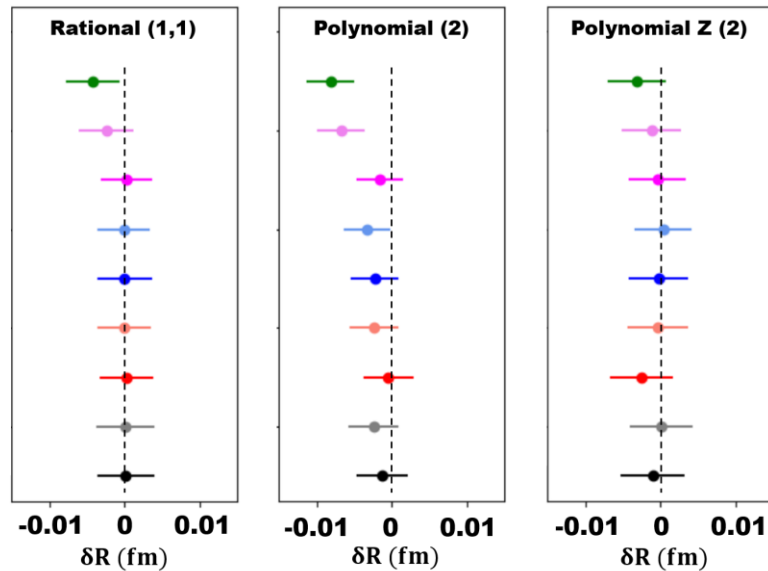
- ❑ Proton from breakup vs elastic recoil deuteron (Electro-disintegration rates are < 6% of the elastic rates)



- ❑ Møller event vs e-d elastic event



# The robust fitter study for PRad vs DRad



X. Yan *et al.* PRC98,025204 (2018)

Ye-2018

Bernauer-2014

Alarcón-2017

Arrington-2007

Arrington-2004

Kelly-2004

Gaussian

Monopole

Dipole

- Used **9** models to reflect various reasonable **approximations to the unknown true function** of  $G_E^p$
- Fitters: dipole, monopole, gaussian, rational, polynomial, poly-z, and continued fraction...
- Best fitter for PRad and PRad-II:

$$f_{\text{Rational}(1,1)}(Q^2) = p_0 \frac{1 + p_1^a Q^2}{1 + p_1^b Q^2} \quad r_{\text{fit}} = \sqrt{6(p_1^a - p_1^b)}$$

- $\text{Rational}(1,1)$  does not match  $G_C^d$  data at higher  $Q^2$  range  $\rightarrow$  search for possible new fitters
- Limited number of data-driven  $G_C^d$  parameterizations  $\rightarrow$  generalize the robustness test method

**Abbott I:**  $G_C^d(Q^2) = G_{C,0} \cdot \left[ 1 - \left( \frac{Q}{Q_C^0} \right)^2 \right] \cdot \left[ 1 + \sum_{i=1}^5 a_{ci} Q^{2i} \right]^{-1}$

**Parker:**  $G_C^d(Q^2) = G_{C,0} \cdot \left[ 1 - \left( \frac{Q}{Q_C^0} \right)^2 \right] \cdot \left[ \prod_{i=1}^5 (1 + |a_i| Q^2) \right]^{-1}$

**Abbott II:**

$$G_C^d(Q^2) = \frac{G^2(Q^2)}{(2\tau + 1)} \cdot \left[ \left( 1 - \frac{2}{3}\tau \right) g_{00}^+ + \frac{8}{3}\sqrt{2\tau} g_{+0}^+ + \frac{2}{3}(2\tau - 1) g_{+-}^+ \right]$$

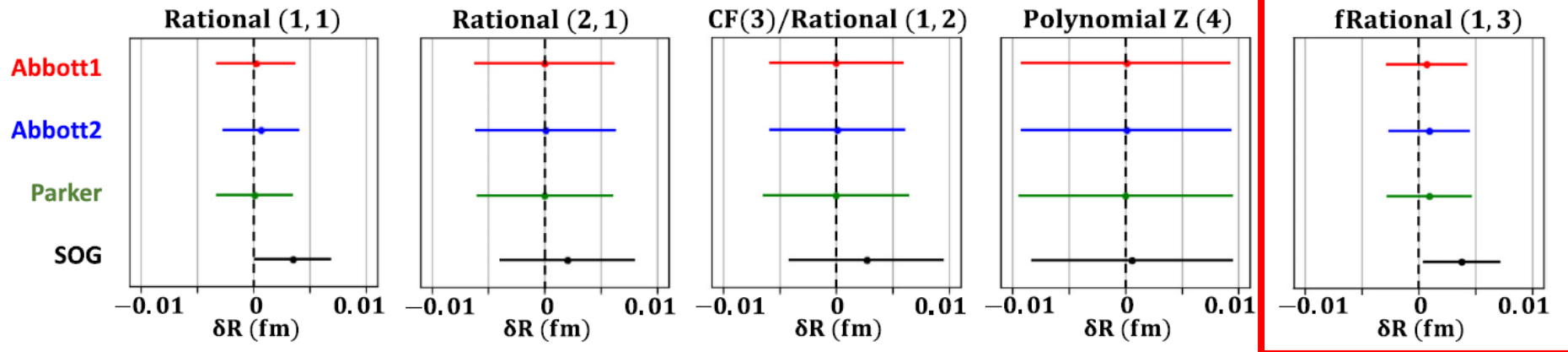
$$g_{00}^+ = \sum_{i=1}^n \frac{a_i}{\alpha_i^2 + Q^2} \quad g_{+0}^+ = Q \sum_{i=1}^n \frac{b_i}{\beta_i^2 + Q^2} \quad g_{+-}^+ = Q^2 \sum_{i=1}^n \frac{c_i}{\gamma_i^2 + Q^2}$$

**Sum-of-Gaussian(SOG):**

$$G_C^d(Q^2) = G_{C,0} \cdot e^{-\frac{1}{4}Q^2\gamma^2} \cdot \sum_{i=1}^N \frac{A_i}{1 + 2R_i^2/\gamma^2} \cdot \left[ \cos(QR_i) + \frac{2R_i^2}{\gamma^2} \frac{\sin(QR_i)}{QR_i} \right]$$



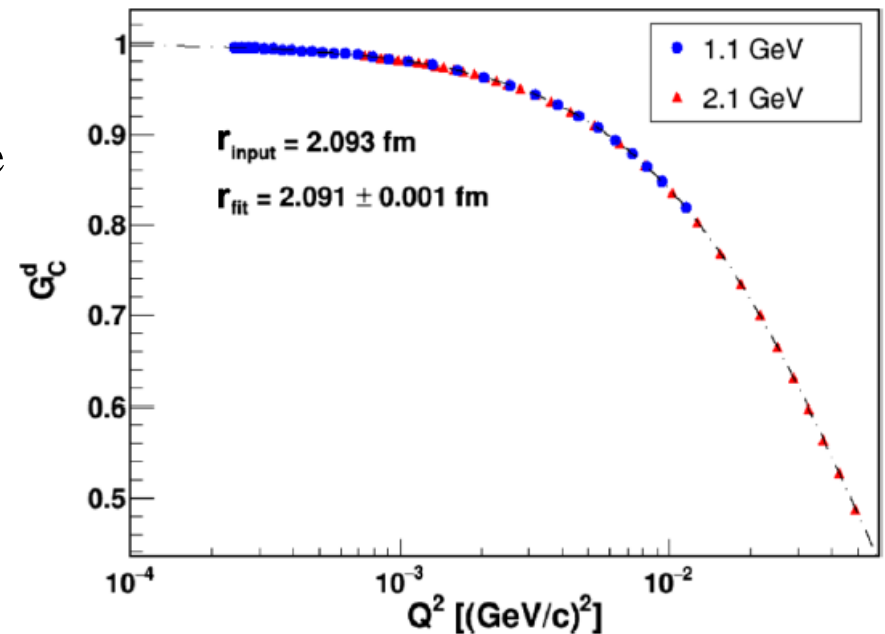
# Model dependent study in $r_d$ extraction



- Various functional forms were tested with modern parameterizations of the deuteron form factors, using DRad kinematic range and uncertainties.
- Fixed Rational (1,3) was identified as a robust fitter with lowest uncertainties

$$f_{\text{fixed Rational}(1,3)}(Q^2) = p_0 \frac{1 + a_1 Q^2}{1 + b_1 Q^2 + b_{2,\text{fixed}} Q^4 + b_{3,\text{fixed}} Q^6}$$

$$r_{\text{fit}} = \sqrt{6(a_1 - b_1)}$$



J.Zhou *et al.* PRC103, 024002 (2021)



# Radiative Correction (RC) Calculations

