Dilution Factor Determination for RG-C

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This is an extract from the Run Plan for RG-C, focusing on the determination of the proper dilution factor for each Physics channel under investigation.

CONTENTS

I. OVERVIEW

Hall-B Run Group C ran from June 8, 2022, to March 20, 2023. The run used 2.2 GeV (1-pass) and 10.5 GeV (5-pass) electron beam with high polarization on polarized ammonia and deuterated ammonia targets (5 cm long cells immersed in liquid ^{4}He), as well as several auxiliary targets. The run had four distinct parts:

1. Commissioning at 2.2 GeV (4 days at the beginning) - this was used to commission the raster system and target operations, and to measure the product of beam and target polarization through elastic and quasi-elastic scattering. We also used these 4 days to measure spin structure functions at lower Q^2 to cross-check with earlier measurements (CLAS6 run group EG1b).

2. June 13 – August 31 we ran with the standard "FTOn" configuration and 5 pass beam, using a 5 cm long, 1.5 cm diameter target cell filled with either ammonia (NH_3) or deuterated ammonia (ND_3) for 15 PAC days each. Due to the smaller size of the FTOn Möller cone, the raster radius was reduced to about 6 mm and the beam current to 4 nA, for a luminosity of about 4×10^{34} nucleons/cm^2 times electrons/s. We ran for 69 days (nominally 35 PAC days but, because of the much lower efficiency than expected, only 28 PAC days of real data).

3. After about 1 week of configuration change, we began running the FTout part of RG-C with the new bespoke Möller cone (named "ELMO" for "Extra Large MOller shield") that can accommodate a 8-9 mm raster on a 2 cm diameter target cell (also 5 cm long) and 7-8 nA beam for a luminosity of roughly 8×10^{34} nucleons/cm^2 times electrons/s. Again, this period was split evenly between ammonia and deuterated ammonia. Deviating from the originally agreed upon run plan, this part of the run was first foreseen to end December 18, 2022, for a total of 107 calendar days or 53 PAC days (instead of the originally planned 90 PAC days); due to the failure of the solenoid magnet power supply on November 11, this was shortened to 70 days (35 PAC days, although the efficiency was significantly lower, resulting in an effective number of 25.4 PAC days).

4. Starting January 30, 2022, for the remainder of the scheduled runtime (7 weeks until 3/20/23), we reverted to the FTon configuration (see item 2).

In this document, we discuss how to use the measurements on the auxiliary targets taken during the experiment (Carbon-12, Polyethylene = CH_2, deuterated polyethylene = CD_2, "Empty" = MT target filled only with an empty target cell as well as liquid Helium-4, and "Foils only" without even the liquid Helium) to extract the fraction of the observed events of interest stemming from the nucleon (H) or nuclear (D) species of interest during the ammonia runs, the so-called dilution factor. After a short introduction, the "experimental method" is explained in more detail and the relevant equations are given. For more background information, see the original "Runplan", and the updates in "RunPlanFTout" and "RunPlanFTon2" (accessible from the RG-C wiki).

II. INTRODUCTION

For any observable that is proportional to the target polarization (target single-spin and double spin observables), we must know the fraction of events that come from the polarized target of interest, and correct for the contribution of all other nuclear species in the beam that cannot be separated by a vertex cut. This fraction is called the "dilution factor" DF, and it depends both on the kinematics (dependent on *all* kinematic variables used to bin the data) and the specific reaction under study. If we call A_{phys} the desired Physics asymmetry on a polarized proton or deuteron, then the measured asymmetry within a given kinematic bin is given as

$$A_{meas} = A_{phys} * (P_b)P_t * DF, \tag{1}$$

where P_b is the (kinematics-independent) beam polarization (for the case of a measurement of a double-spin asymmetry) and P_t is the target polarization (also independent of kinematics or reaction under study), while

$$DF = DF(x, Q^2, t, \phi, ...) \tag{2}$$

is the dilution factor. In the following, we use as an illustration DIS, where we measure the double spin asymmetry on a proton or deuteron for inclusive electron scattering. In that case, the only relevant kinematic variables are x and Q^2. However, all equations and most numeric values in what follows can be applied to any other reaction of interest, as long as we bin the events from all targets in the same kinematic variables.

To determine DF means to calculate which fraction of the overall observed counts in a kinematic bin, due to all material in the beam (and not excluded by cuts), comes from the H or D atoms in the (deuterated) ammonia beads.

Strictly speaking, there are additional backgrounds one has to consider, in particular *polarized* target species that can contribute to the measured asymmetries. For instance, the ^{14}N nuclei in ammonia are slightly polarized themselves, so there is some (deuterium-like) asymmetry that has to be accounted for. Similarly, there may be some H atoms replacing D in ND_3 (isotopic impurity) and vice versa, which will also contribute a unwanted asymmetry. Finally, there may be target components that are not intended and may not be easy to detect, e.g. frozen water ice inside the target cells due to exposure to air, or frozen storage coolant (Argon or Nitrogen). Presumably, all these contributions may require small corrections to the measured asymmetries, but are not further considered here (refer to earlier experiments like EG1b and EG1-DVCS for information on how they have been dealt with before).

The ideal method for determining DF for any channel would be to create a complete (and completely accurate) generator for all reactions that can occur on all target nuclei, including the Aluminum foils, liquid ^{4}He, and both Nitrogen and H or D in ammonia. Such a generator must properly account for radiative effects (both external radiative losses before the scattering and internal radiative corrections to the Born cross section) as well as Fermi motion, EMC effect and other effects in nuclear targets (e.g., p_T enhancement or z-dependent depletion of outgoing mesons in SIDIS). Such a generator can be tested and even tuned by checking that it can properly describe the observed count rates from *all* auxiliary targets discussed below. After that, it is a simple matter of tagging the reconstructed GEMC events with the target nucleon/nucleus from which they originated to calculate DF.

It is important to note here that even if the densities and thicknesses of all target materials are well-known, there is still one unknown that would enter such a simulation-based extraction of DF: the packing fraction PF of the ammonia beads inside the target cell. This is due to the fact that the frozen ammonia that serves as our source of polarized H and D atoms is broken into small beads of a few mm in size which are then filled into the 5-cm long target cells that are inserted into the nose cone of the polarized target. Since these beads have irregular sizes and shapes, it is impossible to determined the exact volume fraction occupied by beads (PF) vs. interstitial voids which, during target operation, will be filled with liquid (superfluid) ^{4}He. Since these beads would evaporate outside a liquid coolant bath (liquid nitrogen or argon), it is difficult to measure the amount of ammonia in any given target cell. Hence, the PF must be extracted from measurement - either using experimental data alone (see next Section) or by comparing the total counts predicted from the ammonia MC, as a function of that PF, with the measured counts (assuming that the generator is properly tuned to represent all nuclear species in question).

In the following, we explain a (preliminary) method to extract the desired information directly from measured count rates alone, using the auxiliary targets. One caveat: The equations below simply use the measured number of counts from each target type, and hence ignore potentially (slightly) different radiative effects for different targets. While we attempted to keep external radiative effects similar by giving each target roughly the same thickness in terms of radiation length (RL), a correction may still be needed. In particular this is of course the case for the empty and foils-only target runs, since the density of liquid ^{4}He alone is a lot smaller than all of the other nuclear species and molecular compounds (ammonia). Reasonable approximations of these corrections could be gotten from a "good-enough" generator that can be used to correct the measured counts with ratios of radiative corrections for the targets as used vs. for the ammonia targets of interest.

III. DETERMINING THE RATES FROM DIFFERENT TARGETS

As a first step, we write down expressions for the observed counts within a given kinematic bin from each of the target configurations that we used during RG-C. We designate the total run time for a given part of the experiment with T and the fraction of that time spent for a specific configuration "Y" as X_Y. Practically speaking, we can think of T as the number of PAC days for a given Physics observable (e.g., DVCS on p or DVCS on n), and X_Y as the fraction of this time spent on a specific target / detector configuration "Y" that enters the extraction of that observable.

1. Configurations

For the following, we assume that the observable is a double-spin or single-target-spin asymmetry on either polarized hydrogen or polarized deuterium. To extract the desired Physics asymmetry from the measurement on the full target, one has to account for contributions from non-hydrogen target components (see Introduction), typically expressed in the form of a dilution factor: $A_{meas} = A_{phys} * P_b P_t * DF$ where $P_b P_t$ is the product of beam and target polarization (or only the target polarization for single-spin asymmetries) and DF is the dilution factor, which is defined as the number of counts from the desired hydrogen species divided by all counts from all target components.

RG-C took data with the following different configurations:

NH/ND: Standard running with polarized e^- on the target cell filled with the polarized ammonia species of interest (NH_3 or ND_3). The total number of counts within a given bin ($\Delta Q^2, \Delta x$,...) and within cuts can be written as

$$N_A = X_A f_c \left[l_F \rho_F \Delta\sigma_F + (L - l_A)\rho_{He}\Delta\sigma_{He} + l_A \rho_A \left(\frac{7}{6}\Delta\sigma_C + 3\Delta\sigma_H \right) \right] \quad (3)$$

Here, all lengths l_i are measured in cm, all densities ρ_i in mol/cm^3 (=ρ_i in g/cm^3 divided by the atomic number of the species in question), and all cross sections are in cm^2. The factor f_c converts the fraction of the full beam time into the product of total number of incident electrons times Avogadro's number (to relate the number of mols/cm^2 to the number of nuclei/cm^2) and also includes the efficiency of CLAS and other proportionality factors. Note that all cross sections are per nucle**us**, so that the cross section for, e.g., ^{12}C is roughly 6 times that for deuterium.

The terms inside the brackets describe various contributions to the measured counts:

(a) Contribution from foils: $l_F \rho_F \Delta\sigma_F$. There are a total of four Al foils within vertex cuts - the beam entrance foil into the liquid helium bath, the entrance foil into the target cell, and the exit foils from both. The total amount of material from these foils is about $l_c\rho_c = 0.027$ g/cm^2 = 0.001 mol/cm^2, which is a small fraction of the overall target density (about 3 g/cm^2). It will suffice to measure the rate from these foils (with an empty target cell installed and no liquid He filled) once for each beam energy and torus configuration and then subtract this contribution from all other measured rates. This should take only a few hours with high beam current (the target will have low luminosity); however, some corrections will have to be applied. In particular, the rate from the exit foils will have to be corrected for pre-scattering external bremsstrahlung effects which will be more prominent for "full" targets, and the entrance foils will need to have a similar correction for after-scattering external bremsstrahlung (which can be at least partially suppressed by correcting the scattered electron energy by "adding in" the energy of bremsstrahlung photons detected in the same direction).

(b) Contribution from liquid He coolant: $(L - l_A)\rho_{He}\Delta\sigma_{He}$. Here, L is the total length of the liquid helium bath from entrance to exit window, and l_A is the effective length of the ammonia beads, i.e., the product of target cell length (5 cm) times packing fraction PF (of order 60%).

(c) Contribution from the nitrogen part of ammonia: $l_A\rho_A\frac{7}{6}\Delta\sigma_C$. This ansatz assumes that the cross section on ^{14}N is about 7/6 that on ^{12}C, given that they both have the same n/p ratio and similar binding energies. It could be refined for specific reactions where this assumption is not entirely correct.

(d) Contribution from the hydrogen (deuteron) part of ammonia: $l_A\rho_A 3\Delta\sigma_H$. The factor 3 accounts for the three hydrogen/deuteron atoms per ammonia molecule. The quantity $\Delta\sigma_H$ is to be understood as the generic cross section on those atoms - in the case of inclusive scattering on the Deuteron, it would of course be equal to $\Delta\sigma_D$. In general, the ratio $\frac{\Delta\sigma_H}{\Delta\sigma_D}$ can range from as little as 1/2 (for DIS on the proton at small x, where cross sections on protons and neutrons are similar) to values greater than 1, which can happen for exclusive channels with tight exclusivity cuts, where Fermi smearing in the deuteron would reduce the cross section in the denominator relative to that in the numerator. (A similar modification can happen for other ratios, as well, where, e.g., the cross section on ^{12}C **within cuts** might be much smaller than that on the deuteron times 6. On the other hand, for SIDIS channels at high P_t, the cross section on deuterium might be even smaller than 1/6 of that on ^{12}C, due to momentum broadening of outgoing hadrons in the nuclear medium.)

The dilution factor is the ratio of the last of these 4 contributions divided by the sum from all 4. To evaluate this expression, one needs to know the value of all parameters that appear in the Eq. 3. All of the densities are either well-known or can be measured precisely. Of the relevant lengths, only l_A is uncertain and can vary from one target cell to the next, depending on how densely the ammonia beads are packed. The remaining unknowns (after subtracting the measured contribution from the foils) are the cross sections for Helium, Carbon and Hydrogen (Deuterium). This indicates that at least 3 additional measurements must be made to solve for all unknowns. (In principle, there is a fifth unknown, namely the conversion factor f_c, but it will cancel in the calculation of DF).

The three auxiliary measurements needed are discussed next:

MT: Run on empty target cell (with foils) and filled liquid He bath, but no other target components. The total number of counts in the given bins and within cuts is

$$N_{MT} = X_{MT} f_c \left[l_F \rho_F \Delta\sigma_F + L\rho_{He}\Delta\sigma_{He} \right] \quad (4)$$

This equation contains no additional unknowns and can be solved for $\Delta\sigma_{He}$, modulo the factor f_c.

C: Run on a target cell (with foils) containing 3 ^{12}C foils with a combined radiation thickness equal to that of the ammonia target, and filled liquid He bath. The total number of counts in the given bins and within cuts is

$$N_C = X_C f_c \left[l_F \rho_F \Delta\sigma_F + (L - l_C)\rho_{He}\Delta\sigma_{He} + l_C \rho_C \Delta\sigma_C \right] \tag{5}$$

This equation can be solved for $\Delta\sigma_C$, modulo the factor f_c, using as input the result from the MT run and the total thickness l_C of carbon, which can be measured accurately.

CH: Run on a target cell (with foils) containing 3 Polyethylene ($(^{12}CH_2)_n$) foils with a combined radiation thickness equal to that of the ammonia target, and filled liquid He bath. The total number of counts in the given bins and within cuts is

$$N_{CH} = X_{CH} f_c \left[l_F \rho_F \Delta\sigma_F + (L - l_{CH})\rho_{He}\Delta\sigma_{He} + l_{CH} \rho_{CH} \left(\Delta\sigma_C + 2\Delta\sigma_H \right) \right] \tag{6}$$

This equation can be solved for $\Delta\sigma_H$, modulo the factor f_c, using as input the result from the previous two measurements. Both the total thickness l_{CH} and the density of polyethylene can be measured accurately. Ideally, we would use both CH_2 and CD_2 as targets for extraction of proton and deuteron asymmetries; however, if the latter are not available, we can use the measurements on CH_2 and scale the cross section $\Delta\sigma_H$ with the ratio $\Delta\sigma_D/\Delta\sigma_H$. This ratio is well-known over a wide kinematic range, in particular for inclusive scattering. For SIDIS, this could be augmented by isospin-symmetry arguments. For nDVCS, $\Delta\sigma_H$ could be related to $\Delta\sigma_n$, given that the cross section is dominated by Bethe-Heitler processes which can be calculated accurately.

The 4 equations 3–6 above can be solved for the four unknowns and the dilution factor expressed in terms of the measurement results. The resulting algebra is rather lengthy and unwieldy; it has been programmed in a Mathematica notebook. Here is the result:

$$DF = \frac{9(n_A - n_{MT})\rho_A(l_C L(-n_{CH} + n_{MT})\rho_C + l_C l_{CH}(n_F - n_{MT})(\rho_C - \rho_{CH}) + l_{CH} L(n_C - n_{MT})\rho_{CH})}{n_A(9 l_C L(-n_{CH} + n_{MT})\rho_A\rho_C + 2 l_{CH} L(n_C - n_{MT})\rho_A\rho_{CH} + l_C l_{CH}(n_F - n_{MT})(9\rho_A\rho_C - 2(\rho_A + 3\rho_C)\rho_{CH}))}. \tag{7}$$

Here, the lower case $n_i = N_i/X_i$ to account for the fraction of total running time X_i spent on each target. This assumes that all 4 targets will be run with the same beam current (otherwise one has to define X_i as the integrated charge on each of the various targets). Note that only the "MT" target will run with significantly higher beam current than the others, which doesn't matter much anyway because (as it turns out) very little running time on that target is required (of order 1-2%). We give a numerical estimate for the factors in Eq. 7 below (Section III 2).

For comparison or combination with different analysis methods, we also give the result for the Packing Fraction PF:

$$PF = \frac{6 l_C \rho_C l_{CH} \rho_{CH}(n_A - n_{MT})}{2 l_{CH}\rho_{CH} L \rho_A (n_{MT} - n_C) + 9 l_C \rho_C L \rho_A (n_{CH} - n_{MT}) + l_C l_{CH}(n_{MT} - n_F)(9\rho_A\rho_C - 2(\rho_A + 3\rho_C)\rho_{CH})}. \tag{8}$$

2. Example for Dilution Factor extraction

Within the model for the dilution factor *df* described here, we can write down a formula how to calculate it once we have data on all auxiliary targets. (For the final analysis, we will have to refine this by correcting for different radiation lengths of the different target components). We are making the following assumptions (from the target drawings and photos):

- $l_C = 1.678$ cm
- $L = 5.86$ cm
- $l_{CH} = 3.18$ cm
- $\rho_{He} = 0.145/4.0026 = 0.03623$ mol/cm^2
- $\rho_C = 1.79261/12 = 0.1494$ mol/cm^2

- $\rho_{CH} = 0.9425/14.0266 = 0.06719$ mol/cm^2
- $\rho_A = 0.92/17$ (still needs confirmation)

This leads to the following equation for the dilution factor:

$$DF = 0.19573 \frac{(n_A - n_{MT})(6.62379 r_{CH} - 5.64604 n_C - 1.97775 n_F + n_{MT})}{n_A(1.29647 n_{CH} - 0.245577 n_C - 0.0508949 n_F - n_{MT})} \tag{9}$$

For a reasonable set of assumptions, and assuming that $\Delta\sigma_p = 0.7\Delta\sigma_D$, we find a dilution factor of 0.20 for polarized hydrogen and 0.27 for polarized deuterium.

Meanwhile we get for the packing fraction:

$$PF = 0.582331 \frac{n_A - n_{MT}}{1.29647 n_{CH} - 0.245577 n_C - 0.0508949 n_F - n_{MT}}. \tag{10}$$

Note that in this equation, PF is defined relative to the entire length of the liquid Helium "bathtub", $L = 5.86$ cm. If you want to know PF for just the target cell itself, this result would have to be multiplied with $L/5cm = 1.17$:

$$PF = 0.682492 \frac{n_A - n_{MT}}{1.29647 n_{CH} - 0.245577 n_C - 0.0508949 n_F - n_{MT}}. \tag{11}$$