DRAFT
SANE-HMS Nitrogen Correction

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1 Introduction

Nitrogen correction, C_N is the factor to eliminate the nitrogen polarization. Unlike RSS using ^{15}N, SANE has ^{14}N for ammonia. According to an RSS technote [1], the nitrogen correction is

$$C_N = 1 + \frac{A_{15}}{A_1}\frac{f_{15}}{f_1}\frac{P_{15}}{P_1}, \tag{1}$$

where A's are asymmetries, f's are dilution factors, and P's are polarizations, with subscript denoting the mass number of each nucleus. In this formula, the ratio of asymmetries is approximated by shell model.

SANE should use different model for nitrogen asymmetry. And nitrogen polarization should be re-calculated by equal spin temperature (EST) model. SMC and CLAS EG1-DVCS have their own approximations for it, and actually CLAS followed the SMC's analysis[2, 3]:

$$C_N = 1 + \frac{\eta_N}{\eta_p}\frac{A_N\sigma_N}{A_p\sigma_p}\frac{P_N}{P_p}, \tag{2}$$

where η is the number of nuclei, and σ the cross section per nucleus, with subscript indicating the name of nucleus. So $\frac{\eta_N}{\eta_p}$ is 1/3 for ammonia. Considering spin-1 nitrogen-14 nucleus as a spinless carbon surrounded by extra proton and neutron,

$$\sigma_N A_N = -\frac{1}{3}(\sigma_p A_p + \sigma_n A_n) \approx -\frac{1}{3}\sigma_d A_d, \tag{3}$$

where subscript n means neutron, d deuteron. So, the ratio of (Asymmetry) $\times$ (Cross Section) of deuteron and proton becomes important. Of course, polarization ratio of nitrogen to proton should be known also.

2 Asymmetry and Cross Section

$$\sigma^{unpol.} = \frac{d^2\sigma^{unpol.}}{d\Omega dE'} = \frac{2\alpha^2 E'}{Q^2 E}\frac{F_1}{MD'}, \tag{4}$$

where $D' = \frac{1-\epsilon}{1+\epsilon R}$ as ϵ is the virtual photon polarization, while F_1 and R are unpolarized structure functions [4]. So, the ratio of cross sections per nucleon of deuteron and proton is

$$\frac{\sigma^d}{\sigma^p} = \frac{F_1^d D'^p}{F_1^p D'^d}, \tag{5}$$

where superscript p is for proton and d for deuteron.

The following formulae are originally from Ref.[5] and modified in Hoyoung's Thesis[4]. (A_{180}, A_{80}) and (g_1, g_2) have the following relation:

$$A_{180} = \frac{-D'_{180}}{F_{1,180}}[-\frac{E_{180} + E'_{180}\cos\vartheta_{180}}{E_{180} - E'_{180}}g_1 + \frac{Q^2_{180}}{(E_{180} - E'_{180})^2}g_2], \tag{6}$$

$$\begin{aligned} A_{80} = \frac{-D'_{80}}{F_{1,80}}[&\frac{E_{80}\cos 80^\circ + E'_{80}(\sin\vartheta_{80}\cos\varphi_{80}\sin 80^\circ + \cos\vartheta_{80}\cos 80^\circ)}{E_{80} - E'_{80}}g_1 \\ &+ \frac{2E_{80}E'_{80}(\sin\vartheta_{80}\cos\varphi_{80}\sin 80^\circ + \cos\theta_{80}\cos 80^\circ - \cos 80^\circ)}{(E_{80} - E'_{80})^2}g_2]. \end{aligned} \tag{7}$$

So, when we calculate $\frac{\sigma^d}{\sigma^p}\frac{A_d}{A_p}$, $\frac{\sigma^d}{\sigma^p}$ is cancelled out by the factor of $\frac{A_d}{A_p}$, i.e. $\frac{D'^d F_1^p}{F_1^d D'^p}$.

In the SANE-HMS resonance region, at least, it seems like $A_{180} \propto g_1$ and $A_{80} \propto g_2$, roughly. It was discussed in Ref.[4], as following Table 1 and Figure 1.

So, if we assume $A_{180} \propto g_1$ and $A_{80} \propto g_2$,

$$\frac{\sigma^d}{\sigma^p}\frac{A_d}{A_p} \approx \frac{g_1^d}{g_1^p}, \tag{8}$$

for A_{180}, and

$$\frac{\sigma^d}{\sigma^p}\frac{A_d}{A_p} \approx \frac{g_2^d}{g_2^p}, \tag{9}$$

for A_{80}.

But it needs further investigation. Before that, it would be better to use world data of unpolarized cross sections and asymmetries of proton and deuteron.

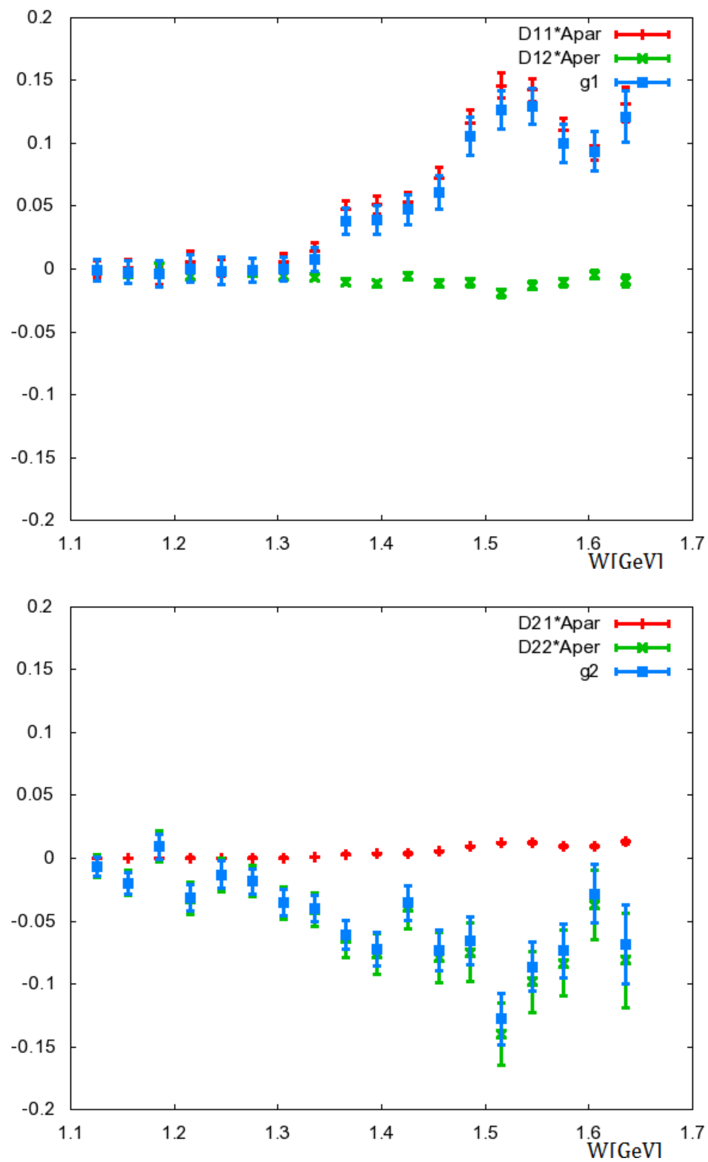

Figure 1: (Top) g_1 as a sum of $D_{11}A_{180}$ and $D_{12}A_{80}$, (Bottom) g_2 as a sum of $D_{21}A_{180}$ and $D_{22}A_{80}$
, where $A_{par} = A_{180}$ and $A_{per} = A_{80}$

Table 1: Matrix elements of the transformation from (A_{180}, A_{80}) to (A_1, A_2) and (g_1, g_2), i.e. $(A_1, A_2) = C(A_{180}, A_{80})$ and $(g_1, g_2) = D(A_{180}, A_{80})$.

	C				D			
$W(GeV)$	C_{11}	C_{12}	C_{21}	C_{22}	D_{11}	D_{12}	D_{21}	D_{22}
1.095	1.5826	-2.2007	2.0173	2.7772	0.0048	0.0012	0.0002	0.0057
1.125	1.6152	-2.1441	2.0129	2.7642	0.0236	0.0056	0.0012	0.0284
1.155	1.6555	-2.0796	2.0178	2.7377	0.0671	0.0155	0.0036	0.0805
1.185	1.6900	-2.0113	2.0123	2.7087	0.1453	0.0325	0.0081	0.1739
1.215	1.7013	-1.9493	1.9760	2.6935	0.2026	0.0448	0.0117	0.2472
1.245	1.7026	-1.8849	1.9277	2.6773	0.1886	0.0417	0.0113	0.2359
1.275	1.7093	-1.8143	1.8847	2.6521	0.1648	0.0361	0.0102	0.2101
1.305	1.7179	-1.7545	1.8449	2.6403	0.1598	0.0348	0.0102	0.2085
1.335	1.7299	-1.6973	1.8088	2.6316	0.1685	0.0365	0.0111	0.2248
1.365	1.7382	-1.6487	1.7704	2.6346	0.1830	0.0397	0.0125	0.2512
1.395	1.7540	-1.5989	1.7399	2.6336	0.2024	0.0437	0.0143	0.2843
1.425	1.7777	-1.5605	1.7179	2.6502	0.2332	0.0502	0.0171	0.3359
1.455	1.8222	-1.5368	1.7153	2.6914	0.2956	0.0633	0.0225	0.4358
1.485	1.8760	-1.5283	1.7214	2.7597	0.4172	0.0892	0.0330	0.6326
1.515	1.8767	-1.4768	1.6784	2.7518	0.4560	0.0974	0.0375	0.7116
1.545	1.8302	-1.3896	1.5958	2.6717	0.3911	0.0833	0.0335	0.6275
1.575	1.8109	-1.3230	1.5399	2.6211	0.3770	0.0794	0.0337	0.6196
1.605	1.8224	-1.2815	1.5117	2.6132	0.4105	0.0851	0.0384	0.6909
1.635	1.8503	-1.2590	1.4977	2.6426	0.4860	0.0995	0.0479	0.8425

3 Polarization

For the ratio of polarizations, $\frac{P_N}{P_p}$, it has been known that EST matches well with actual polarization measurements [6, 7, 2].

$$P_p = \tanh\left(\frac{\hbar\omega_p}{2kT_s}\right), \tag{10}$$

$$P_N = \frac{r^2 - 1}{r^2 + r + 1}, \tag{11}$$

$$r = \exp\left(\frac{\hbar\omega_N}{kT_s}\right), \tag{12}$$

where ω's are the Larmor angular frequencies with subscript denoting species of nucleus, and T_s equal spin temperature. $\omega_p = 267.513 \times 10^6 B$ and $\omega_N = 19.331 \times 10^6 B$, where B is the magnetic field in Tesla [8].

As SANE has average proton polarization of 68 % , $P_p = 0.68$, T_s should be 0.00616, which is 6.16 mK. In that case, $P_N = 0.0797$. If we assume 10 % error for EST method [3], $\frac{P_N}{P_p} = 0.12 \pm 0.01$.

References

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