Preliminary Design of SoLID Ecal Shashlyk Module

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$$(1\,psi = 4.448\,N/0.000645\,m^2 = 6894\,N/m^2)$$

Material Properties:

Basic properties of materials used in the calculation

density of lead: $\rho_L = 0.011340\,kg/cm^3$
Young's modulus of lead: $E_L = 2560\,ksi(kilo-psi) = 1.765\times10^{10}\,N/m^2$
density of scintillator: $\rho_S = 0.001220\,kg/cm^3$
tensile yield strength of brass rods: $Y_{T,brass} = 18000\,psi = 1.24\times10^8\,N/m^2$
shear yield strength of brass: $Y_{S,brass} = 9000\,psi = 6.2\times10^7\,N/m^2$
Young's modulus of brass: $E_{T,brass} = 15000\,ksi = 1.0\times10^{10}\,N/m^2$
Strength of stainless steel is about factor 2 higher than brass

Properties of pure lead from http://www.matbase.com/ or http://www.azom.com/article.aspx?ArticleID=2863#13

maximum tensile load of lead: 14-32 MPa
tensile yield strength of lead: 5-19 MPa

tensile yield strength of aluminum alloy 7075: $Y_{T,Al7075} = 390-450\,MPa$
shear yield strength of aluminum alloy 7075: 300 MPa
shear modulus of aluminum alloy 7075: $E_{S,Al7075} = 27500\,MPa$
Young's modulus of aluminum alloy 7075 $E_{Al7075} = 70000\,MPa$

Material properties used but need to be checked:

Young's modulus of scintillator: $E_S = 460\,ksi(kilo-psi) = 3.17\times10^9\,N/m^2$
yield strength of scintillator: unknown
coefficient of static friction: $\mu = 0.1$

Properties of module:

Geometry of module:

side of hexagon: $b = 62.5\,mm$
cross-sectional area of module: $A_m = 100\,cm^2$
distance between two 2.5-mm rods in opposite corners: $a_2 = 2\times(56.25\,mm)$
distance between two 2.5-mm rods on one side of the hexagon: $a_1 = 2\times(28.14\,mm)$
number of layers of lead: $n_L = 194$
number of layers of scintillator: $n_S = 194$
number of layers of paper: $n_P = 2\times194 = 388$
thickness of lead layers: $t_L = 0.5\,mm$

thickness of scintillator layers: $t_S = 1.5\,mm$
thickness of paper: $t_P = 0.12\,mm$
diameter of rods: $D_{rod} = 2.5\,mm$
cross-sectional area of rods: $A_{rod} = \pi (D_{rod}/2)^2 = 0.049\,cm^2$

Calculated properties of module:
weight of module: $W = (n_L t_L \rho_L + n_S t_S \rho_S) A_m g = 142.7\,N$ (counting only lead and scintillator)
length of module: $L = n_L t_L + n_S t_S + n_P t_P = 481\,mm$

Module assembly design

First, all layers need to be sandwiched between two “endplates”. At least one endplate need to have a large hole in the center for all WLS fibers to come out (I will call this “back endplate”). Both endplates need to have the six big holes at the corners for the 2.5-mm diameter rods. To fix the module layers and the end plates together, I am proposing the use of both the rods and the side “sheets”. The endplates will be used to compress the layers during the initial assembly (first load). Then, a higher compression (second load) is applied by adding compression on the endplate, and side sheets are attached to the endplates using 3 screws per sheet per side. This second load is then reduced from the endplates, and elongation of the side sheets should produce enough compression such that once the module is placed horizontally, the static friction between layers is large enough to balance the weight of the module. [A moderate compression can still be applied after assembly if the modules are transported long distance or for storage, to reduce the long-term stress on the components.]

At this moment, the compression should be provided mostly by the side sheets. Provided we design the side sheets properly (holes for fixing to the endplates should NOT be pre-drilled, but rather drilled when being attached), the compression will not depend on the position of any turnable component. Thus we expect this compression to be stable during the rest of the construction, installation, and experimental running. The module is then attached (cantilevered) by the top three rods to the back supporting plate. The cantilevering force on these three rods are comparable to the final load for the top single rod only, and less for the other 5 rods.

We calculate the resulting stress on all screws, nuts, and the six rods in this design throughout the proposed assembling and installation stages. As mentioned above, the weight of the module is balanced by static friction such that no shear stress is placed on the side sheets or the rods, and we will also calculate the consequence of lose compression to demonstrate this point. (as you will see, the main problem is in the resulting shear deformation of the side sheets, which will cause top module to lean on the bottom module.)

The proposed module structure is shown in a figure that I will send along with this document.

Calculation of the first load

During the first stage of assembling the module, all layers are stacked together first. The endplate with the big hole should be at the bottom, and the endplate without hole should be at the top for applying compression. A few stainless steel pins can be inserted into the WLS fiber holes to keep all layers aligned. Six 2.5-mm diameter rods will be inserted into the six big holes and nuts attached to the end. A first load of

$$F_{1st\,load}=500\,kg=4900\,N$$

can be applied directly on the top endplate. The compression applied on the lead and the scintillator layers is:

$$\frac{F_{1st\,load}}{A_m}=\frac{4900\,N}{100\,cm^2}=4.9\times10^5\,N/m^2$$

This is 1/10 of the tensile yield strength of lead so should be okay. *We need to know the strength of the scintillator to make sure it's okay there too.*

Note that we could rely on turning the nuts to apply this compression but it will depend on the material of the rod+nut (Al7075 is okay, brass is not): If we turn the nuts to apply the 4900N of force, we need to tighten the nuts along the direction of the rod by

$$dL_{nut,1st\,load}=\frac{L}{E_{brassrod}}\frac{F_{first\,load}}{6\,A_{rod}}=\frac{481\,mm}{1.0\times10^{10}\,N/m^2}\frac{4900\,N}{6\times0.049\,cm^2}=8.02\,mm$$

The resulting shear stress on the nuts is:

$$\left(\frac{F}{A}\right)_{nut,first\,load}=\frac{4900\,N}{6\times D_{nut}(3/32\,inch)}=\frac{4900\,N}{6\times2.5\,mm\times2.4\,mm}=1.36\times10^8\,N/m^2$$

where (3/32inch) is the thread depth. This stress is higher than the yield strength of brass and will damage the thread. If we use Al7075 nuts then there will be no problem since this is factor of ~2 less than the Al7075's shear strength, but compressing by applying the load on the endplate is still a safer bet.

Under this first load, the layers will be compressed by $\frac{\Delta L_S}{L_S}=\frac{1}{E_S}\frac{F_{first\,load}}{A_m}$ and $\frac{\Delta L_L}{L_L}=\frac{1}{E_L}\frac{F_{first\,load}}{A_m}$ for the scintillator and the lead, respectively, giving:

$$\Delta L_S=(n_S t_S)\frac{1}{E_S}\times(4.9\times10^5\,N/m^2)=\frac{194\times1.5\,mm}{3.17\times10^9\,N/m^2}\times(4.9\times10^5\,N/m^2)=0.044\,mm$$

and

$$\Delta L_L=(n_L t_L)\frac{1}{E_L}\times(4.9\times10^5\,N/m^2)=\frac{194\times0.5\,mm}{1.765\times10^{10}\,N/m^2}\times(4.9\times10^5\,N/m^2)=0.0027\,mm$$

Calculation of the second and the final load

I assume the first load will flatten all layers. Then, we apply a second load on the endplates in order to attach the side sheets. The purpose is such that, once the second load is released from the endplates, the side sheets will elongate. Eventually the elongation of the side sheets and the compression of the layers will balance each other. The resulting compression should reach a minimum of

$$F_{final\,load,\,min}=\frac{W}{\mu}=\frac{142.7\,N}{0.1}=1427\,N$$

which can be provided by an elongation of

$$\Delta L_{sheets}=\frac{L}{E_{sheet}}\frac{F_{final\,load}}{A_{sheet}}$$

ALICE used steel plate, but I am worried about magnetism. So I will study brass and aluminum here (but you will see Al is better). If using 1mm-thick Al7075 sheets:

$$\Delta L_{AL7075\,sheets}=\frac{481\,mm}{7\times10^{10}\,N/m^2}\frac{1427\,N}{6\times62.5\,mm\times1\,mm}=0.026\,mm$$

if using 1mm-thick brass:

$$\Delta L_{brass\,sheets}=\frac{481\,mm}{1.0\times10^{10}\,N/m^2}\frac{1427\,N}{6\times62.5\,mm\times1\,mm}=0.18\,mm$$

We also need to calculate the compression of the layers for this final load. As can be seen from the previous section, deformation of the lead sheet is much less than the scintillator sheet. So to the first order we can assume the deformation is only in the scintillators:

$$\Delta L_{S,\,final\,load}=\frac{194\times1.5\,mm}{3.17\times10^{9}\,N/m^2}\frac{1427\,N}{100\,cm^2}=0.013\,mm$$

This means if we want to reach a final compression of 1427N, we need to apply a second compression load so the total layer deformation under this second compression, for the aluminum sheet case:

$$0.013\,mm+0.026\,mm=0.040\,mm$$

As one can see, if we want to avoid further compress the layers from the first load, we should use aluminum side sheets than brass. In this case, the compression from the first load (0.044mm on scintillators) is already good enough for attaching the side sheets with a safety factor of 1.1. On the other hand if we prefer to use a safety factor 2, then we need to apply a second compression load of

$$N_{second\,load}=\frac{2\times0.040\,mm}{0.044\,mm}\times4900\,N=8910\,N$$

which is probably still very okay on the scintillator and the lead layers. The resulting final load is

$$F_{final\,load}=2\times F_{final\,load,\,min}=2854\,N$$

which is about 476N per side sheet.

Calculation of the stress on screws for the final load

The side sheets are attached to the endplates, using three screws per side per sheet. Total would be 18 screws on one endplate. I start from diameter 3-mm screws. The shear stress on the screw in the horizontal direction is

$$\frac{F_{final\ load}}{18\times\pi\left(\frac{D_{screw}}{2}\right)^2}=\frac{2854\,N}{18\times\pi(3\,mm/2)^2}=2.22\times10^7\,N/m^2$$

This is about factor 2 smaller than the shear yield strength of brass but factor >10 less than that of Al7075. So I propose we use Al screws and Al side sheets.

Calculation of side sheets if supporting module weight (avoid!)

If for any reason the compression is lost, and the full weight is supported by the bottom two aluminum side sheets, the normal force would be

$$N=W/\sqrt{3}=82.4\,N$$

Now we need to consider the shear stress on the screws along the direction of the thread. Assuming a thread depth of (3/32in=2.4mm), the stress is

$$\frac{W/\sqrt{3}/6}{\pi D_{screw}(3/32\,inch)}=\frac{82.4\,N/6}{\pi(3\,mm)(2.4\,mm)}=6.1\times10^5\,N/m^2$$

which is much lower than the shear yield strength of brass or stainless steel or aluminum so should not be a problem.

In the case that the weight is supported only by the bottom aluminum sheets, we also need to consider shear deformation of the sheet. At the center of the module, the sagging would be:

$$\Delta L_{sheets,shear,weight-bearing}=\frac{\frac{L}{2}}{E}\frac{82.4\,N}{(62.5\,mm)\times(1\,mm)}=\frac{\frac{481\,mm/2}{27500\,MPa}\times82.4\,N}{62.5\,mm^3}=240\,mm\times(4.8\times10^{-2})=11.52\,mm$$

This apparently will be a problem, since there is no way we can leave 12mm of room between each modules and the top module will lean on the bottom module in the case of lost module compression, and will collapse the whole assembly. Therefore, although it may be okay to not compress the module to use friction for prototyping, we definitely should use the 2854N compression for the full assembly case for SoLID.

Calculation of rod stress for cantilevering

If we cantilever the module by the top three rods to a back supporting plate, the cantilevering force would be the largest for the top single rod (F2) and less for the middle two rods (F1). They can be calculated as:

(zero torque) $$\frac{W\times L}{2}=2F_1(a_1/2)+F_2(a_2/2)$$

$$\frac{F_2}{a_2}=\frac{F_1}{a_1}$$

(uniform elongation along cross section of module)

giving

$$F_2 = \frac{WL}{a_2 + 2a_1 \frac{a_1}{a_2}} = \frac{142.7\,N \times 481\,mm}{2 \times 56.25\,mm + \frac{2 \times 28.14\,mm \times 28.14\,mm}{56.25\,mm}} = 488\,N$$

$$F_1 = \frac{28.14\,mm}{56.25\,mm} \times 488\,N = 244\,N$$

This will cause an elongation of the top rod:

$$\Delta L_{toprod\,,\,cantilevering} = \frac{L}{E} \frac{488\,N}{\pi(2.5\,mm/2)^2} = \frac{481\,mm}{7 \times 10^{10}\,N/m^2} \frac{488\,N}{4.9\,mm^2} = 0.68\,mm$$

and an elongation of the middle top two rods:

$$\Delta L_{middle-top\,two\,rod\,,\,cantilevering} = \frac{L}{E} \frac{244\,N}{\pi(2.5\,mm/2)^2} = 0.34\,mm$$

The shear stress on the top nut, using a thread (nut) diameter of 2.5mm and assuming a thread depth of (3/32 in=2.4mm), is

$$\frac{488\,N}{\pi(2.5\,mm)(2.4\,mm)} = 2.59 \times 10^7\,N/m^2$$

which is a factor of 10 less than the shear yield strength of Al7075, but only factor 2 less than that of brass. So again we should use Al here.

Also, unlike the rod-only design, cantilevering is provided by the rod only, and compression is provided by the side sheet only. So there will be no additional load on the side sheet due to cantilevering and vice versa. We do need to make sure that the nut for fixing the rods to the endplates are not tightened too much, though, otherwise it will impose additional load on the top rod in addition to cantilevering.

Summary of Assembling Process and Loads:

Step 1: stack all layers between two 6mm-thick Al endplates, insert alignment pins, insert six Al rods and fix to endplates using Al nuts. The rods should be longer than the module and have longer thread on one end for future attachment to the back supporting structure. At this stage, the bottom endplate should have the WLS fiber routing hole, while the top endplate should have no hole.
Step 2: Apply a first load of 4900N (500kg) uniformly on the endplates, monitor total thickness and wait to stablize (no change in total thickness over 24 hours);
Step 3: increase load to 1000kg (second load) by applying additional compression on the endplate, monitor total thickness and wait to stablize (no change in total thickness over 24 hours);
Step 4: wrap module side with black tape;
Step 5: attach six 1mm-thick aluminum side sheets, use Al screws to attach to Al endplates, 3 screws per hexagon side per sheet. There should be no pre-drilled holes on the side sheets so the screws can fix the sheets to the compressed layers. Re-inforcement may be considered to make sure the hole shape does not deform over time.

Step 6: remove load on endplate. The final compression load should be balanced at 2854N along the module length;
Step 7: Tighten the 12 nuts on the six rods to snug, then turn slightly to tighten further;
Step 8: insert WLS fibers, mount PMT (for prototyping);
Step 9: place the module in the horizontal direction. Insert the end of the top 3 rods into the back supporting structure. Attach additional nuts to the back of the support (opposite side from the module), turn to snug and tighten further. The top nut should be tighten by 0.68mm along the direction of rod, and the middle-top two nuts should be tightened by 0.34mm. Tighten further is okay but should not exceed factor of 2 of these values.
Step 10: Let module cantilevering on top 3 rods, the tension from the rods/nuts should be strong enough to support the module in this position. Can monitor elongation of rods when cantilevering. If the rods are tightened properly, the rod length should not lengthen by more than 0.68mm (for top) or 0.34mm (for middle-top two rods).